

Name:

Solutions

MA 160 Dr. Katiraie Quiz 4 Sections 3.1 - 3.3 (20 POINTS + 3PTS Extra Credit)

1. Find the derivative of each function. (3 Points)

(a) $f(x) = 6e^x - \frac{x^2}{3}$ $f(x) = 6e^x - \frac{1}{3}x^2$

$$f'(x) = 6e^x - \frac{2}{3}x$$

(b) $g(w) = \frac{w^4 - 8w^2 - 4}{w^2} = \frac{w^4}{w^2} - \frac{8w^2}{w^2} - \frac{4}{w^2} = w^2 - 8 - 4w^{-2}$

$$g'(w) = 2w + 8w^{-3}$$

$$g'(w) = 2w + \frac{8}{w^3}$$

- (c)
- $y = (3x^2)^4$
- (Hint: First simplify using properties of exponents.)

$$y = 81x^8$$

$$y' = 8 \cdot 81x^7 = 648x^7$$

2. If
- $f(x) = \frac{4}{\sqrt{x}}$
- find
- $f(x) = 4x^{-1/2}$
- (3 Points)

(a) $f'(x) = -\frac{1}{2} \cdot 4x^{-1/2-1}$

$$= -\frac{1}{2} \cdot 4x^{-3/2} = -2x^{-3/2} = \frac{-2}{\sqrt{x^3}}$$

(b) $f'(4) = \frac{-2}{\sqrt{4^3}} = \frac{-2}{\sqrt{64}} = \frac{-2}{8} = -\frac{1}{4}$

- (c) Write the equation of the tangent line to
- $f(x) = \frac{4}{\sqrt{x}}$
- at
- $x = 4$
- .

$$f(4) = \frac{4}{\sqrt{4}} = \frac{4}{2} = 2 \text{ and } m = -\frac{1}{4} \text{ and use } y - y_1 = m(x - x_1)$$

$$y - 2 = -\frac{1}{4}(x - 4) \Rightarrow y = -\frac{1}{4}x + 1 + 2 \Rightarrow y = -\frac{1}{4}x + 3$$

3. The equation of motion of a moving object is $s(t) = 6t^2 + 2t^{3/2}$ where $s(t)$ is measured in feet and t is the time in seconds. Find each of the following and **use appropriate units in your answers.**

(a) Find the velocity after 4 seconds

(2 Points)

$$s'(t) = 12t + 3t^{1/2}$$

$$s'(4) = 12(4) + 3(4)^{1/2} = 12(4) + 3(2) = 48 + 6 = 54 \frac{\text{ft}}{\text{sec}}$$

(b) Find the acceleration after 4 seconds

(2 Points)

$$a(t) = 12 + \frac{3}{2}t^{-1/2}$$

$$a(4) = 12 + \frac{3}{2}(4)^{-1/2} = 12 + \frac{3}{2} = 12.75 \text{ ft/sec}^2$$

4. The average price for a major league baseball game x years after 1990 can be modeled by $p(x) = 9.41 - 0.19x + 0.09x^2$.

(a) Use the model to find the instantaneous rate of change of the average ticket price in 2010. $x = 2010 - 1990 = 20$ (2 Points)

$$p'(x) = -0.19 + 2(0.09)x \quad p'(20) = -0.19 + 0.18(20) = \$3.41/\text{year}$$

(b) In a sentence, explain the meaning of your answer to part (a). **Use appropriate units.** (2 Points)

The average price for a major league baseball game in 2010 was increasing at \$3.41 per year.

5. Use the product rule to find the derivative.

(2 Points)

$$f(x) = x^3 e^x$$

$$f'(x) = 3x^2 e^x + x^3 e^x$$

$$= x^2 e^x (3 + x)$$

6. Differentiate the following using quotient rule and simplify your result. (2 Points)

$$f(x) = \frac{3x+1}{3x-1}$$

$$f'(x) = \frac{3(3x-1) - 3(3x+1)}{(3x-1)^2} = \frac{9x-3-9x-3}{(3x-1)^2}$$

$$= \frac{-6}{(3x-1)^2}$$

7. Find the equation of the tangent line to the function

$$f(x) = \frac{x^2+1}{x-1} \text{ when } x = 3 \text{ on the curve. (2 Points)}$$

at $x=3$ $y = \frac{3^2+1}{3-1} = \frac{10}{2} = 5$

$$y-5 = \frac{1}{2}(x-3)$$

$$y = \frac{1}{2}x - \frac{3}{2} + 5$$

$$f'(x) = \frac{2x(x-1) - 1(x^2+1)}{(x-1)^2}$$

$$f'(3) = \frac{2 \cdot 3(3-1) - (3^2+1)}{(3-1)^2} = \frac{12-10}{4} = \frac{2}{4} = \frac{1}{2}$$

$$y = \frac{1}{2}x + \frac{7}{2}$$

Extra Credit ☺

8. Suppose that the cost, in dollars, for a company to produce x pairs of a new line of jeans is $C(x) = 1700 + 4x + 0.09x^2 + 0.0007x^3$. (3 Points)

(a) Find the average cost function.

Please include Units

$$\frac{C(x)}{x} = \frac{1700 + 4x + 0.09x^2 + 0.0007x^3}{x} \text{ (dollars/Jeans)}$$

(b) Find $C'(100)$

Please include Units

$$C'(100) = 4 + 2(0.09)(100) + 3(0.0007)(100)^2 = 43 \text{ dollars/pair of jeans}$$

(c) What does this $C'(100)$ predict? Please pick one of the following:

- i) The approximate cost of the 101st pair of jeans
- ii) The exact cost of the 100th pair of jeans.
- iii) The approximate cost of the 100th pair of jeans.
- iv) The exact cost of the 99th pair of jeans.