MA160 Quiz 6 Form A Sections 4.2, 4.3 and 4.5 Name: Solutions
Dr. Katiraie

1. ***End behavior of a function***
Investigating the ***end behavior of a function*** means determining how the function behaves for x very large in the positive and negative directions, that is, determine $\lim_{x\to\infty} f(x)$ *and* $\lim_{x\to-\infty} f(x)$. We will only be considering the end behavior of a polynomial function.

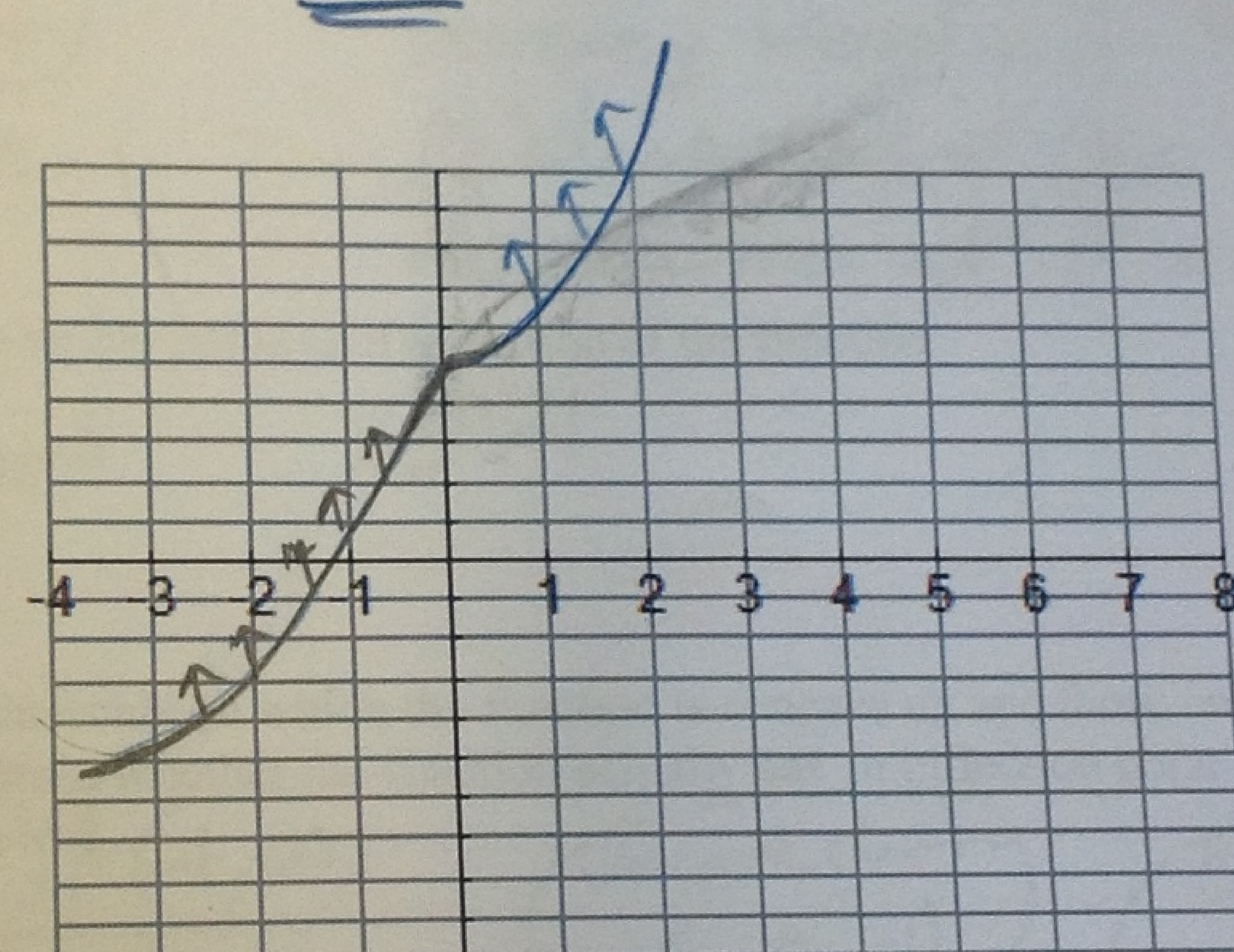
For a polynomial function, the end behavior is determined by the behavior of the term of highest degree.

In order to determine the end behavior of $f(x)=-3x^4-4x^2+7x+2$**, the end behavior is determined by the term** $-3x^4$**. Please answer the following end behavior questions:** **(1 Point)**

a) As $x\to\infty$, f(x) → $\boxed{-\infty}$

b) As $x\to-\infty$, f(x) → $\boxed{-\infty}$

2. Sketch the graph of a differentiable function which is increasing and concave up on $(-\infty,0)$ and increasing and concave up on $(0,\infty)$. **(2 Points)**



3. Suppose that a function f is differentiable for all x and $f(3)=6,\ f'(3)=0,\ and\ f''(3)=-1$. **(1 Point)**
Only one of the following is true. Which is it?

(a) f has a relative minimum at (3, 0)

(b) f has a relative minimum at (3, 6)

(c) f has a relative maximum at (3, 6)

(d) f has a relative maximum at (3, 0)

(e) f has a point of inflection at (3, 6)

(f) f has a point of inflection at (0, 6)

4. Let $f(x) = x^3 + 3x^2 - 24x$.

(a) Find $f'(x) = 3x^2 + 6x - 24$

Find $f''(x) = 6x + 6$ **(1 Point)**

(1 Point)

(b) List the critical values of the function. **(2 Point)**

$3x^2+6x-24 = 3(x^2+2x-8) = 0$

$3(x+4)(x-2) = 0$ $\quad x=-4 \quad x=2$

(c) Determine the intervals on which the function is increasing and those on which it is decreasing and then state whether the function has a relative maximum, a relative minimum, or neither at each critical value listed in part (b). **(1 Point)**

	$(-\infty,-4)$	$x=-4$ $(-4,2)$	$x=2$ $(2,\infty)$
$f'(x)=$	$+$	$-$	$+$
$f(x)=$	$\nearrow$	$\searrow$	$\nearrow$

Increasing Interval $= (-\infty,-4) \cup (2,\infty)$

Decreasing Interval $= (-4, 2)$

Relative Max at $(x=-4, 80)$

Relative min at $(x=2, -28)$

(d) Determine the possible point(s) of inflection of the function. **(1 Point)**

$6x+6=0 \quad x=-1$

(e) Determine the intervals on which the function is concave up and those on which it is concave down and then state whether the function actually has an inflection point at each point in part (d). **(2 Points)**

	$(-\infty,-1)$	$x=-1$ $(-1,\infty)$
$f''(x)$	$-$	$+$
$f(x)$	C.D	C.U

Concave down $(-\infty,-1)$

Concave up $(-1,\infty)$

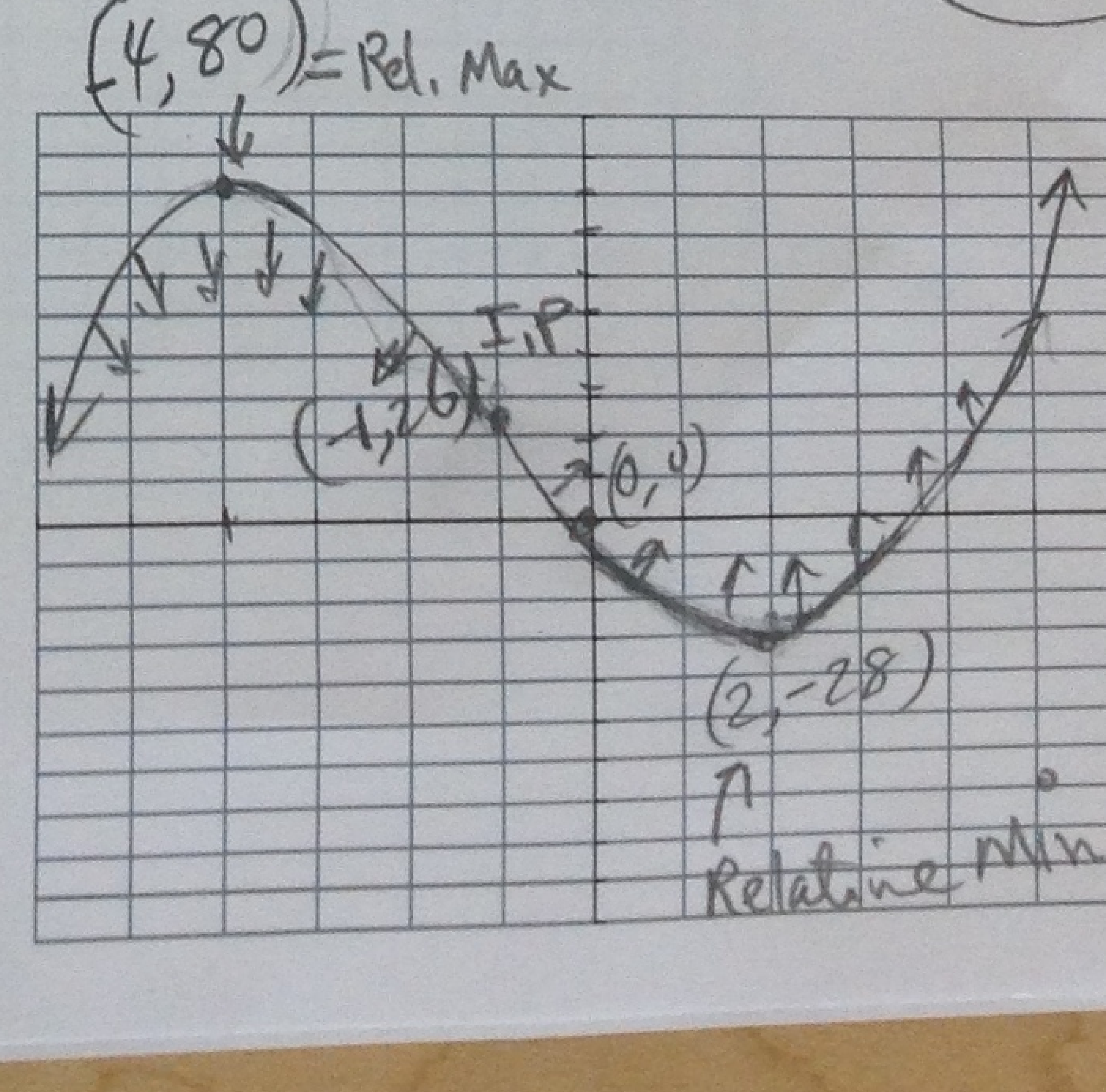
Inf. Point $(-1, 26)$

(f) Determine the end behavior of the function. **(1 Point)**

a) As $x \to \infty$, f(x) → $\boxed{\infty}$

b) As $x \to -\infty$, f(x) → $\boxed{-\infty}$

(g) Using the information above, sketch a graph of the function, showing any local extreme points and inflection points. **(2 Points)**



5. Sketch the graph of a differentiable function that satisfies all of the given conditions.

(5 Points)

$f'(0) = f'(2) = f'(4) = 0$

$f'(x) > 0$ *if* $x < 0$ or $2 < x < 4$

$f'(x) < 0$ *if* $0 < x < 2$ or $x > 4$

$f''(x) > 0$ *if* $1 < x < 3$

$f''(x) < 0$ *if* $x < 1$ or $x > 3$

		$X=1$	$X=3$
$f''(x)$	$-$	$+$	$-$
$f(x)$	CD	CU	C.D

	$X=0$	$X=2$	$X=4$
$f'(x) = +$	$-$	$+$	$-$
↗	↘	↗	↘