

# Quiz 3

MONTGOMERY COLLEGE  
Department of Mathematics  
Rockville Campus

Solution  
By Dr. Katiraie

MA 160 Dr. Katiraie

Sections 2.3 - 2.4 (20 POINTS)

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

$$f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

1. To find the derivative of the function  $f(x) = x^2 - 8x + 9$  at  $x=2$  (4 points)

a. Find  $f(2) = 2^2 - 8(2) + 9 = 4 - 16 + 9 = -3$

b.  $f(2+h) = (2+h)^2 - 8(2+h) + 9$   
 $= 4 + 4h + h^2 - 16 - 8h + 9 = h^2 - 4h - 3$

- c. Find the average rate of change  $\frac{f(2+h) - f(2)}{h}$  and simplify

$$\frac{f(2+h) - f(2)}{h} = \frac{h^2 - 4h - 3 - (-3)}{h} = \frac{h(h-4)}{h} = h-4$$

- d. Take the limit as  $h$  approaches 0  $\lim_{h \rightarrow 0} (h-4) = -4$

2. Use any of the definitions of derivatives (provided on top of this page) to find the instantaneous rate of change of function

$f(t) = t^2$  at  $t = 7$

$t$	$f(t)$
7	49
$t$	$t^2$

(2 points)

$$\lim_{t \rightarrow 7} \frac{t^2 - 49}{t - 7} = \lim_{t \rightarrow 7} \frac{(t+7)(t-7)}{(t-7)} = 7+7 = 14$$

3. Use any of the definitions of derivatives (provided on top of this page) to find the instantaneous rate of change of function

$f(x) = 7x^2 - x$  at  $x=3$

(2 points)

$$\lim_{h \rightarrow 0} \frac{7(3+h)^2 - (3+h) - (7(3)^2 - 3)}{h} = \lim_{h \rightarrow 0} \frac{7(9+6h+h^2) - 3h - 63 + 3}{h}$$

$$\lim_{h \rightarrow 0} \frac{63 + 42h + 7h^2 - 3h - 63 + 3}{h} = \lim_{h \rightarrow 0} \frac{h(41+7h)}{h} = 41$$

4. Suppose it starts snowing at 6 PM. Interpret each of the following, where  $H(t)$  is the height of snow in inches as a function of time  $t$  in hours after 6 PM. (5 points)

(a)  $H(6) = 4$

At 12 midnight  
There is 4 inches of snow on the ground.

(b)  $H'(6) = 1.5$

At 12 midnight  
Snow is accumulating at a rate of 1.5 inches per hour.

(c)  $H(20) = 13$

at 2:00pm next day, there is 13 inches of snow on the ground

(d)  $H'(20) = 0$

At 2:00pm next day, it has stopped snowing

(e)  $H'(23) < 0$

at 5:00pm next day, snow is melting.

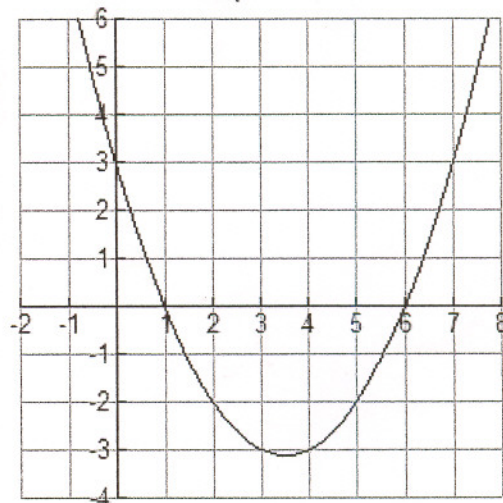
5. Let  $h(t)$  be a person's height in inches at age  $t$  years. Write a sentence, using appropriate units, explaining the meaning of each of the following. (2 points)

(a)  $h(12) = 56$  A 12 year old person is 56 inches tall.

(b)  $h'(12) = 2.5$  This 12 year old person is growing at a rate of 2.5 inches per year.

6. **Using the Graph of the Derivative:** The graph shown is the graph of  $f'$  the **derivative** of a function  $f$ . **Note that the graph of  $f$  is not shown.** If the function  $f$  is defined for all  $x$ , use this graph to answer the following questions. (5 points)

Graph of  $f'$



a. On what interval(s) is the function  $f$  increasing?

$$(-\infty, 1) \cup (6, \infty)$$

b. On what interval(s) is the function  $f$  decreasing?

$$(1, 6)$$

c. At what value(s) of  $x$ , if any, does  $f$  have a local maximum?

$$x = 1$$

d. At what value(s) of  $x$ , if any, does  $f$  have a local minimum?

$$x = 6$$

e. Suppose it is also known that  $f$  goes through the point  $(0, 0)$ . Based on all of the above information, sketch a possible graph of the function  $f$ .

