MA 160 Test #3 – Form B - Summer 2008

(100 points) Name Solutions

Show all of your work on the test paper. All of the problems must be solved symbolically using Calculus. You may use your calculator to confirm your answers, but full credit is not given unless the answer follows from the symbolic work shown.

1. Differentiate each function and simplify your answer.

(a) (10 points) $y = \frac{3x+5}{2x-9}$

$$y' = \frac{(2x-9)3-(3x+5)2}{(2x-9)^2} = \frac{6x-27-6x-10}{(2x-9)^2} = \frac{-37}{(2x-9)^2}$$

(b) (12 points) $f(x) = x^4(5x-3)^7$

$$f'(x) = x^4 7(5x-3)^6 5 + (5x-3)^7 4x^3$$
$$= 35x^4(5x-3)^6 + 4x^3(5x-3)^7$$
$$= x^3(5x-3)^6(35x+4(5x-3)) = x^3(5x-3)^6(55x-12)$$

2. (16 points) Let $f(x) = x^2 + \frac{54}{x}, \ x > 0$

(a) Find $f'(x)$

$$f(x) = x^2 + \frac{54}{x} = x^2 + 54x^{-1}$$

$$f'(x) = 2x - 54x^{-2} = 2x - \frac{54}{x^2}$$

(b) Find any critical numbers of $f(x)$ on the given domain.

$$2x - \frac{54}{x^2} = 0$$

$$2x = \frac{54}{x^2} \ so \ 2x^3 = 54, \ x^3 = 27 \ and \ x = \sqrt[3]{27} = 3$$

(c) Use the First or Second Derivative Test to determine if the function $f(x)$ has a relative maximum or a relative minimum at any critical number you found in part (b). ***State your answer clearly, that is, does the function have a relative maximum or does it have a relative minimum at any critical number you found in part (b)?***

$$f''(x) = 2 + 108x^{-3} = 2 + \frac{108}{x^3} > 0 \ if \ x > 0$$

Since the second derivative is >0, f is concave up and has a relative minimum at x = 3.

3. (13 points) Let $f(x) = \ln(e^{4x} + 2)$. Find the equation of the tangent line to this function when x = 0 on the graph. Write your answer using exact values, not approximations.

$$f(x) = \ln(e^{4x} + 2)$$

$$f'(x) = \frac{4e^{4x}}{e^{4x} + 2}$$

$$x = 0, \quad y = f(0) = \ln(e^0 + 2) = \ln(1+2) = \ln(3)$$

$$m_{\tan} = f'(0) = \frac{4e^0}{e^0 + 2} = \frac{4}{3}$$

$$y - \ln(3) = \frac{4}{3}(x - 0) \ \ so \ \ y = \frac{4}{3}x + \ln(3)$$

4. (12 points) For what value or values of x does the function $f(x) = x^4 \ln x, \ x > 0$ have a horizontal tangent line? Write your answer using exact values, not approximations.

$$f(x) = x^4 \ln x$$

$$f'(x) = x^4 \frac{1}{x} + \ln x(4x^3) = x^3 + 4x^3 \ln x = x^3(1 + 4\ln x)$$

$$x^3(1 + 4\ln x) = 0$$

$$x^3 = 0 \quad 1 + 4\ln x = 0$$

$$x = 0, \quad 4\ln x = -1 \ \ so \ \ \ln x = -1/4 \ \ and \ \ x = e^{-1/4}$$

The function is not defined for $x = 0$, *so the only answer is* $x = e^{-1/4}$.

5. (13 points) When a company produces and sells x units of a certain product each week, its profit (in dollars) that week is $P(x) = -.01x^2 + 40x - 3000$. The company is increasing production each week so that the production level t weeks from now will be $x = 400 + 30t$.

(a) Find the marginal profit $\frac{dP}{dx}$.

$$P(x) = -.01x^2 + 40x - 3000$$

$$\frac{dP}{dx} = -.02x + 40$$

(b) Find the time rate of change of profit $\frac{dP}{dt}$.

$$\frac{dP}{dt} = \frac{dP}{dx} \cdot \frac{dx}{dt} = (-.02x + 40)(30) = -.6x + 1200$$

(c) How fast (with respect to time) are profits changing when t = 10? ***Write a sentence in everyday language to explain the meaning of your answer. Be sure to use appropriate units.***

If t = 10, x = 400 + 30(10) = 700 so $\frac{dP}{dt} = -.6(700) + 1200 = 780$

After 10 weeks, profits are increasing at a rate of $780 per week.

6. (24 points) Choose 2 of the following 3 problems. You may do all of them for up to 5 points extra credit.

For each problem, be sure to answer the question or questions asked.

Problem #1:
A large closed rectangular storage box with a square base is to be constructed using two different types of wood. The base is made of wood costing $7 per square foot and the top and sides are made of wood costing $2 per square foot. Suppose that the amount available to spend is $600. Find the dimensions of the box of greatest volume that can be constructed.

$$Cost = 7x^2 + 2x^2 + 2(4xh) = 9x^2 + 8xh$$

$$9x^2 + 8xh = 600$$

$$8xh = 600 - 9x^2$$

$$h = \frac{600 - 9x^2}{8x}$$

$$Volume = x^2 h = x^2\left(\frac{600 - 9x^2}{8x}\right)$$

$$V = x\left(\frac{600 - 9x^2}{8}\right) = \frac{1}{8}(600x - 9x^3)$$

$$V' = \frac{1}{8}(600 - 27x^2) = 0$$

$$600 = 27x^2 \quad so \quad x^2 = \frac{600}{27} \quad and \quad x = \sqrt{\frac{600}{27}} \approx 4.7\ ft$$

$$h = \frac{600 - 9(4.7)^2}{8(4.7)} \approx 10.7\ ft.$$

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Problem #2: An appliance manufacturer is marketing a new mini-refrigerator. It determines that in order to sell x of these refrigerators, the price per refrigerator must be $p = 378 - 0.3x$.

(a) Write the revenue function $R(x)$ for the product.

$R(x) = x(378 - 0.3x) = 378x - 0.3x^2$

(b) Suppose the total cost of producing x refrigerators is given by $C(x) = 5000 + 0.6x^2$. Find and simplify the profit function $P(x)$.

$$P(x) = 378x - 0.3x^2 - (5000 + 0.6x^2)$$
$$= 378x - 0.3x^2 - 5000 - 0.6x^2$$
$$= -0.9x^2 + 378x - 5000$$

(c) How many refrigerators must the company produce and sell in order to maximize profit?

$$P'(x) = -1.8x + 378 = 0 \quad so \quad x = \frac{378}{1.8} = 210$$

(d) What price per refrigerator must be charged in order to maximize profit?

$p = 378 - 0.3(210) = \$315$

Problem #3:

A fast food restaurant is establishing its inventory policy for ordering frozen french fries. In the coming year, they expect to sell 2500 pounds of french fries. It costs $4 to place an order and the carrying costs for a year are $2 per pound based on the average amount in storage.

(a) Determine the economic order quantity for the french fries, that is, the order size which minimizes the inventory cost.

x = number of pounds of french fries per order
r = number of orders per year
$xr = 2500$

$$C = 4r + 2\cdot\frac{x}{2} = 4r + x = 4\cdot\frac{2500}{x} + x = \frac{10000}{x} + x$$

$$C' = -\frac{10000}{x^2} + 1 = 0$$

$$1 = \frac{10000}{x^2} \quad so \quad x^2 = 10000 \quad and \quad x = 100 \quad pounds \ per \ order$$

(b) How many times a year should the fast food restaurant order french fries in order to minimize inventory cost?

$$r = \frac{2500}{100} = 25 \quad orders \ per \ year$$