MONTGOMERY COLLEGE
Department of Mathematics
Rockville Campus

MA 160 Dr. Katiraie Sections 1.3 -- 1.6 (30 POINTS)

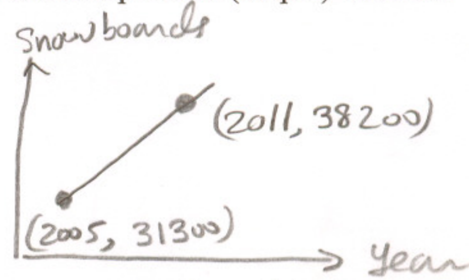
1. A company that produces snowboards has seen its annual sales increase linearly. In 2005, it sold 31,300 snowboards, and it sold 38,200 snowboards in 2011. (3 POINTS)

a) Suppose you want to describe this situation with a linear function. What two variables will you use? Which variable is the independent variable (input) and which variable is the dependent (output) variable?

(2005, 31300) (2011, 38200)

input is the year, output is No of snowboards sold.

b) Draw two axes, labeling them with your variable names from part a). You do not need to put a scale on your axes. Then plot and label the two points you were given, drawing a line between the two points.



c) Recall that the slope of a line between two points is given by $m = \dfrac{\Delta y}{\Delta x} = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{\text{change in output}}{\text{change in input}}$

Calculate the slope of your line, and interpret it as a rate of change.

$m = \dfrac{38200 - 31300}{2011 - 2005} = 1150$; Every year, the number of snowboard sales increses by 1150 snowboards

2. As dry air moves upward, it expands and cools. Suppose that the ground temperature is 20°C and the temperature at a height of 1 km is 10°C. (4 POINTS)

a) You want to describe this situation with a function. Determine the input and output variables, draw and label two axes, then plot and label the two points you were given.

(0 Km, 20°C)

(1Km, 10°C)

b) Determine the slope of the line between your two points, and interpret the slope.

$\dfrac{10-20}{1-0} = -10\ {}^\circ C / 1Km$ For Every 1Km increase in height, the temperature drops 10°C

c) Use the point-slope form of the equation of a line $y - y_1 = m(x - x_1)$ or use the slope-intercept form of the equation of a line $y = mx + b$ to find a linear equation for your function.

$y - 20°C = -10(x - 0)$

$y = -10x + 20$

$T = -10x + 20$

d) Use your equation to determine the air temperature at an elevation of 2.5 km.

$T = -10(2.5) + 20$

$= -25 + 20$

$= \boxed{-5\ °C}$

3. Assume that the population of bacteria doubles every hour. The colony of bacteria starts out with 100 bacteria. Let f(t) represent the population of bacteria at time t, where **t is in hours.** (2 points)

a. Find the formula for f(t) $= 100(2)^t$

b. Predict when there will be 100,000 bacteria.

$100\,000 = 100(2)^t$

$1000 = 2^t \Rightarrow \log 1000 = t \log 2$

$t = \frac{\log 1000}{\log 2} = 9.97$ HRS

4. The following table represents an exponential function of the form y = C a^x.
Find the value of C and a, and write the formula for the function in the form y = C a^x.
(Please show all the mathematical steps very clearly) (2 points)

x	y
1	10
2	2
3	2/5
4	2/25
5	2/125

$* \frac{1}{5}$ $*$ 1/5 $*$ 1/5

$$\frac{2 = \cancel{c} a^2}{10 = \cancel{c} a^1} \Rightarrow \frac{1}{5} = a$$

$10 = c\left(\frac{1}{5}\right)$

$c = 50$

$y = 50\left(\frac{1}{5}\right)^x$

5. Let $f(x) = (5)^x$ Evaluate f at the indicated values. (2 points)

a. $f(0) = 5^0 = 1$

b. Find x when $f(x) = \frac{1}{25}$

$\frac{1}{25} = 5^x$

$x = \frac{\log(1/25)}{\log(5)} = -2$

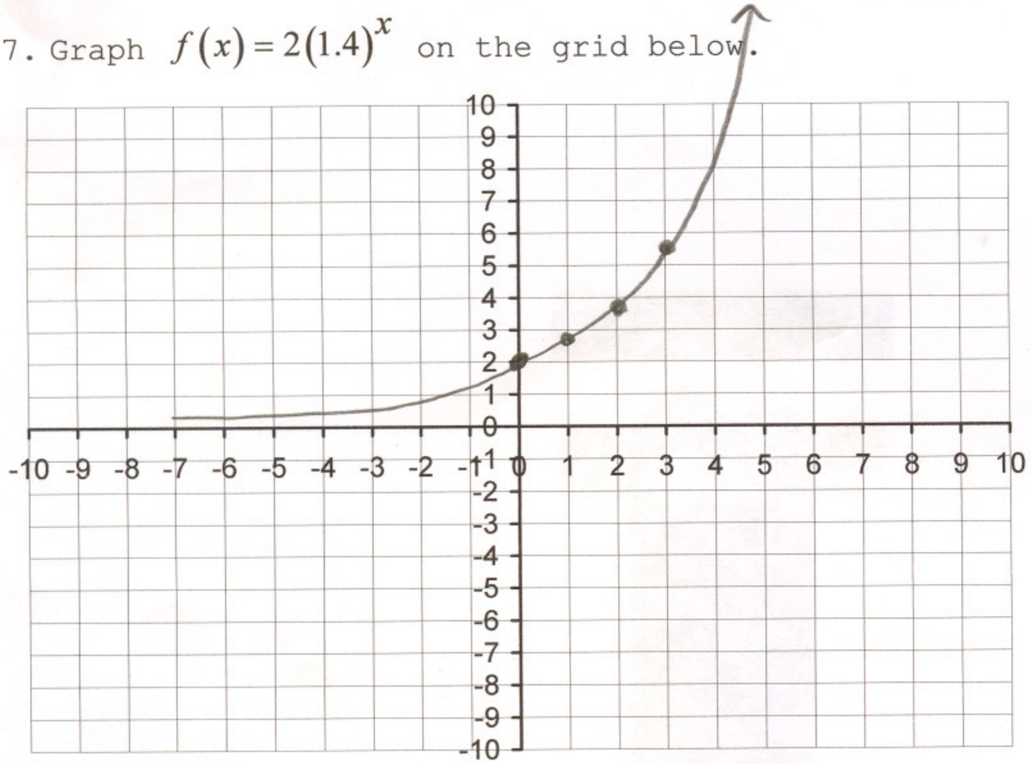
6. Some values for the function f is shown in the table below. (2 points)

x	1	2	3	4	5	6	7
$f(x)$	0	1	2	6	15	37	90

a. Find $f(2) = 1$

b. Find $f^{-1}(6) = 4$

7. Graph $f(x)=2(1.4)^x$ on the grid below. (2 points)



X	y
0	2
1	2.8
2	3.92
3	5.488

8. Assume that the growth of the population of bacteria triples every hour. The colony of bacteria starts out with 100 bacteria. Let f(t) represent the population of bacteria at time t, where **t is in hours.** (4 points)

a. Find an equation for f(t) $= \boxed{100(3)^t}$

b. Predict the number of bacteria after 2 hours.

$= 100(3)^2 = 900$ Bacteria

c. Predict the number of bacteria after 150 **minutes** $\times \frac{1\text{ HR}}{60\text{ min}} = \boxed{2.5\text{ HR}}$

$=100(3)^{2.5} \approx 1559$ Bacteria

d. Predict when there will be 500,000 bacteria.

$500000 = 100(3)^t \Rightarrow 5000 = 3^t \Rightarrow t = \frac{\log 5000}{\log 3} = \boxed{7.75\text{ HRS}}$

9. Assume that the population of bacteria doubles every hour. The colony of bacteria starts out with 30 bacteria. Let f(t) represent the population of bacteria at time t, where **t is in hours.** (1 point)

a. Find the formula for f(t) $= 30(2)^t$

b. Predict when there will be 5,000 bacteria.

$5000 = 30(2)^t \Rightarrow \frac{5000}{30} = 2^t$

$t = \frac{\log\left(\frac{5000}{30}\right)}{\log 2} = \boxed{7.38\text{ HR}}$

10. The following table represents an exponential function of the form $y = C\,a^x$. Find the value of a and b, and write the formula for the function in the form $y = C\,a^x$.
(Please show all the mathematical steps very clearly)

(2 points)

x	y
1	$\frac{5}{3}$
2	$\frac{5}{9}$
3	$\frac{5}{27}$
4	$\frac{5}{81}$
5	$\frac{5}{243}$

$* \frac{1}{3}$

$* 1/3$

$$\frac{\frac{5}{9} = \cancel{c}a^2}{\frac{5}{3} = \cancel{c}a^1} \Rightarrow \boxed{\frac{1}{3} = a}$$

$$\frac{5}{3} = c\left(\frac{1}{3}\right) \Rightarrow \boxed{c = 5}$$

$$\boxed{y = 5\left(\frac{1}{3}\right)^a}$$

11. Let $f(x) = (0.5)^x$ Evaluate f at the indicated values. (2 points)

a. $f(0)$	b. $f^{-1}(4096)$	c. Find x when $f(x) = \frac{1}{32}$
$f(0) = (0.5)^0 = 1$	$4096 = 0{,}5^x$ $\log 4096 = x \log 0.5$ $x = \frac{\log 4096}{\log 0.5} = -12$	$\frac{1}{32} = 0{,}5^x$ $\log(\frac{1}{32}) = x \log(0.5)$ $x = \frac{\log(1/32)}{\log(0.5)} = \boxed{5}$

12. Solve the following algebraically. (4 points)

a) $5(3)^x = 10935$

$$3^x = \frac{10935}{5}$$

$$x \log 3 = \log\left(\frac{10935}{5}\right)$$

$$x = \frac{\log\left(\frac{10935}{5}\right)}{\log 3} = \boxed{7}$$

b) $6(5)^x - 750 = 0$

$$6(5)^x = 750$$

$$5^x = \frac{750}{6}$$

$$x = \frac{\log(750/6)}{\log 5} = \boxed{3}$$

c) $2\ln(x) = 3$

$$\ln(x) = \frac{3}{2}$$

$$x = e^{3/2} \cong \boxed{4.48}$$

d) $2e^x - 3 = 0$

$$e^x = \frac{3}{2}$$

$$x = \ln\left(\frac{3}{2}\right) \cong \boxed{0.405}$$