Name: Solutions

Dr. Katiraie **MA160** **Test II** (Sections 2.3---3.6 plus 25 points from Test I Concepts) Spring 2013
Total Possible Points = 100 Points + 10 Extra Credit for WebAssign Homework (IF > 95% COMPLETED)

Show all your work.

1. Given $f(x) = x^2 - 8x + 2$, find the following and simplify your answer (8 points)

a) $f(x+h) = (x+h)^2 - 8(x+h) + 2$

$= x^2 + 2xh + h^2 - 8x - 8h + 2$

b) $f(x+h) - f(x) = (\cancel{x^2} + 2xh + h^2 \cancel{-8x} - 8h \cancel{+2}) - (\cancel{x^2} \cancel{-8x} \cancel{+2})$

$= 2xh + h^2 - 8h$

c) $\dfrac{f(x+h)-f(x)}{h} = \dfrac{h(2x+h-8)}{h} = 2x + h - 8$

d) $\lim_{h \to 0} \dfrac{f(x+h)-f(x)}{h} = \lim_{h \to 0} 2x + h - 8 = \boxed{2x - 8}$

2. Find the following limit: (6 Points)

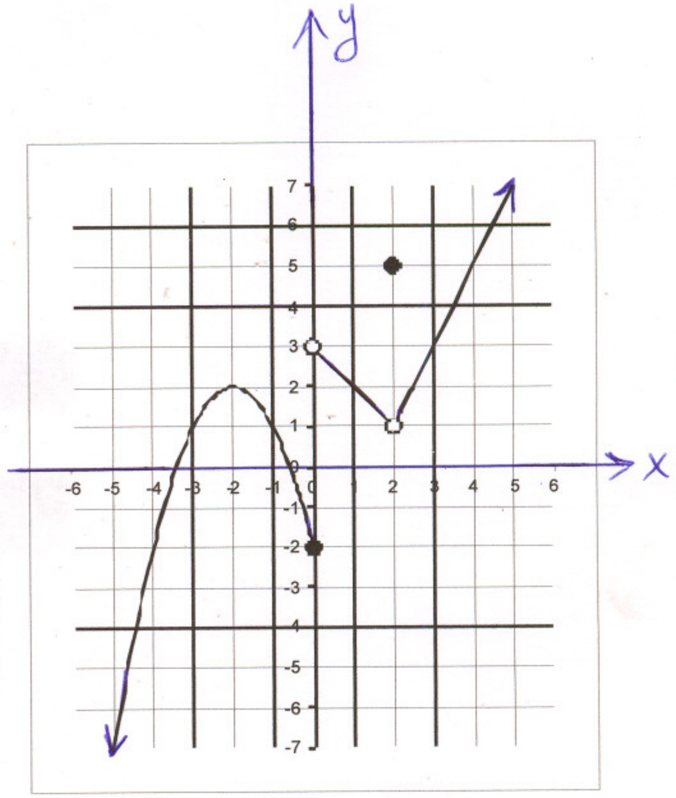
a) $\lim_{t \to 5} \dfrac{t^2-25}{t-5} = \lim_{t \to 5} \dfrac{(t+5)\cancel{(t-5)}}{\cancel{(t-5)}} = \lim_{t \to 5} t+5$

$= \lim_{t \to 5} t+5 = 5+5 = \boxed{10}$

b) $\lim_{x \to -5} \dfrac{\frac{1}{5}+\frac{1}{x}}{5+x} = \lim_{x \to -5} \dfrac{\frac{x+5}{5x}}{5+x} = \lim_{x \to -5} \dfrac{x+5}{5x} \cdot \dfrac{5+x}{1}$

$= \lim_{x \to -5} \dfrac{x+5}{5x} \cdot \dfrac{1}{5+x} = \dfrac{1}{5(-5)} = \boxed{\dfrac{-1}{25}}$

3. Let f(x) be the graph to the right. Use this function to answer each of the questions below. (5 points)



(a) $f(0) = \boxed{-2}$ $\lim\limits_{x\to 0^-} f(x) = \boxed{-2}$

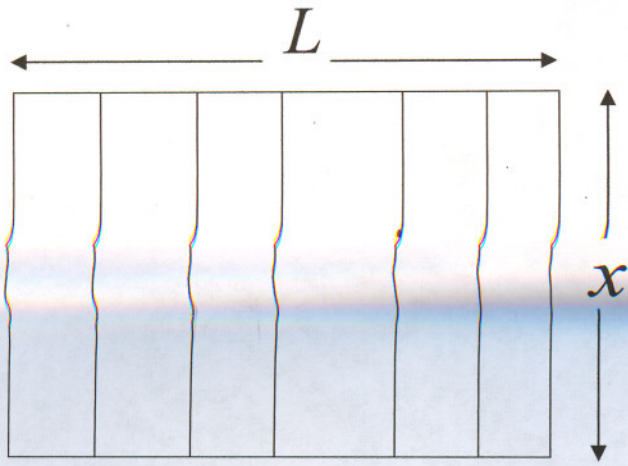
$\lim\limits_{x\to 0^+} f(x) = \boxed{3}$ $\lim\limits_{x\to 0} f(x) = \boxed{\text{DNE}}$

(b) Is f(x) continuous or discontinuous at x = 0? Why or why not?

Not Continuous Because at $x = 0$ $\lim\limits_{x\to 0} f(x) \neq f(0)$

4. A farmer wants to enclose a rectangular area and then divide it into ~~seven~~ Six smaller rectangular regions by putting up fencing parallel to one side of the rectangle. The total amount of fencing available is 2100 feet. An illustration is shown below.

A) Express the total area of the enclosed region as a function of x and simplify your answer. (4 points)



$$2L + 7X = 2100$$

$$L = \frac{2100 - 7X}{2}$$

$$A(x) = XL$$

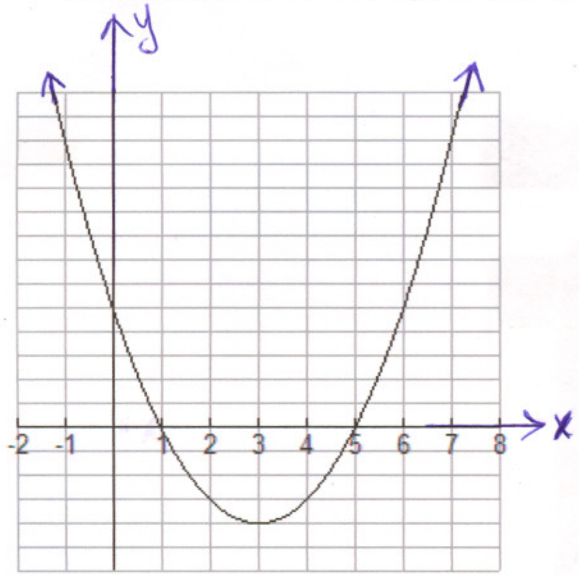
$$= X\left(\frac{2100 - 7X}{2}\right) = \boxed{1050X - 3.5X^2 \ \text{ft}^2}$$

B) State the Domain of the above function. (Hint: what are the possible values of x) (2 points)

$$2100 - 7X = 0 \Rightarrow 7X = 2100 \Rightarrow X = 300$$

$0 < X < 300$ feet

5. The graph of the function $f(x)$ is shown. For each value x = a in the chart below, indicate whether $f(a)$ is positive, negative, or zero, and whether $f'(a)$ is positive, negative, or zero. (5 points)



a	$f(a)$	$f'(a)$
-1	positive	negative
0	Positive	Negative
1	Zero	Negative
3	Negative	Zero
5	Zero	Positive
6	Positive	Positive

6. The table shows the estimated percentage P of the population of Brazil that are mobile-phone subscribers. (End of year estimates are given.)

Year	1997	1999	2001	2003	2005	2007
P	2.7	8.8	16.3	25.6	46.3	63.1

Find the average rate of percentage P of the population of Brazil that are mobile-phone subscribers from 1997 to 2005. What is the unit? (5 points)

$(1997, 2.7)$
$(2005, 46.3)$ $\Longrightarrow m = \dfrac{46.3 - 2.7}{2005 - 1997} = \boxed{5.45 \ \dfrac{\text{Percent}}{\text{year}}}$

The average rate of percentage of Population of Brazil that were mobile-phone subscribers from 1997 to 2005 was $\boxed{5.45 \ \dfrac{\text{Percent}}{\text{year}}}$

7. If f is a differentiable function of x and $g(x) = \sqrt{x}\, f(x)$ $= x^{1/2} f(x)$

(a) Find an expression for the derivative of $g(x)$ in terms of $f(x)$ *and* $f'(x)$. (4 points)

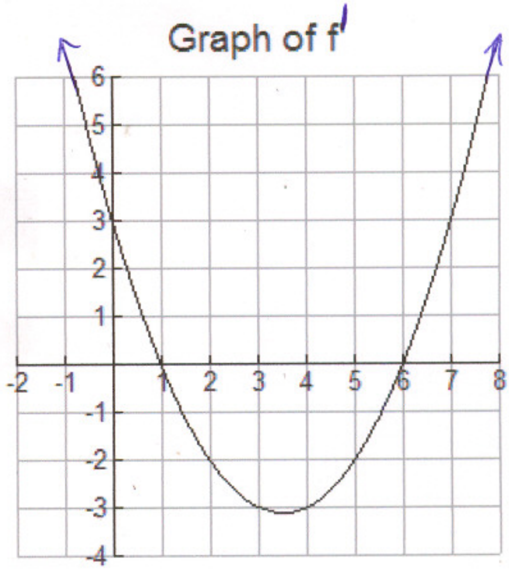
$$\boxed{f'(x) = \frac{1}{2}x^{-\frac{1}{2}} f(x) + x^{\frac{1}{2}} f'(x)}$$

(b) If it is known that $f(8) = 12$ *and* $f'(8) = 5$, find $g'(8)$. (4 points)

$$f'(8) = \frac{1}{2}(8)^{-\frac{1}{2}} f(8) + (8)^{1/2} f'(8)$$
$$= \frac{1}{2}(8)^{-\frac{1}{2}}(12) + 8^{1/2}(5) \boxed{\cong 16.26}$$

8. Using the Graph of the Derivative

The graph shown is the graph of f', the ***derivative*** of a function f. Note that the graph of f is not shown.



If the function f is defined for all x, use this graph to answer the following questions.

(10 points)

a. On what interval(s) is the function f increasing?

$(-\infty, 1) \cup (6, \infty)$

b. On what interval(s) is the function f decreasing?

$(1, 6)$

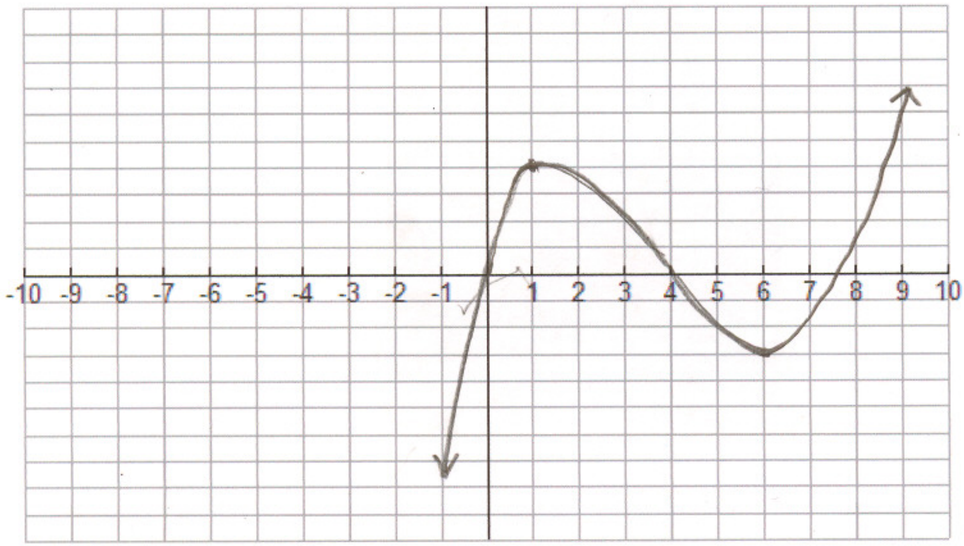
c. At what value(s) of x, if any, does f have a local maximum?

at $x = 1$

d. At what value(s) of x, if any, does f have a local minimum?

at $x = 6$

Suppose it is also known that f goes through the point (0,0). Based on all of the above information, sketch a possible graph of the function f.



9. Find the derivative of each of the following functions (18 points)

(a) $g(x)=\frac{-6}{x^4} = -6x^{-4}$

$$g'(x) = 24x^{-5} = \boxed{\frac{24}{x^5}}$$

(b) $f(x)=7e^x+\frac{x^2}{3} = 7e^x + \frac{1}{3}x^2$

$$\boxed{f'(x) = 7e^x + \frac{2}{3}x}$$

(c) $f(x)=x^2e^x$

$$\boxed{f'(x) = 2xe^x + x^2e^x}$$

(d) $f(x)=\frac{x^3-5e^x}{2x-3}$

$$\boxed{f'(x) = \frac{(3x^2-5e^x)(2x-3)-2(x^3-5e^x)}{(2x-3)^2}}$$

(e) $y=\ln(5x+2)$

$$y' = \frac{1}{5x+2}\cdot 5$$

$$= \boxed{\frac{5}{5x+2}}$$

(f) $y=\ln(x^3+2)$

$$y' = \frac{1}{x^3+2}\cdot(3x^2)$$

$$= \boxed{\frac{3x^2}{x^3+2}}$$

10. If $f(x)=6\sqrt{x}-\frac{4}{\sqrt{x}}$ find $f(x) = 6x^{1/2} - 4x^{-\frac{1}{2}}$

(a) $f'(x) = \boxed{\frac{6}{2}x^{-\frac{1}{2}} + 2x^{-\frac{3}{2}}}$ (3 points)

(b) $f'(4) = 3(4)^{-\frac{1}{2}} + 2(4)^{-\frac{3}{2}} = \boxed{1.75}$ (3 points)

11. The equation of motion of a moving object is $s(t) = 2t^2 + t$, where s(t) is position of the object measured in feet and t is the time in seconds. Find each of the following and use appropriate units in your answers.

(a) The velocity after 4 seconds $\Rightarrow v(t) = 4t + 1$ (2 points)

$$v(4) = 4(4)+1 = \boxed{17 \frac{ft}{sec}}$$

(b) The acceleration after 4 seconds (2 points)

$$a(t) = \boxed{4 \ ft/sec^2}$$

12. The average price for a major league baseball game x years after 1990 can be modeled by $p(x) = 9.41 - 0.19x + 0.09x^2$.

(a) Use the model to find the instantaneous rate of change of the average ticket price in 2005. (3 points)

$t = 2005 - 1990 = 15$

$$p'(x) = -0.19 + 0.18x \Rightarrow p'(15) = \frac{\$2.51}{year}$$

(b) In a sentence, explain the meaning of your answer to part (a). Use appropriate units. (2 points)

The instantaneous rate of change of average ticket price in 2005 was $\frac{\$2.51}{year}$.

13. Find the equation of the tangent line to the function $f(x) = \frac{x^2+1}{x-1}$ when x = 3 on the curve. (5 points)

$$f'(x) = \frac{2x(x-1) - 1(x^2+1)}{(x-1)^2} = \frac{2x^2 - 2x - x^2 - 1}{(x-1)^2} = \frac{x^2 - 2x - 1}{(x-1)^2}$$

at $x=3$ $y = \frac{10}{2} = 5$, $m = \frac{9-6-1}{4} = \frac{2}{4} = \frac{1}{2}$

$y - 5 = \frac{1}{2}(x-3)$

$y = \frac{1}{2}x - \frac{3}{2} + 5 \Rightarrow \boxed{y = \frac{1}{2}x + \frac{7}{2}}$

14. The radioactive substance Bismuth-210 has a half-life of 5.0 days.

(a) A sample originally has a mass of 1000 mg. Find a formula for the mass remaining after t days (Hint: find an exponential function of the type $A(t) = Ce^{kt}$ that models the above situation)

(4 points)

$A(t) = ce^{kt}$

$500 = 1000e^{k(5)} \Rightarrow \frac{500}{1000} = e^{5k} \Rightarrow \ln(\frac{1}{2}) = \cancel{\ln e}^{5k}$

$\ln(\frac{1}{2}) = 5k \Rightarrow \boxed{k = \frac{\ln(1/2)}{5}}$

$\boxed{A(t) = 1000 e^{\frac{\ln(\frac{1}{2})}{5}t}}$ (mg)

(b) Find the mass remaining after 10 days.

(2 points)

$A(10) = 1000e^{\frac{\ln(1/2)}{5}(10)} = \boxed{250 \text{ mg}}$

(c) When is the mass reduced to 100 mg?

(3 points)

$100 = 1000 e^{\frac{\ln(1/2)}{5}t}$; Solve for t

$\frac{100}{1000} = e^{\frac{\ln(1/2)}{5}t}$

$\ln(\frac{100}{1000}) = \ln e^{\frac{\ln(\frac{1}{2})}{5}t}$

$\ln(\frac{100}{1000}) = \frac{\ln(1/2)}{5}t \Rightarrow t = \frac{\ln(\frac{100}{1000})}{\left(\frac{\ln(1/2)}{5}\right)} = 16.6$ Days