MA 160 Practice Test #4 Summer 2008

(100 points) Name Solutions

Show all of your work on the test paper. Full credit is not given unless the answer follows from the work shown.

1. (6 points) The value of a painting is increasing exponentially and satisfies the differential equation

$$P'(t) = .08P(t),$$

where t is measured in years and $P(t)$ is the value of the painting in millions of dollars. **Use the differential equation** to determine how fast the value will be increasing when the value reaches $5 million. Write your answer in a sentence, and use appropriate units.

$P'(t) = .08(5) = .4$

When the painting's value reaches $5 million, the value will be increasing at a rate of $.4 million ($400,000) per year.

2. (12 points) Let $P(t)$ be the population (in millions) of a certain city t years after 1980. Suppose that $P(t)$ satisfies the differential equation $P'(t) = 0.04P(t)$ and that $P(0) = 2.8$.

(a) Write the formula for $P(t)$. $P(t) = 2.8e^{0.04t}$

(b) What was the population in 1995? $P(15) = 2.8e \wedge (0.04 * 15) \approx 5.1 \textit{ million}$

(c) In what year will the population be 10 million?

$2.8e^{0.04t} = 10$

$e^{0.04t} = 10/2.8$

$\ln(e^{0.04t}) = \ln(10/2.8)$

$0.04t = \ln(10/2.8) \textit{ so } t = \ln(10/2.8)/0.04 \approx 31.8 \textit{ or } 32$

The year is 1980 + 32 = 2012

3. (9 points) A radioactive substance is decaying exponentially. If there are 50 grams of the substance present at time t = 0, and 20 grams present 6 days later,

(a) Find the exponential decay constant for this substance correct to four decimal places.

$P(t) = 50e^{-\lambda t}$

$P(6) = 50e^{-\lambda 6} = 20$

$e^{-6\lambda} = 20/50$

$\ln(e^{-6\lambda}) = \ln(20/50)$

$\lambda = \ln(20/50)/-6 = .1527$

(b) Write the formula for this situation. $P(t) = 50e^{-.1527t}$

4. (16 points) Find the derivative of each function. You do not have to simplify your answer.

(a) $f(x) = x^3 e^{4x^2}$

$f'(x) = x^3 e^{4x^2} 8x + e^{4x^2} 3x^2 = 8x^4 e^{4x^2} + 3x^2 e^{4x^2}$

(b) $g(x) = \ln(2x^2 + 7)$

$g'(x) = \dfrac{4x}{2x^2 + 7}$

5. (24 points) Determine the following integrals. Simplify your answers and write your answers with no negative exponents.

(a) $\int (4x - 6x^2 + 3)dx = \frac{4}{2}x^2 - \frac{6}{3}x^3 + 3x + C = 2x^2 - 2x^3 + 3x + C$

(b) $\int \left(\frac{x^5}{4} + \frac{4}{x^5} \right) dx = \int \left(\frac{1}{4}x^5 + 4x^{-5} \right) dx = \frac{1}{4} \cdot \frac{1}{6} x^6 + \frac{4}{-4} x^{-4} + C = \frac{1}{24}x^6 - \frac{1}{x^4} + C$

(c)

$\int \left(6\sqrt{x} + \frac{7}{x} \right) dx = \int \left(6x^{1/2} + 7 \cdot \frac{1}{x} \right) dx = \frac{6}{3/2} x^{3/2} + 7\ln|x| + C = \frac{2}{3} \cdot 6x^{3/2} + 7\ln|x| + C = 4x^{3/2} + 7\ln|x| + C$

6. (13 points) Find the function $f(x)$ if $f'(x) = 12e^{3x} + 5$ and $f(0) = 11$.

$f(x) = \frac{12}{3} e^{3x} + 5x + C = 4e^{3x} + 5x + C$

$f(0) = 4e^0 + 0 + C = 11$

$4 + C = 11$ *so* $C = 7$ *and* $f(x) = 4e^{3x} + 5x + 7$

7. (8 points) Which of the following is $\int x e^x dx$? Show work to substantiate your answer. No credit will be given if you do not show how you arrived at your answer.

(a) $\frac{1}{2}x^2 e^x + C$

Use the Product Rule to find the derivative of this function.

It is $\frac{1}{2}x^2 e^x + e^x 2 \cdot \frac{1}{2}x \neq xe^x$ so this is not the correct answer.

(b) $x e^x + e^x + C$

Use the Product Rule to find the derivative of this function.

It is $xe^x + e^x \cdot 1 + e^x \neq xe^x$ so this is not the correct answer.

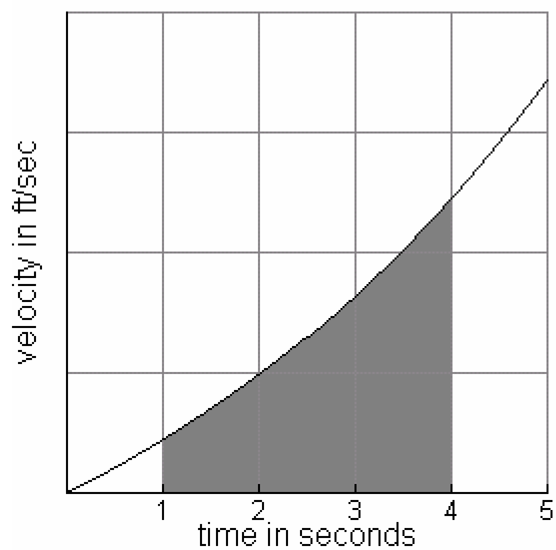
(c) $x e^x - e^x + C$

Use the Product Rule to find the derivative of this function.

It is $xe^x + e^x \cdot 1 - e^x = xe^x$ so this <u>is</u> the correct answer.

8. (4 points) The graph shown represents the velocity (in feet per second) of an object at time t (in seconds). In a sentence, interpret the area of the shaded region.

The area of the shaded region represents the total distance (in feet) traveled by the object from t = 1 to t = 4 seconds.



9. (8 points) Use a Riemann Sum to approximate the area under the graph of the function $f(x) = \sqrt{x}$ on the interval $3 \le x \le 5$, with n = 4 and selected points as left endpoints of subintervals. Do all calculations to at least four decimal places, and write your answer correct to four decimal places.

$$\Delta = \frac{5-3}{4} = \frac{2}{4} = .5$$

$$[f(3) + f(3.5) + f(4) + f(4.5)](.5)$$

$$[1.7321 + 1.8708 + 2 + 2.1213](.5) = (7.7242)(.5) = 3.8621$$