

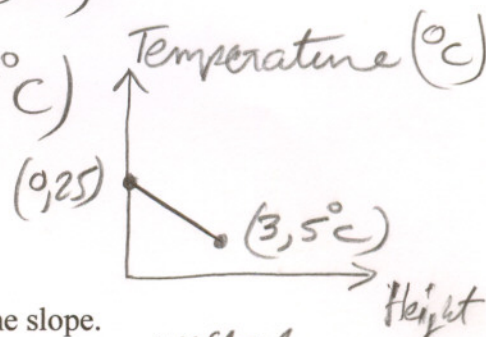
1. As dry air moves upward, it expands and cools. Suppose that the ground temperature is 25°C and the temperature at a height of 3 km is 5°C .

(4 POINTS)

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{change in output}}{\text{change in input}}$$

$(0, 25^{\circ}\text{C})$

$(3, 5^{\circ}\text{C})$



- a) You want to describe this situation with a function. Determine the input and output variables, draw and label two axes, then plot and label the two points you were given.

- b) Determine the slope of the line between your two points, and interpret the slope.

$$m = \frac{25 - 5}{0 - 3} = \frac{-20}{-3} = \frac{20}{3} \frac{^{\circ}\text{C}}{\text{km}}$$

For Every 1 km Altitude the temperature decreases by $\frac{20}{3} \frac{^{\circ}\text{C}}{\text{km}}$

- c) Use the point-slope form of the equation of a line $y - y_1 = m(x - x_1)$ or use the slope-intercept form of the equation of a line $y = mx + b$ to find a linear equation for your function.

$$y - 25 = -\frac{20}{3}(x - 0)$$

$$y = -\frac{20}{3}x + 25 \quad ^{\circ}\text{C}$$

- d) Use your equation to determine the air temperature at an elevation of 2.5 km.

$$y = -\frac{20}{3}(2.5) + 25 = 8.33 \quad ^{\circ}\text{C}$$

2. Assume that the population of bacteria doubles every hour. The colony of bacteria starts out with 1000 bacteria. Let $f(t)$ represent the population of bacteria at time t , where t is in hours.

(3 points)

- a. Find the formula for $f(t)$

$$f(t) = 1000(2)^t$$

- b. Predict when there will be 300,000 bacteria.

$$300000 = 1000(2)^t$$

$$300 = 2^t$$

$$\log 300 = t \log 2$$

$$t = \frac{\log 300}{\log 2} = 8.23 \text{ Hrs}$$

- Predict the population after 4 hours.

$$= 1000(2)^4 = 16000 \text{ Bacteria}$$

3. The following table represents an exponential function of the form $y = C a^x$.
Find the value of C and a , and write the formula for the function in the form $y = C a^x$.

(Please show all the mathematical steps very clearly)

(2 points)

x	y
1	42
2	6
3	6/7
4	6/49

$$a = \frac{1}{7}$$

$$42 = C \left(\frac{1}{7}\right)^1$$

$$C = 294 \Rightarrow$$

$$y = 294 \left(\frac{1}{7}\right)^x$$

4. Let $f(x) = (4)^x$ Evaluate f at the indicated values.

(2 points)

a. $f(0) = 4^0 = 1$

b. Find x when $f(x) = \frac{1}{64}$

$$\frac{1}{64} = 4^x$$

$$\log \frac{1}{64} = x \log 4 \Rightarrow x = \frac{\log(\frac{1}{64})}{\log 4}$$

$$x = -3$$

5. Let $f(x) = (0.2)^x$ Evaluate f at the indicated values.

(3 points)

a. $f(0) = 0.2^0 = 1$

b. $f^{-1}(3125)$

$$3125 = 0.2^x$$

$$\log 3125 = x \log 0.2$$

$$x = \frac{\log 3125}{\log 0.2} = -5$$

c. Find x when $f(x) = \frac{1}{625}$

$$\frac{1}{625} = 0.2^x$$

$$\log \left(\frac{1}{625}\right) = x \log 0.2$$

$$x = \frac{\log \left(\frac{1}{625}\right)}{\log(0.2)} = 4$$

6. Solve the following algebraically
(Please Round your answers to two decimal places)

(6 points)

a) $10(4)^x = 10930$

$$4^x = 1093$$

$$\log 4^x = \log 1093$$

$$x = \frac{\log 1093}{\log 4} = 5.05$$

b) $3(5)^x - 756 = 0$

$$3(5)^x = 756$$

$$5^x = \frac{756}{3} = 252$$

$$x = \frac{\log 252}{\log 5} = 3.44$$

c) $8 \ln(x) = 32$

$$\ln x = 4$$

$$x = e^4 = 54.60$$

d) $20e^x - 30 = 0$

$$20e^x = 30$$

$$e^x = \frac{3}{2}$$

$$x = \ln\left(\frac{3}{2}\right) = 0.41$$