MA 160
Section 1.1

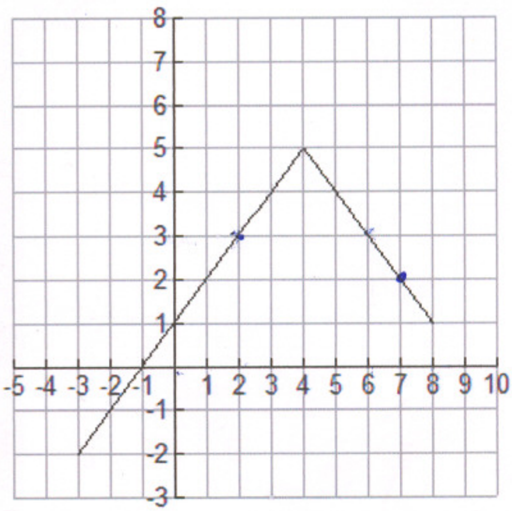
Functions and their Representations

I. Functions

1. The function $N(t)$ represents the number of U.S. cellular phone subscribers. $N(t)$ is measured in millions and t represents years since 2000. In a sentence, interpret the equation $N(4)=182$ in this context. Be sure to use appropriate units.

In the year 2004, there were 182 million U.S. cell phone subscribers.

2. Use the graph shown to answer the questions below.



(a) Find $f(7) = 2$

in 2007, there were 2 million U.S. cell phone subscribers.

(b) Find all x such that $f(x)=3$.

2 & 6

2002 & 2006

(c) State the domain of the function using interval notation.

$[0, 8]$

$[2000, 2008]$

(d) State the range of the function using interval notation.

$[1, 5]$
million million

3. If $f(x)=x^2-2x+5$, find and simplify

(a) f(3)	(b) f(-4)	(c) f(a + 3)
$3^2-2(3)+5$ $=9-6+5$ $=8$	$(-4)^2-2(-4)+5$ $=16+8+5$ $=29$	$(a+3)^2-2(a+3)+5$ $=a^2+6a+9-2a-6+5$ $=a^2+4a+8$

$$f(x) = x^2 - 2x + 5$$

(d) $\frac{f(a+h)-f(a)}{h}$ $= \frac{((a+h)^2 - 2(a+h) + 5) - (a^2 - 2a + 5)}{h}$

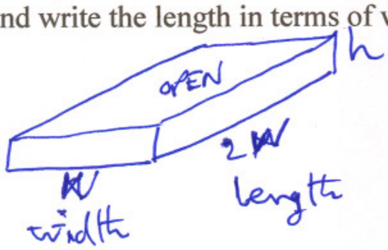
$= \frac{\cancel{a^2} + 2ah + h^2 \cancel{-2a} - 2h \cancel{+5} \cancel{-a^2} \cancel{+2a} \cancel{-5}}{h}$

$= \frac{2ah + h^2 - 2h}{h} = \frac{h(2a+h-2)}{h} = 2a+h-2$

4. Later in the semester, we will use the methods of Calculus to solve problems involving ***optimization;*** that is, finding the ***best*** way of doing something. Many of these problems involve geometric figures in which it is necessary to express one quantity as a function of another. Here is an example of such a problem.

A large rectangular box, open at the top, is to be constructed so that the volume is 20 cubic feet and the base has length equal to twice its width. We will seek a formula for the *surface area* of the box as a function of the width.

(a) First draw a picture of the box and label its dimensions. Use h for the height, w for the width, and write the length in terms of w.



(b) Write an expression in terms of h and w for the surface area S of the box. To do this, you will have to add up the areas of the bottom and the four sides of the box.

Area = Bottom + sides + side + side + side

$= w(2w) + (wh) + w(h) + 2wh + 2wh$

$= 2w^2 + 2wh + 4wh = 2w^2 + 6wh$

(c) Now consider the volume of the box. You are given that it is 20 ft^3. Use this fact to write an equation relating h and w and then solve for h in terms of w.

$w(2w)(h) = 20 \Rightarrow h = \frac{20}{2w^2} = \frac{10}{w^2}$

(d) Substitute the expression for h which you found in (c) into the expression for S in (b) and simplify the result. You should now have an expression ***for surface area as a function of w***. What is a reasonable domain for this function?

$S = 2w^2 + 6wh = 2w^2 + 6(w)\left(\frac{20}{2w^2}\right)$

$= 2w^2 + 6w\left(\frac{10}{w^2}\right)$

$= 2w^2 + \frac{60}{w}$