MA 160

Section 1.3 Linear Models and Rates of Change

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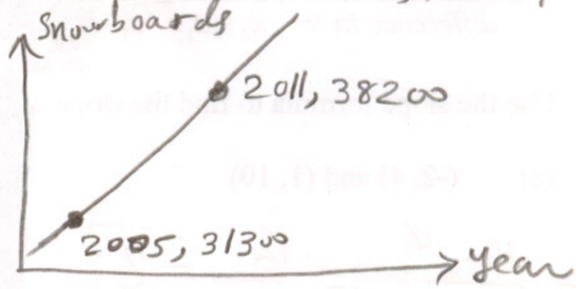
1. A company that produces snowboards has seen its annual sales increase linearly. In 2005, it sold 31,300 snowboards, and it sold 38,200 snowboards in 2011.

a) Suppose you want to describe this situation with a linear function. What two variables will you use? Which variable is the independent variable (input) and which variable is the dependent (output) variable?

Year is the input, No. of snowboards sold is the output

(2005, 31300) (2011, 38200)

b) Draw two axes, labeling them with your variable names from part a). You do not need to put a scale on your axes. Then plot and label the two points you were given, drawing a line between the two points.



c) Recall that the slope of a line between two points is given by $m = \dfrac{\Delta y}{\Delta x} = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{\text{change in output}}{\text{change in input}}$

Calculate the slope of your line, and interpret it as a rate of change.

$m = \dfrac{38200 - 31300}{2011 - 2005} = 1150$; Every year, the number of snowboard sales increases by 1150 snowboards.

2. As dry air moves upward, it expands and cools. Suppose that the ground temperature is 20°C and the temperature at a height of 1 km is 10°C.

a) You want to describe this situation with a function. Determine the input and output variables, draw and label two axes, then plot and label the two points you were given.

$(0, 20^\circ C)$

$(1 km, 10^\circ C)$

b) Determine the slope of the line between your two points, and interpret the slope.

$\dfrac{10-20}{1-0} = -\dfrac{10\ ^\circ C}{1 km}$; for every 1 km increase in height the temp drops $10^\circ C$

c) Use the point-slope form of the equation of a line $y - y_1 = m(x - x_1)$ or use the slope-intercept form of the equation of a line $y = mx + b$ to find a linear equation for your function.

$y - 20 = -10(x - 0) \Rightarrow y = -10x + 20$

$\boxed{T = -10x + 20}$

d) Use your equation to determine the air temperature at an elevation of 2.5 km.

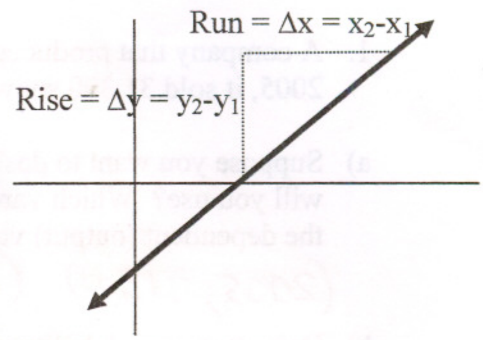
$T = -10(2.5) + 20$

$= -25 + 20$

$= \boxed{-5^\circ C}$

MA 160 Linear and Power Functions
Sections 1.3, 1.4

I. Slope and Linear Functions



1. The **slope** m of the line passing through the points (x_1, y_1) *and* (x_2, y_2) is defined by

$$m = \frac{Rise}{Run} = \frac{vertical\ change}{horizontal\ change} = \frac{\Delta y}{\Delta x}$$

$$= \frac{difference\ in\ y}{difference\ in\ x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2},\ where\ x_1 \neq x_2$$

Use the slope formula to find the slope of the line containing the points

(a) (-2, 4) and (1, 10)

$$m = \frac{10-4}{1--2} = \frac{6}{3} = \boxed{2}$$

(b) (-3, 4) and (-1, -5)

$$m = \boxed{\frac{-5-4}{-1--3} = \frac{-9}{2}}$$

2. You can use either of the following to find the equation of a line.

Slope-Intercept Equation	**Point-Slope Equation**
$y = mx + b\ \ or\ \ f(x) = mx + b$	$y - y_1 = m(x - x_1)$
The line has slope m and y-intercept $(0, b)$.	The line has slope m and contains the point (x_1, y_1).

Write an equation of the line through each pair of points in question #1 above. You may use either equation above, but write the answer in the form $y = mx + b$.

(a)

$y - 4 = 2(x - -2)$

$y - 4 = 2x + 4$

$\boxed{y = 2x + 8}$

(b)

$y - 4 = \frac{-9}{2}(x - -3)$

$y - 4 = \frac{-9}{2}x - \frac{27}{2}$

$y = \frac{-9}{2}x - \frac{27}{2} + 4$

$y = \frac{-9}{2}x - \frac{27}{2} + \frac{8}{2}$

$y = \frac{-9}{2}x - \frac{19}{2}$

3. The slope of a line is a measure of the rate at which a line is changing. For that reason, it is also called the **rate of change** of the line.

In 1991, the cost of tuition and fees at public two-year colleges was $800. This cost had increased to $1300 by 1996. Find the average rate of change of the cost of tuition and fees during this time period. **Write a sentence interpreting your answer. Be sure to use appropriate units.**

$$(1991, 800) \quad (1996, 1300) \Rightarrow m = \frac{1300 - 800}{1996 - 1991} = \frac{500}{5} = \frac{\$100}{\text{year}}$$

Every year the cost of tuition and fees at public two-year colleges increases by $100.

4. **High blood pressure in men**

Age of male	Percentage of males with high blood pressure
30	7.3
40	12.1
50	20.4
60	24.8
70	34.9

(a) Using the points (40, 12.1) and (60, 24.8) from the data above, find a linear function that fits the given data.

$$m = \frac{24.8 - 12.1}{60 - 40} = \frac{12.7}{20} = 0.635$$

$$y - y_1 = m(x - x_1)$$

$$y - 12.1 = 0.635(x - 40)$$

$$y = 0.635x - 25.4 + 12.1 \Rightarrow \boxed{y = 0.635x - 13.3}$$

(b) Use the function to estimate the percentage of 55-year-old men with high blood pressure.

$$y = 0.635(55) - 13.3$$

$y = 21.625$; 21.625 percent of 55 year old men have high blood pressure

II. Power Functions

A function of the form $f(x) = x^a$ is called a power function. In Calculus, we will often have to rewrite expressions involving negative exponents or radicals in the form $f(x) = Cx^a$, where C is a constant, in order to apply certain formulas to the function.

For example, each of the following functions can be rewritten in this form:

- $f(x) = \frac{1}{x^3}$ can be rewritten as $f(x) = x^{-3}$
- $f(x) = \frac{5}{x^4}$ can be rewritten as $f(x) = 5x^{-4}$
- $f(x) = \sqrt[3]{x^2}$ can be rewritten as $f(x) = x^{2/3}$
- $f(x) = -\frac{7}{\sqrt{x}}$ can be rewritten as $f(x) = -7x^{-1/2}$

Rewrite each function below in the form $f(x) = Cx^a$.

5. $f(x) = \frac{1}{x^8}$ $\boxed{= x^{-8}}$	6. $f(x) = \frac{3}{x^2}$ $\boxed{= 3x^{-2}}$	7. $f(x) = 6\sqrt{x}$ $\boxed{= 6x^{1/2}}$
8. $f(x) = \sqrt[3]{x^4}$ $\boxed{= x^{4/3}}$	9. $f(x) = -6\sqrt{x^3}$ $\boxed{= -6x^{3/2}}$	10. $f(x) = \frac{5}{\sqrt[4]{x}}$ $\boxed{= 5x^{-\frac{1}{4}}}$