Dr. Katiraie MA160 Test II Form B (Sections 1.6—2.4 and Sections 0.1---1.5) Summer 08

Name Solution Total Possible Points = 100 Points

Show all your work.

A) Find the derivative of each function below and write your answers with no negative exponents. **(16 pts)**

a) $f(x)=\frac{x^2}{6}-8x+4$ $\boxed{f'(x)=\frac{1}{3}x-8}$	b) $f(x)=x\sqrt[3]{x}$ $f(x)=x\,x^{\frac{1}{3}}=x^{\frac{4}{3}}$ $\boxed{f'(x)=\frac{4}{3}x^{\frac{1}{3}}}$	c) $f(x)=\frac{5}{3x^2}$ $f(x)=\frac{5}{3}x^{-2}$ $f'(x)=\frac{-10}{3}x^{-3}$ $\boxed{f'(x)=\frac{-10}{3x^3}}$	d) $f(x)=\frac{x+2}{7}$ $f(x)=\frac{1}{7}x+\frac{2}{7}$ $\boxed{f'(x)=\frac{1}{7}}$
e) $f(x)=-\pi^2+8x$ $\boxed{f'(x)=8}$	f) $f(x)=-8+4x$ $\boxed{f'(x)=4}$	g) $f(x)=\sqrt[4]{x}+7$ $f'(x)=\frac{1}{4}x^{-\frac{3}{4}}$ $\boxed{f'(x)=\frac{1}{4x^{\frac{3}{4}}}}$	h) $f(x)=\frac{x^3}{\sqrt{x}}$ $f(x)=x^{3-\frac{1}{2}}=x^{\frac{5}{2}}$ $\boxed{f'(x)=\frac{5}{2}x^{\frac{3}{2}}}$

B) Use the definition of the derivative to find $f'(x)$ for the following function.

$f(x)=x^2-6x+3$ **(4 pts)**

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{(x+h)^2-6(x+h)+3-(x^2-6x+3)}{h}$$

$$\lim_{h\to 0}\frac{\cancel{x^2}+2xh+h^2-\cancel{6x}-6h+\cancel{3}-\cancel{x^2}+\cancel{6x}-\cancel{3}}{h}=\lim_{h\to 0}\frac{\cancel{h}(2x+h-6)}{\cancel{h}}=\boxed{2x-6}$$

C) Find the point on the curve $f(x)=\sqrt{x}$ where the tangent line is parallel to the line $f(x)=\frac{x}{6}$ **(5 pts)**

$$f'(x)=\frac{1}{2}x^{-\frac{1}{2}}=\frac{1}{2\sqrt{x}} \qquad f'(x)=\frac{1}{6}$$

$$\frac{1}{2\sqrt{x}}=\frac{1}{6}\Rightarrow 2\sqrt{x}=6 \quad x=9$$

$\boxed{(9,3)}$

Find the derivative.

(2pts) 1) $y = (5x - 3)^5$

$$y' = 5(5x-3)^4(5) = \boxed{25(5x-3)^4}$$

(2pts) 2) $y = \sqrt{-2x + 2} = (-2x+2)^{1/2}$

$$y' = \frac{1}{2}(-2x+2)^{-\frac{1}{2}}(-2) \Rightarrow \boxed{y' = \frac{-1}{(-2x+2)^{1/2}}}$$

(2pts) 3) $y = (3x^2 + 5x + 1)^{3/2}$

$$\boxed{y' = \frac{3}{2}(3x^2+5x+1)^{\frac{1}{2}}(6x+5)}$$

(2pts) 4) $f(x) = \dfrac{-5}{(2x - 3)^4} = -5(2x-3)^{-4}$

$$y' = 20(2x-3)^{-5}(2) = \boxed{\frac{40}{(2x-3)^5}}$$

Solve the problem.

(4pts) 5) If the price of a product is given by $P(x) = \dfrac{1024}{x} + 800$, where x represents the demand for the product, find the rate of change of price when the demand is 16. (i.e. x = 16

$$P'(x) = -1024x^{-2}$$

$$P'(16) = -1024(16)^{-2} = \boxed{-4}$$

(4pts) 6) A ball is thrown vertically upward from the ground at a velocity of 122 feet per second. Its distance from the ground after t seconds is given by $s(t) = -16t^2 + 122t$. How fast is the ball moving 7 seconds after being thrown?

$$s'(t) = -32t + 122$$

$$s'(7) = -32(7) + 122 = \boxed{-102 \text{ ft/sec}}$$

(4pts) 7) The total cost to produce x handcrafted wagons is $C(x) = 110 + 6x - x^2 + 4x^3$. Find the marginal cost when x = 3.

$$C'(x) = 6 - 2x + 12x^2$$

$$C'(3) = -2(3) + 12(3)^2 + 6 = 108$$

(4pts) 8) Exposure to ionizing radiation is known to increase the incidence of cancer. One thousand laboratory rats are exposed to identical doses of ionizing radiation, and the incidence of cancer is recorded during subsequent days. The researchers find that the total number of rats that have developed cancer t months after the initial exposure is modeled by $N(t) = 1.19\,t^{2.1}$ for $0 \le t \le 10$ months. Find the rate of growth of the number of cancer cases at the 8th month.

$$N'(t) = (1.19)(2.1)\,t^{1.1}$$

$$N'(8) = (1.19)(2.1)(8)^{1.1} = 24.61 \approx 25 \frac{\text{cases}}{\text{Month}}$$

(4pts) 9) Suppose the demand for a certain item is given by $D(p) = -4p^2 + 8p + 4$, where p represents the price of the item in dollars. Find D'(12) and interpret your result.

$$D'(p) = -8p + 8$$

$$D'(12) = -8(12) + 8 = -88 \frac{\text{units}}{\text{dollar}}$$

At the price of $12, for every $1 increase in price, the demand decreases by 88 units.

10) The revenue generated by the sale of x bicycles is given by $R(x) = 80.00x - x^2/200$ dollars. Find the marginal revenue when x = 900 units, and interpret your result.

(4pts) $$R'(x) = 80 - \frac{2}{200}x = 80 - \frac{1}{100}x$$

$$R'(900) = 80 - \frac{1}{100}(900) = 80 - 9 = 71 \frac{\$}{\text{Bicycle}}$$

At the production level of 900 bicycles, for every 1 extra bicycle made the revenue increases by $71.