Solution

Total Possible Points = 100

the derivative.

1) $y = \sqrt{-2x+2} = (-2x+2)^{\frac{1}{2}}$

$$y' = \frac{1}{2}(-2x+2)^{-\frac{1}{2}}(-2) = \boxed{\frac{-1}{(-2x+2)^{1/2}}}$$

2) $f(x) = \dfrac{-5}{(2x-3)^4} = -5(2x-3)^{-4}$

$$f'(x) = 20(2x-3)^{-5}(2) = \boxed{\frac{40}{(2x-3)^5}}$$

olve the problem.

ts) 3) The revenue generated by the sale of x bicycles is given by $R(x) = 90.00x - x^2/200$ dollars. Find the marginal revenue when x = 1300 units, and interpret your result.

$$R'(x) = 90 - \frac{1}{100}x$$

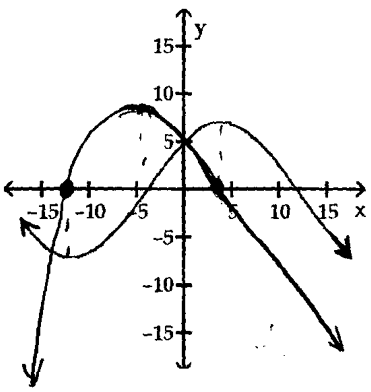
$$R'(1300) = 90 - \frac{1}{100}(1300) = 90 - 13 = 77$$

t production level of 1300 units, for every 1 extra Bicycle that is sold

revenue increases by \$ 77.

Sketch the derivative of the following graph

ts) 4)



The function gives the distances (in feet) traveled in time t (in seconds) by a particle. Find the velocity and acceleration at the given time.

5) $s = 3t^3 + 3t^2 + 4t + 2$

$s'(t) = 9t^2 + 6t + 4$

$s''(t) = 18t + 6$

(2pts) a) Find the velocity at $t = 2$

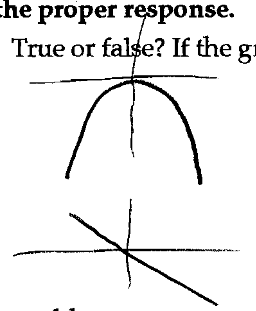
$v(2) = 9(2)^2 + 6(2) + 4 = \boxed{52 \text{ ft/sec}}$

(2pts) b) Find the acceleration at $t = 2$

$a(2) = 18(2) + 6 = \boxed{42 \text{ ft/sec}^2}$

Provide the proper response.

(1pt) 6) True or false? If the graph of a function f is concave down on its entire domain, then f′ is decreasing.



TRUE

Solve the problem.

(10pts) 7) A baseball team is trying to determine what price to charge for tickets. At a price of \$10 per ticket, it averages 45,000 people per game. For every increase of \$1, it loses 5,000 people. What price per ticket should be charged in order to maximize revenue?

$(45000, 10)$

$(40000, 11)$

$m = \dfrac{11-10}{40000-45000} = \dfrac{-1}{5000} = -0.0002$

$y - 10 = -0.0002(x - 45000)$

$y = -0.0002x + 19 \implies \boxed{P = -0.0002x + 19}$

$R(x) = xP = x(-0.0002x + 19) = -0.0002x^2 + 19x$

$R' = -0.0004x + 19 = 0$

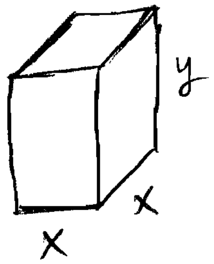
$0.0004x = 19$

$x = 47500$ tickets $\implies P(47500) = -0.0002(47500) + 19$

$= \boxed{\$9.50}$

8) A company is constructing an open-top square-based, rectangular metal tank that will have a volume of 74 ft^3. What dimensions yield the minimum surface area? Round to the nearest tenth, if necessary.

(10 Pts)



$$x^2 y = 74 \Rightarrow y = \frac{74}{x^2}$$

$$SA = x^2 + 4xy = x^2 + 4x\frac{74}{x^2} = x^2 + 296x^{-1}$$

$$(SA)' = 2x - 296x^{-2} = 0$$

$$2x = \frac{296}{x^2} \Rightarrow 2x^3 = 296$$

$$x^3 = 148 \Rightarrow x = 5.3 \text{ feet}$$

$$y = \frac{74}{5.3^2} = 2.6 \text{ feet}$$

Dimensions are 5.3 feet by 5.3 feet by 2.6 feet

The table lists the values of the functions f and g and their derivatives at several points. Use the table to find the indicated derivative.

9)

x	1	2	3	4
f(x)	1	4	3	2
f'(x)	3	-5	1	4
g(x)	2	4	1	3
g'(x)	1	6	-5	1

(4 Pts) a) Evaluate $\frac{d}{dx}(f[g(x)])$ at $x = 3$.

$$f'(g(x)) \cdot g'(x)$$

$$f'(g(3))\,g'(3) = f'(1)\,g'(3) = (3)(-5) = \boxed{-15}$$

(3 Pts) b) Evaluate $\frac{d}{dx}(f(x)g(x))$ at $x = 2$.

$$= f'(2)g(2) + f(2)g'(2) =$$

$$= (-5)(4) + (4)(6) = -20 + 24 = \boxed{4}$$

(3 Pts) c) Evaluate $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)$ at $x = 1$.

$$\frac{f'(1)g(1) - f(1)g'(1)}{(g(1))^2} = \frac{(3)(2) - (1)(1)}{2^2} = \boxed{\frac{5}{4}}$$

Find the derivative.

(4pts) 10) $g(x) = \frac{x^2+5}{x^2+6x}$

$$g'(x) = \frac{2x(x^2+6x) - (2x+6)(x^2+5)}{(x^2+6x)^2}$$

$$g'(x) = \frac{\cancel{2x^3}+12x^2-\cancel{2x^3}-10x-6x^2-30}{(x^2+6x)^2} = \boxed{\frac{6x^2-10x-30}{(x^2+6x)^2}}$$

Find the derivative of the function.

(4pts) 11) $f(x) = (2x^3 + 3)(5x^7 - 7)$

$$f'(x) = 6x^2(5x^7-7) + (2x^3+3)(35x^6)$$

$$= 30x^9 - 42x^2 + 70x^9 + 105x^6$$

$$= \boxed{100x^9 + 105x^6 - 42x^2}$$

Find the derivative.

(4pts) 12) $y = -9e^{10x}$

$$\boxed{y' = -90e^{10x}}$$

Find the derivative of the function.

(4pts) 13) $y = x + e^{4x^2}$

$$\boxed{y' = 1 + 8x\,e^{4x^2}}$$

(4pts) 14) $y = \ln(6 + x^2)$

$$y' = \frac{1}{6+x^2} \cdot (0+2x) = \boxed{\frac{2x}{6+x^2}}$$

Show all of your work on the test paper. All of the problems must be solved symbolically using Calculus. You may use your calculator to confirm your answers, but full credit is not given unless the answer follows from the symbolic work shown.

15. Let $g(x)=\ln(e^{4x}+2)$. Find the equation of the tangent line to this function when $x=0$ on the graph. Write your answer using exact values, not approximations. (8 Points)

$$g'(x)=\frac{1}{e^{4x}+2}\cdot 4e^{4x}=\left.\frac{4e^{4x}}{e^{4x}+2}\right|_{x=0}=\frac{4e^{0}}{e^{0}+2}=\frac{4}{3}$$

at $x=0$ $\quad y=\ln(e^{0}+2)=\ln 3$; $\quad m=\frac{4}{3}$ $\quad x=0,\ y=\ln 3$

$$y-\ln 3=\frac{4}{3}(x-0)$$

$$\boxed{y=\frac{4}{3}x+\ln 3}$$

16. For what value or values of x does the function $f(x)=x^4\ln(x),\ \ x>0$, have a horizontal tangent line? Write your answer using exact values, not approximations. (8 Points)

$$f'(x)=4x^3\ln x+x^4\frac{1}{x}$$

$$=4x^3\ln x+x^3$$

$$x^3(4\ln x+1)=0$$

$x=0$ OR $4\ln x+1=0$

$$\ln x=\frac{-1}{4}$$

$$\boxed{x=e^{\frac{-1}{4}}}$$

17. A fast food restaurant is establishing its inventory policy for ordering frozen french fries. In the coming year, they expect to sell 800 pounds of french fries. It costs $16 to place an order and the carrying cost for a year is $4 per pound based on the average amount in storage.

let r = No of orders

X = Amount in each order (order size)

a) Determine the economic order quantity for the french fries, that is, the order size which minimizes the inventory cost. **(7 Points)**

$$xr = 800 \implies r = \frac{800}{x}$$

$$\text{Cost} = 16r + 4\left(\frac{X}{2}\right)$$

$$= 16\left(\frac{800}{X}\right) + 2X$$

$$\text{Cost} = 12800X^{-1} + 2X$$

$$(\text{Cost})' = -12800X^{-2} + 2 = 0$$

$$\frac{-12800}{X^2} = -2$$

$$-2X^2 = -12800$$

$$X^2 = 6400$$

$$\boxed{X = 80 \text{ lbs per order}}$$

b) How many times a year should the fast food restaurant order french fries in order to minimize inventory cost? **(2 Points)**

$$r = \frac{800}{X} = \frac{800}{80} = \boxed{10 \text{ orders per year}}$$