Dr. Katiraie MA160 Summer 2008 Test 4 Form B

Name: Solutions

Show all of your work on the test paper. All of the problems must be solved symbolically using Calculus. You may use your calculator to confirm your answers, but full credit is not given unless the answer follows from the symbolic work shown.

1. Let $g(x)=\ln(e^{5x}+3x)$. Find the equation of the tangent line to this function when $x=0$ is on the graph. Write your answer using exact values, not approximations. (8 Points)

$g'(x)=\frac{1}{e^{5x}+3x}(5e^{5x}+3)$ and $g'(0)=\frac{5e^{0}+3}{e^{5(0)}+3(0)}=\frac{8}{1}=8 \Rightarrow \boxed{m=8}$

$g(0)=\ln(e^{0}+3(0))=\ln(1)=0$

$y-y_1=m(x-x_1) \Rightarrow y-0=8(x-0)$

$\boxed{y=8x}$

2. The table lists the values of the functions f and g and their derivatives at several points. Use the table to find the indicated derivatives. (8 Points)

x	4	3	2	1	0
$f(x)$	2	3	4	1	3
$f'(x)$	5	1	-5	3	2
$g(x)$	3	2	4	1	4
$g'(x)$	2	-3	7	0	1

(use chain Rule)

a) Evaluate $\frac{d}{dx}(f[g(x)])$ at $x=2$

$= f'(g(x))g'(x)$

at $x=2 \Rightarrow f'(g(2))g'(2)$

$=f'(4)g'(2)$

$=(5)(7)=\boxed{35}$

b) Evaluate $\frac{d}{dx}(f(x)g(x))$ at $x=0$ (use Product Rule)

$=f'(0)g(0)+f(0)g'(0)$

$=(2)(4)+(3)(1)$

$=8+3=\boxed{11}$

3. A fast food restaurant is establishing its inventory policy for ordering frozen french fries. In the coming year, they expect to sell 20000 pounds of french fries. It costs \$2 to place an order and the carrying cost for a year is \$4 per pound based on the average amount in storage.

a) Determine the economic order quantity for the french fries, that is, the order size which minimizes the inventory cost. (7 Points)

let X = order size

r = number of orders per year.

$$Xr = 20000 \Rightarrow r = \frac{20000}{X}$$

$$Cost = 2r + 4\left(\frac{X}{2}\right) = 2r + 2X$$

$$Cost = 2\left(\frac{20000}{X}\right) + 2X = \frac{40000}{X} + 2X = 40000X^{-1} + 2X$$

$$(Cost)' = -40000X^{-2} + 2 = 0 \qquad \frac{-40000}{X^2} = -2 \Rightarrow 2X^2 = 40000$$

$$X^2 = 20000$$

$$X = \sqrt{20000}$$

$$X = 141.42 \text{ lbs}$$

b) How many times a year should the fast food restaurant order french fries in order to minimize inventory cost? (2 Points)

$$r = \frac{20000}{X} = \frac{20000}{141.42} = \boxed{141.42 \text{ orders}}$$

4) Solve the following equations for x. (5 Points Each)

a) $\ln x^5 - 3\ln x = 2$	b) $\ln(x-5) - \ln(x+3) = 3$
$\ln(x^5) - \ln(x^3) = 2$ $\ln\left(\frac{x^5}{x^3}\right) = 2$ $\ln(x^2) = 2$ $x^2 = e^2 \Rightarrow \boxed{x = e}$	$\ln\left(\frac{x-5}{x+3}\right) = 3$ $\frac{x-5}{x+3} = \frac{e^3}{1}$ $e^3 x + 3e^3 = x - 5$ $x(e^3 - 1) = -3e^3 - 5$ $\boxed{x = \frac{-3e^3 - 5}{(e^3 - 1)}}$ But since $x =$ Negative No Solutions

5) A sample of 100 grams of radioactive material is placed in a vault. Let P(t) be the amount remaining after t years. And suppose that P(t) satisfies the differential equation $P'(t) = -.031P(t)$ (2 points Each)

a) Find the formula for P(t) $= \boxed{100e^{-0.031t}}$ (grams)

b) How long will it take for the radioactive material to disintegrate to 40 grams?

$40 = 100e^{-0.031t} \Rightarrow 0.4 = e^{-0.031t} \Rightarrow \ln(0.4) = \cancel{\ln} e^{-0.031t}$

$t = \frac{\ln(0.4)}{-0.031} = \boxed{29.56 \text{ years}}$

c) Use the differential equation to determine how fast the sample is disintegrating when just three grams remains.

$P'(t) = -0.031(3) = \boxed{-0.093 \ \frac{\text{grams}}{\text{year}}}$

d) What amount of radioactive material remains when it is disintegrating at the rate of 0.2 grams per year?

$-0.2 = -0.031\,P(t) \Rightarrow P(t) = \frac{-0.2}{-0.031} = \boxed{6.45 \text{ grams}}$

e) Find the half-life of the radioactive material.

$P(t) = 100\,e^{-0.031t}$

$50 = 100\,e^{-0.031t}$

$\frac{50}{100} = e^{-0.031t} \Rightarrow 0.5 = e^{-0.031t} \Rightarrow \ln(0.5) = -0.031t$

$t = \frac{\ln(0.5)}{-0.031} \cong \boxed{22.4 \text{ Years}}$

6) Six thousand dollars is deposited into a saving account at 4.5% interest compounded continuously. (2 points Each)

a) What is the formula for A(t), the balance after t years?

$$\boxed{A(t) = 6000e^{0.045t}}$$

b) What differential equation is satisfied by A(t), the balance after t years.

$$A'(t) = \boxed{0.045\,A(t)}$$

c) How much money will be in the account after 2 years?

$$A(2) = 6000e^{0.045(2)} \boxed{= \$6565.06}$$

d) When will the investment triple?

$$A(t) = 6000\,e^{0.045t}$$

$$3 * 6000 = 6000\,e^{0.045t}$$

$$\frac{3 * \cancel{6000}}{\cancel{6000}} = e^{0.045t} \Rightarrow 3 = e^{0.045t}$$

$$t = \frac{\ln(3)}{0.045} \boxed{\cong 24.41 \text{ years}}$$

e) How fast is the balance growing when it reaches $7000?

$$A'(t) = 0.045\,A(t)$$

$$= 0.045(7000)$$

$$\boxed{= 315 \ \frac{\text{Dollars}}{\text{year}}}$$

7) Determine the following: (6 points each)

$\int\left(\frac{2}{x}+\frac{x}{2}\right)dx$ $= 2\ln\lvert x\rvert + \frac{1}{2}\frac{x^2}{2} + c$ $= \boxed{2\ln\lvert x\rvert + \frac{1}{4}x^2 + c}$	$\int x\sqrt{x}dx = \int x^{3/2}dx$ $= \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + c$ $= \boxed{\frac{2}{5}x^{\frac{5}{2}} + c}$
$\int(x-2x^2+\frac{1}{3x})dx = \int(x-2x^2+\frac{1}{3}\frac{1}{x})dx$ $\boxed{\frac{x^2}{2} - \frac{2x^3}{3} + \frac{1}{3}\ln\lvert x\rvert + c}$	$\int 3e^{-2x}dx$ $= \frac{3e^{-2x}}{-2} + c$ $= \boxed{\frac{-3}{2}e^{-2x} + c}$

8) A ball is thrown upward from a height of 256 feet above the ground, with an initial velocity of 96 feet per second. The velocity at time t is
$v(t) = 96 - 32t$ feet per second (4 points each)

a) Find $s(t)$, the function giving the height of the ball at time t.

$s(t) = 96t - \frac{32t^2}{2} + c \Rightarrow$ But $s(0) = 256$

$\Rightarrow \boxed{s(t) = 96t - 16t^2 + 256}$ (feet)

b) How long will the ball take to reach the ground?

let $96t - 16t^2 + 256 = 0 \Rightarrow 16t^2 - 96t - 256 = 0$

$16(t^2 - 6t - 16) = 0 \Rightarrow 16(t-8)(t+2) = 0$

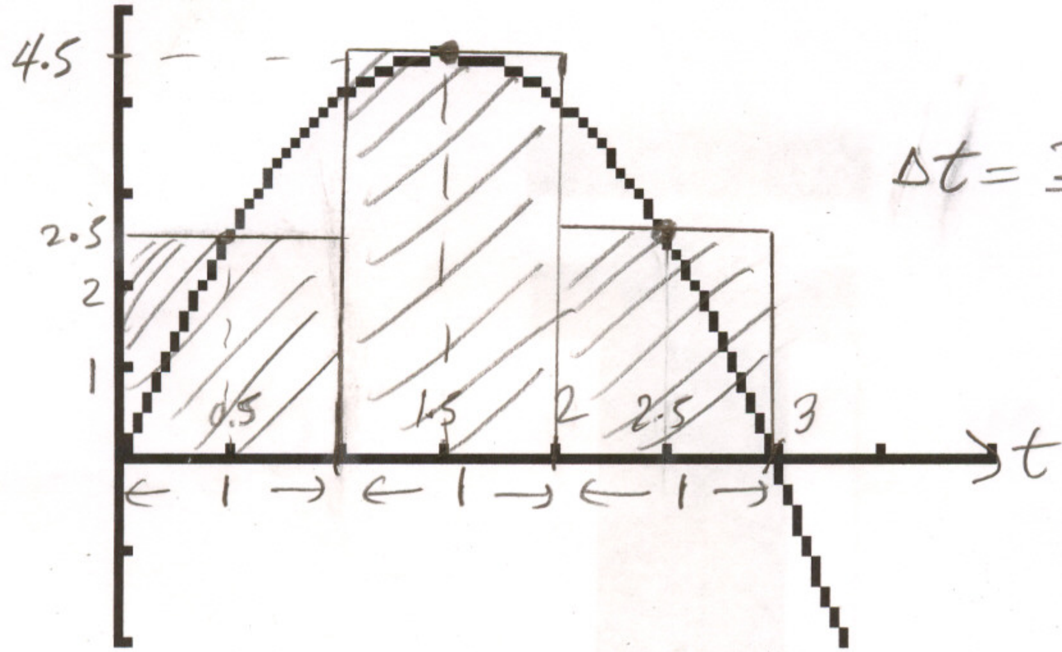
$\boxed{t = 8}$ seconds $\quad \cancel{t = -2}$

c) How high will the ball go?

let $v(t) = 0 \Rightarrow 96 - 32t = 0 \Rightarrow t = \frac{96}{32} = 3$ seconds

$s(3) = 96(3) - 16(3)^2 + 256 = \boxed{400 \text{ feet}}$

9) The graph of $v(t) = -2t^2 + 6t$ (meters per second) gives the velocity of an object at time t.



$\Delta t = \frac{3-0}{3} = 1$

a) Sketch proper boxes to estimate the Riemann sum with n = 3 and selected points as midpoints to find the distance the object travels from t = 0 to t = 3 seconds (4.5 points)

$= (1)(2.5) + (1)(4.5) + (1)(2.5)$

$= \boxed{9.5}$ meters

b) Find the distance the object travels from t = 0 to t = 3 (Please remember the proper units). (4.5 points)

$v(t) = -2t^2 + 6t$

$\int_0^3 (-2t^2 + 6t)\,dt$

$= \frac{-2t^3}{3} + \frac{6t^2}{2} \Big|_0^3 = \frac{-2}{3}(3)^3 + 3(3)^2 - 0 = 9 \text{ meters}$

In addition, please practice question numbers 14 – 28 from final exam review packet.
http://myspace.montgomerycollege.edu/fred.katiraie/MA160rockville.pdf