Name: Solutions

MATH 150 Dr. Katiraie (100 points) Test #4 Form A 2018

(100 points Plus 10 points extra credit for HW completion)

Show all of your work on the test paper. All of the problems must be solved symbolically using Calculus. You may use your calculator to confirm your answer, but full credit is not given unless the answer follows from the symbolic work shown.

1) A bacterial culture has an initial population of 500. If its population grows to 2000 in 4 hours,

a) Find a formula of the type $A(t) = Ce^{kt}$ for the population of bacteria after t hours. (3 points)

$$2000 = 500e^{4K}$$

$$4 = e^{4K} \Rightarrow K = \frac{\ln(4)}{4}$$

$$\boxed{A(t) = 500\, e^{\frac{\ln(4)}{4}t}}$$

b) What will be the population of the bacteria at the end of 6 hours? (3 points)

$$500\, e^{\frac{\ln(4)}{4} * 6} = \boxed{4000}$$

c) When will the population of bacteria reach 6000? (4 points)

$$6000 = 500\, e^{\frac{\ln(4)}{4}t}$$

$$12 = e^{\frac{\ln(4)}{4}t}$$

$$t = \frac{\ln(12)}{\left(\frac{\ln(4)}{4}\right)} = \boxed{7.17 \text{ HRS}}$$

2) A baseball team plays in a stadium that holds 59,000 spectators.

With ticket prices at \$14, the average attendance had been 49,000.
When ticket prices were lowered to \$10, the average attendance rose to 51,000.

Hint: Let x be the number of spectators, and y be the ticket prices.

(a) Find the demand function (which means price as a function of number of attendants) and assume that the price function is linear. (4 points)

$$(49000, 14), \ (51000, 10) \Rightarrow m = \frac{10-14}{51000-49000} = -0.002$$

$$y - 14 = -0.002(x - 49000)$$

$$\boxed{y = -0.002x + 112} \quad \boxed{P = -0.002X + 112} \text{ dollars}$$

(b) How should ticket prices be set to maximize revenue? (3 points)

$$R(x) = xP = -0.002x^2 + 112x$$

$$R' = -0.004x + 112 = 0$$

$$x = \frac{112}{0.004} = 28000$$

$$\text{Price} = -0.002(28000) + 112 = \boxed{\$56}$$

3) A manufacturer of bicycle helmets estimates that the demand (price) of each helmet is $p(x) = 19.2 - 0.005x$ (8 points)

(a) Write the revenue function R(x) of the product.

$R(x) = x(19.2 - 0.005x) \Rightarrow R(x) = -0.005x^2 + 19.2x$

(b) Suppose the total cost of producing x helmets is given by $C(x) = 6200 + 8x + 0.002x^2$
Find and simplify the profit function P(x)

$P(x) = R - C$

$P(x) = -0.005x^2 + 19.2x - (6200 + 8x + 0.002x^2)$

$P(x) = -0.007x^2 + 11.2x - 6200$

(c) How many helmets must the company produce and sell in order to maximize profit?

$P'(x) = -0.014x + 11.2 = 0 \Rightarrow \boxed{x = 800 \text{ Helmets}}$

(d) What price per helmet must be charged in order to maximize profit?

$\text{Price} = 19.2 - 0.005(800) \Rightarrow \boxed{\text{Price} = \$15.20}$

4) The table lists the values of the functions f and g and their derivatives at several points. Use the table to find the indicated derivatives. (14 Points)

x	4	3	2	1	0
$f(x)$	2	3	5	1	3
$f'(x)$	5	1	-5	3	2
$g(x)$	3	2	3	1	4
$g'(x)$	2	-3	7	0	1

a) Evaluate $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)$ at $x = 3$

$\frac{f'g - fg'}{g^2}$ at $x = 3$

$= \frac{1(2) - (3)(-3)}{2^2} = \frac{2+9}{4}$

$= \boxed{\frac{11}{4}}$

b) Evaluate $\frac{d}{dx}(f(x)g(x))$ at $x = 2$

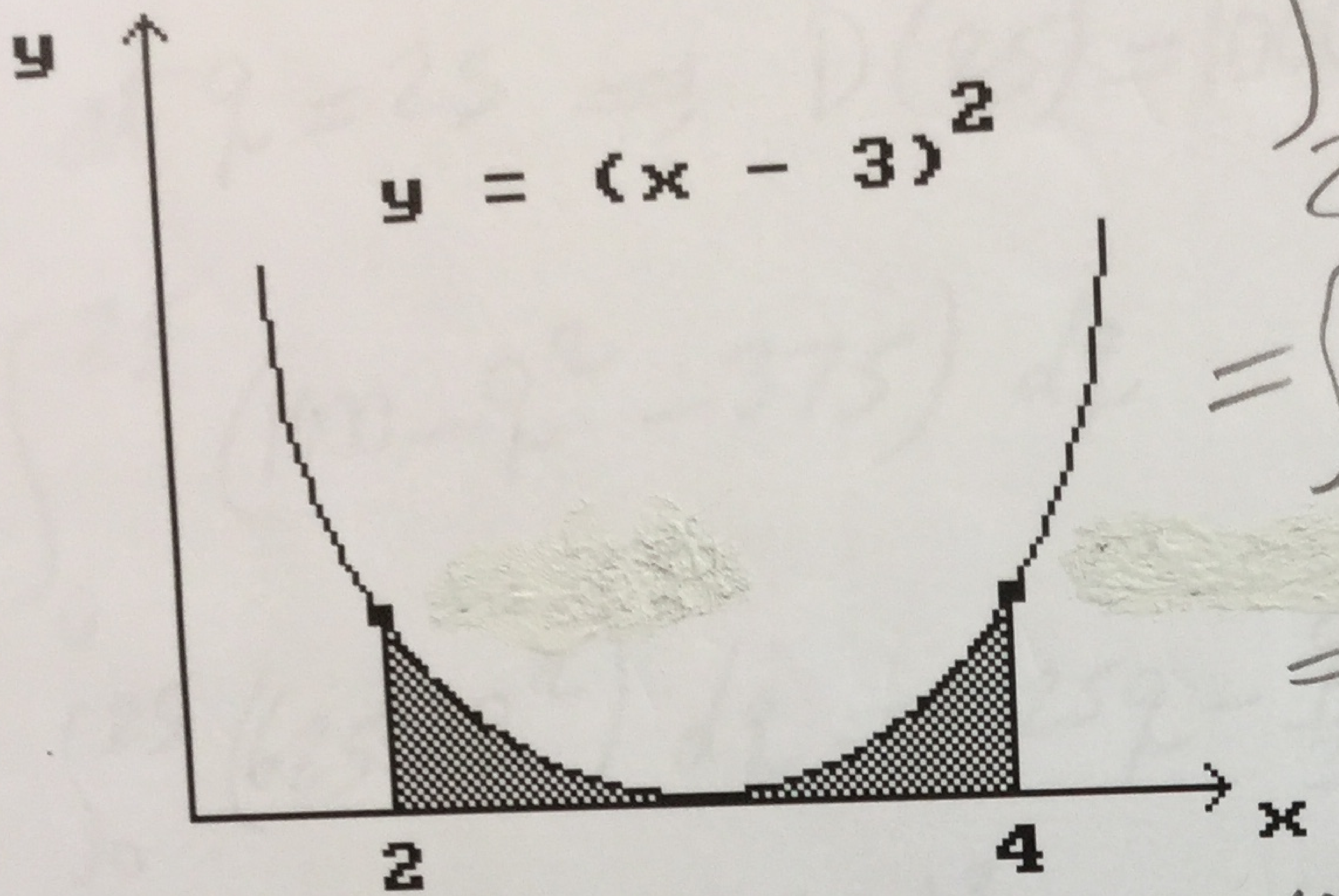
$= f'g + fg'$ at $x = 2$

$= (-5)(3) + 5(7)$

$= -15 + 35$

$= \boxed{20}$

5) Find the shaded area under the given curve. (2 Points)



$$\int_2^4 (x-3)^2\,dx$$

$$=\int_2^4 (x^2-6x+9)\,dx$$

$$=\frac{x^3}{3}-\frac{6x^2}{2}+9x\Big|_2^4$$

$$=0.67 \approx \boxed{\frac{2}{3}}$$

6) A ball is thrown upward from a height of 40 feet above the ground, with an initial velocity of 15 feet per second. The velocity at time t is $v(t)=15-32t$ feet per second (5 points each)

$s(0)=40$ feet

a) Find $s(t)$, the function representing the height of the ball at time t.

$$s(t)=\int(15-32t)\,dt = 15t-\frac{32t^2}{2}+40$$

$$\boxed{s(t)=-16t^2+15t+40 \text{ (feet)}}$$

b) How long will the ball take to reach the ground?

$$-16t^2+15t+40=0$$

$$t=\frac{-15\pm\sqrt{15^2-4(-16)(40)}}{-32}$$

$\boxed{2.12 \text{ seconds}}$

~~-1.18 seconds~~

c) How high will the ball go?

$15-32t=0 \Rightarrow t=\frac{15}{32}$

$$s\left(\tfrac{15}{32}\right)=-16\left(\tfrac{15}{32}\right)^2+15\left(\tfrac{15}{32}\right)+40=\boxed{43.52 \text{ feet}}$$

7) Given the demand function is given by $D(q) = 1000 - q^2$ Dollars, Find the consumer surplus when the sales level is $q = 25$ Gas Stoves (7 Points)

at $q = 25 \Rightarrow D(25) = 1000 - 25^2 = 375$ dollars

$$\int_0^{25} (1000 - q^2 - 375)\, dq$$

$$\int_0^{25} (625 - q^2)\, dq = 625q - \frac{q^3}{3} \Big|_0^{25}$$

$$= 625(25) - \frac{(25)^3}{3} = \boxed{10416.67 \text{ dollars}}$$

8) Find the average of the following function on the given interval.

$f(x) = \frac{3}{x} - 5e^{0.4x}$ [2, 6] (4 Points)

$$\frac{1}{6-2} \int_2^6 \left(\frac{3}{x} - 5e^{0.4x}\right) dx$$

$$= \frac{1}{4}\left(3\ln x - \frac{5e^{0.4x}}{0.4}\right)\Big|_2^6 = \cancel{-18.6033551}$$

$\boxed{-26.7}$

9) Find the function $f(x)$ such that) $f'(x) = 15x^3 - 6x + 3$ and $f(1) = 8$. (5 Points)

$$f(x) = \frac{15x^4}{4} - \frac{6x^2}{2} + 3x + C$$

$$8 = \frac{15}{4} - \cancel{3} + \cancel{3} + C$$

$-\frac{15}{4}$ $-\frac{15}{4}$

$$C = \frac{17}{4} = 4.25$$

$C = \frac{17}{4}$

$$\boxed{f(x) = \frac{15}{4}x^4 - 3x^2 + 3x + \frac{17}{4}}$$

(3 points each)

10) Determine the following:

$\int\left(\frac{3}{x^2}+\frac{x^2}{2}\right)dx = \int 3x^{-2}+\frac{1}{2}x^2\,dx$

$= -3x^{-1}+\frac{1}{2}\frac{x^3}{3}+c$

$= \frac{-3}{x}+\frac{x^3}{6}+c$

$\int x\sqrt{x}dx = \int x^{\frac{3}{2}}dx$

$= \frac{x^{\frac{5}{2}}}{\frac{5}{2}}+c$

$= \frac{2}{5}x^{\frac{5}{2}}+c$

$\int(-2x^2+x+\frac{1}{2x})dx$

$= \frac{-2x^3}{3}+\frac{x^2}{2}+\frac{1}{2}\ln|x|+c$

$\int 20e^{-4x}dx$

$= \frac{20e^{-4x}}{-4}+c$

$= -5e^{-4x}+c$

11) Find the area bounded by the curves $y=6x-x^2$ and $y=x$

Hint: Sketch the above functions, algebraically find the points of intersections. Set up the integral and find the area bounded by the two curves.

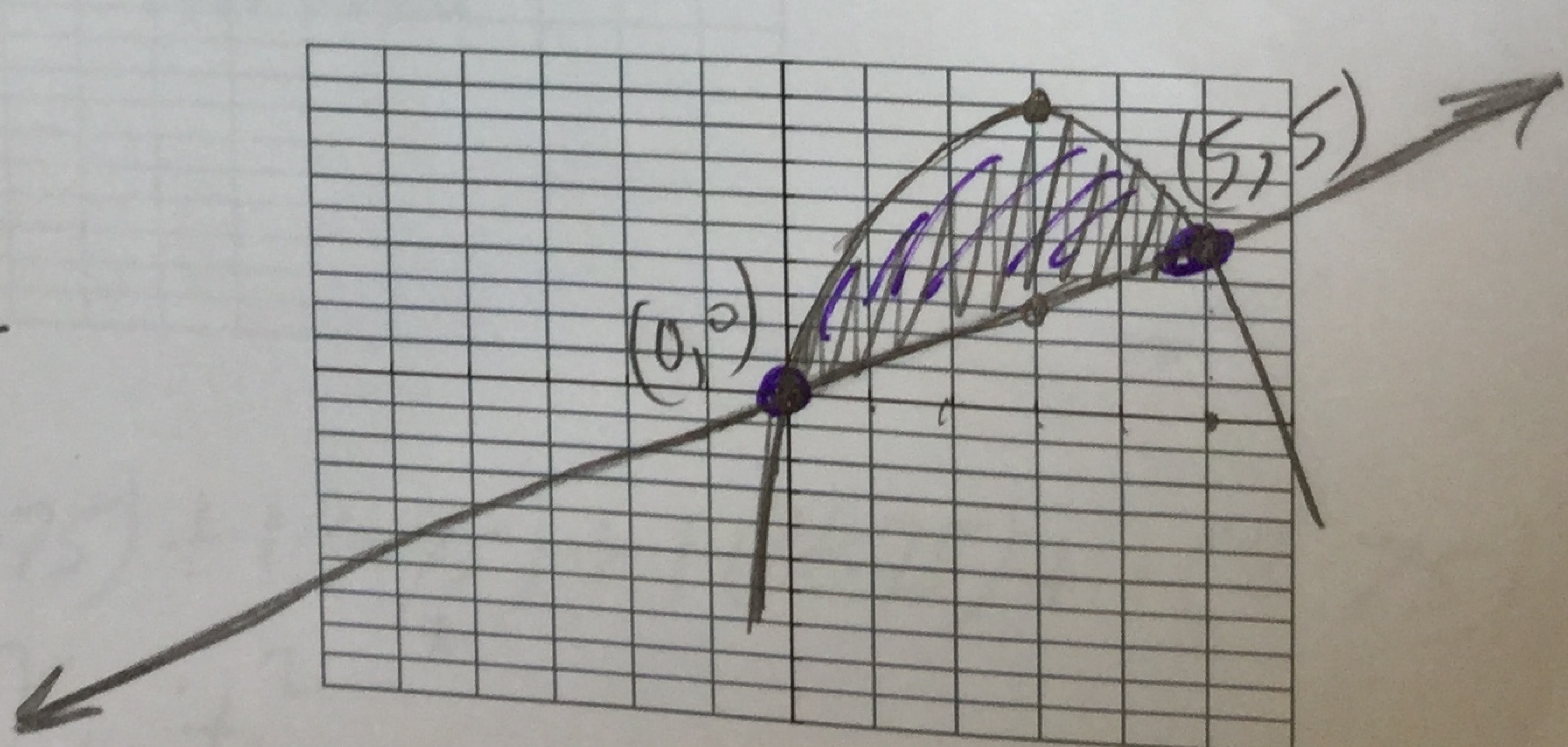
(8 points)

$-x^2+6x=x$

$0=x^2-5x$

$x(x-5)=0 \quad x=0$

$x=5$



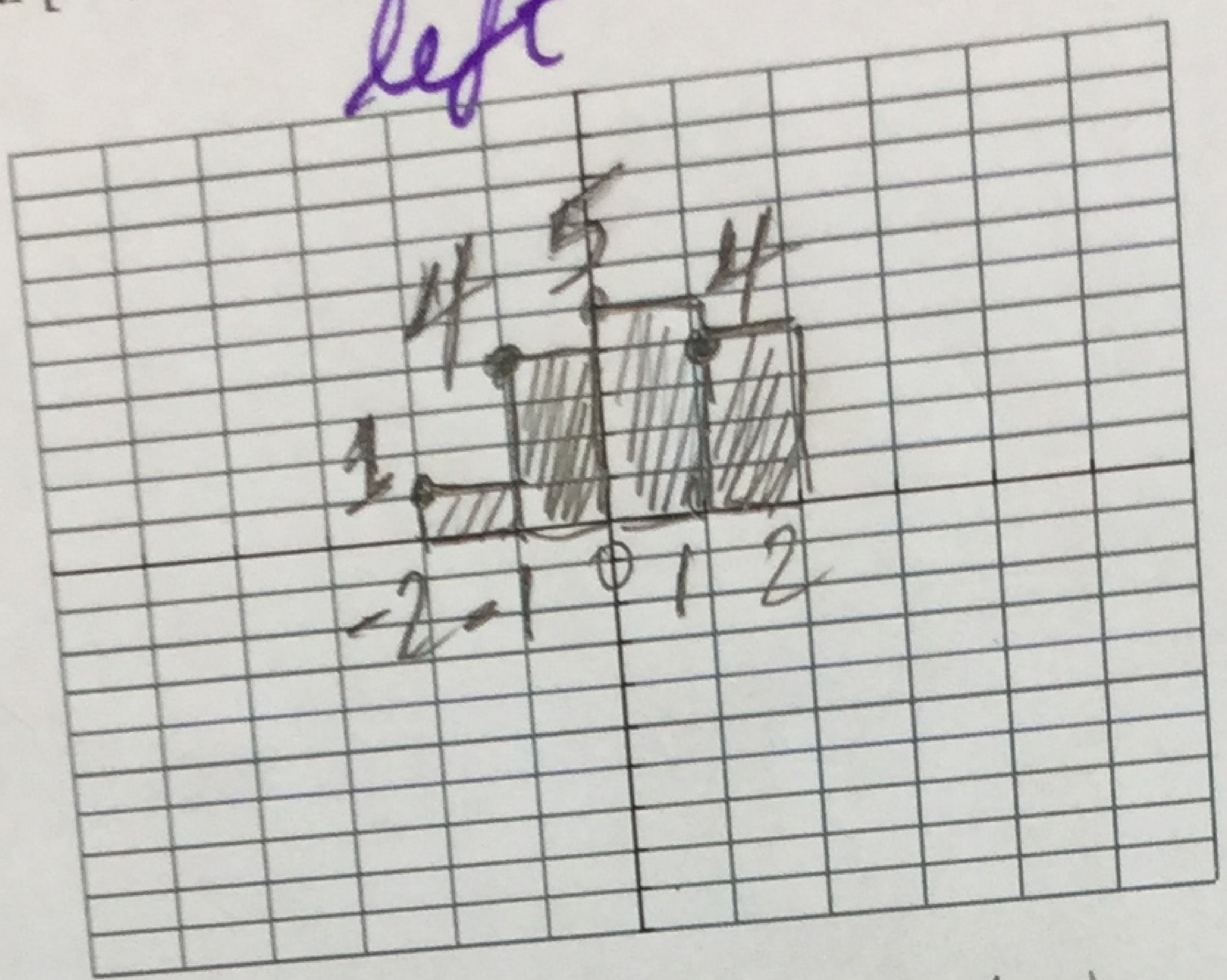
$\int_0^5(6x-x^2-x)\,dx$

$= \int_0^5(-x^2+5x)\,dx = -\frac{x^3}{3}+\frac{5x^2}{2}\Big|_0^5 \cong 20.833$

12A) Use 4 approximating rectangles and left endpoints to estimate the area under the graph of $f(x) = 5 - x^2$ on the interval [-2 , 2]. (4 Points)

$\Delta X = \frac{2 - -2}{4} = 1$

left



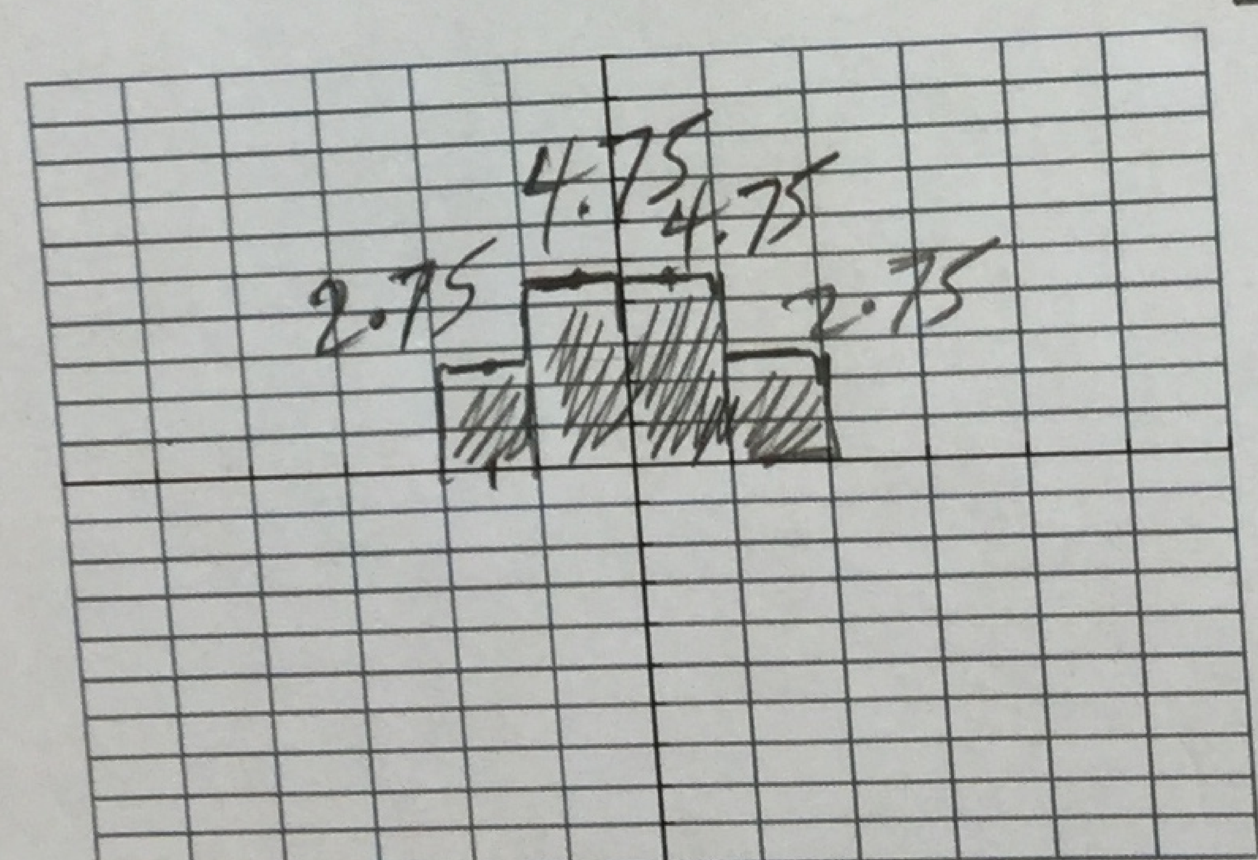
X	$f(x)$
-2	1
-1	4
0	5
1	4

Area $= 1(1) + 1(4) + 1(5) + 1(4)$

$= \boxed{14}$ units^2

12B) We usually get a better approximation using midpoints instead of left or right endpoints. To see this, use 4 approximating rectangles and midpoints to estimate the area under the graph of $f(x) = 5 - x^2$ on the interval [-2 , 2]. (4 Points)

$\Delta X = \frac{2 - -2}{4} = 1$



X	$f(x)$
-1.5	2.75
-0.5	4.75
0.5	4.75
1.5	2.75

$= 1(2.75) + 1(4.75) + 1(4.75) + 1(2.75)$

$= \boxed{15}$ units^2