MA 160   Dr. Katiraie   The Derivative of the Natural Logarithm Function
Section 3.5

1. Consider the function $f(x) = \ln x$. So far, we have not found a formula for the derivative of this function. To find such a formula, recall that the natural logarithmic function can be rewritten as an exponential function.

   Since $f(x) = \ln x$ means $f(x) = \log_e x$, it can be rewritten in exponential form as $e^{f(x)} = x$. Differentiate this function, remembering to use the Chain Rule. Solve for $f'(x)$ and write your answer so that it is in terms of x only (not f(x)).

2. Use the formula that you developed above to differentiate each of the following functions and simplify your answer.

(a) $y = x(\ln x)$	(b) $y = \frac{\ln x}{x^2}$	(c) $y = (\ln x)^5$

3. Write the equation of the tangent line to the function $y = 5\ln x + 3x^2$ when x = 1 on the curve.

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4. Suppose we have a composite function of the form $h(x) = \ln(g(x))$. Let

$f(x) = \ln(x)$. Then $h(x) = f(g(x)) = \ln(g(x))$.

By the Chain Rule, the derivative of this function is

$$h'(x) = f'(g(x)) \cdot g'(x) = \frac{1}{g(x)} \cdot g'(x) = \frac{g'(x)}{g(x)}.$$

Thus, the derivative of the natural logarithm of a function is the derivative of the function divided by the function.

For example, if $y = \ln(x^2 + 3)$, then $y' = \frac{1}{x^2+3} \cdot \frac{d(x^2+3)}{dx} = \frac{1}{x^2+3} \cdot 2x = \frac{2x}{x^2+3}$

5. Use the formula from #4 to differentiate the following functions.

(a) $y = \ln(5x + 2)$	(b) $y = \ln(x^3 + 2)$

Selected answers:

2. (a) $1 + \ln x$ (b) $\frac{1 - 2\ln x}{x^3}$ (c) $\frac{5(\ln x)^4}{x}$

3. $y = 11x - 8$

5. (a) $\frac{5}{5x+2}$ (b) $\frac{3x^2}{x^3+2}$