

Show all of your work on the test paper. Full credit is not given unless the answer follows from the work shown.

1. (6 points) The value of a painting is increasing exponentially and satisfies the differential equation

$$P'(t) = .08P(t),$$

where t is measured in years and $P(t)$ is the value of the painting in millions of dollars. **Use the differential equation** to determine how fast the value will be increasing when the value reaches \$5 million. Write your answer in a sentence, and use appropriate units.

2. (12 points) Let $P(t)$ be the population (in millions) of a certain city t years after 1980. Suppose that $P(t)$ satisfies the differential equation $P'(t) = 0.04P(t)$ and that $P(0) = 2.8$.

- (a) Write the formula for $P(t)$.
- (b) What was the population in 1995?
- (c) In what year will the population be 10 million?

3. (9 points) A radioactive substance is decaying exponentially. If there are 50 grams of the substance present at time $t = 0$, and 20 grams present 6 days later,

- (a) Find the exponential decay constant for this substance correct to four decimal places.

- (b) Write the formula for this situation.

4. (16 points) Find the derivative of each function. You do not have to simplify your answer.

(a) $f(x) = x^3 e^{4x^2}$

(b) $g(x) = \ln(2x^2 + 7)$

5. (24 points) Determine the following integrals. Simplify your answers and write your answers with no negative exponents.

(a) $\int (4x - 6x^2 + 3) dx = - \quad -$

(b) $\int \left(\frac{x^5}{4} + \frac{4}{x^5} \right) dx = \int (\quad) \quad - \quad - \quad - \quad - \quad -$

(c) $\int \left(6\sqrt{x} + \frac{7}{x} \right) dx = \int \quad - \quad - \quad || \quad - \quad || \quad ||$

6. (13 points) Find the function $f(x)$ if $f'(x) = 12e^{3x} + 5$ and $f(0) = 11$.

7. (8 points) Which of the following is $\int x e^x dx$? Show work to substantiate your answer. No credit will be given if you do not show how you arrived at your answer.

(a) $\frac{1}{2} x^2 e^x + C$

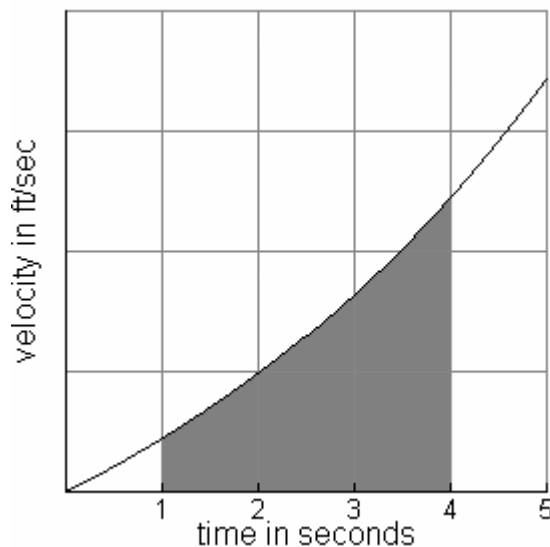
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(b) $x e^x + e^x + C$

(c) $x e^x - e^x + C$

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8. (4 points) The graph shown represents the velocity (in feet per second) of an object at time t (in seconds). In a sentence, interpret the area of the shaded region.



9. (8 points) Use a Riemann Sum to approximate the area under the graph of the function $f(x) = \sqrt{x}$ on the interval $3 \leq x \leq 5$, with $n = 4$ and selected points as left endpoints of subintervals. Do all calculations to at least four decimal places, and write your answer correct to four decimal places.

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