

1. Consider the function $f(x) = \ln x$. So far, we have not found a formula for the derivative of this function. To find such a formula, recall that the natural logarithmic function can be rewritten as an exponential function.

Since $f(x) = \ln x$ means $f(x) = \log_e x$, it can be rewritten in exponential form as $e^{f(x)} = x$. Differentiate this function, remembering to use the Chain Rule. Solve for $f'(x)$ and write your answer so that it is in terms of x only (not $f(x)$).

$$f(x) = \ln x \Rightarrow \ln x = f(x) \Rightarrow x = e^{f(x)}$$

$$1 = e^{f(x)} f'(x)$$

$$\Rightarrow 1 = e^{\ln x} f'(x)$$

$$1 = x f'(x) \Rightarrow f'(x) = \frac{1}{x}$$

$$\boxed{(\ln x)' = \frac{1}{x}}$$

2. Use the formula that you developed above to differentiate each of the following functions and simplify your answer.

<p>(a) $y = x(\ln x)$</p> $y' = 1 \ln x + x \frac{1}{x}$ $= \ln x + 1$	<p>(b) $y = \frac{\ln x}{x^2}$</p> $y' = \frac{\frac{1}{x} \cdot x^2 - 2x \ln x}{(x^2)^2}$ $y' = \frac{x - 2x \ln x}{x^4}$	<p>(c) $y = (\ln x)^5$</p> $y' = 5(\ln x)^4 \left(\frac{1}{x}\right)$ $y' = \frac{5(\ln x)^4}{x}$
$y' = \frac{x(1 - 2 \ln x)}{x^3} = \frac{1 - 2 \ln x}{x^3}$		

3. Write the equation of the tangent line to the function $y = 5 \ln x + 3x^2$ when $x = 1$ on the curve.

$$y' = 5 \frac{1}{x} + 6x \quad @ \quad x = 1 \Rightarrow y' = \frac{5}{1} + 6 = 11$$

$$y = 5 \ln 1 + 3(1)^2 = 3$$

$$y - 3 = 11(x - 1)$$

$$y = 11x - 11 + 3$$

$$\boxed{y = 11x - 8}$$

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By Dr. KATIRAI

4. Suppose we have a composite function of the form $h(x) = \ln(g(x))$. Let $f(x) = \ln(x)$. Then $h(x) = f(g(x)) = \ln(g(x))$.

By the Chain Rule, the derivative of this function is

$$h'(x) = f'(g(x)) \cdot g'(x) = \frac{1}{g(x)} \cdot g'(x) = \frac{g'(x)}{g(x)}$$

Thus, the derivative of the natural logarithm of a function is the derivative of the function divided by the function.

For example, if $y = \ln(x^2 + 3)$, then $y' = \frac{1}{x^2 + 3} \cdot \frac{d(x^2 + 3)}{dx} = \frac{1}{x^2 + 3} \cdot 2x = \frac{2x}{x^2 + 3}$

5. Use the formula from #4 to differentiate the following functions.

<p>(a) $y = \ln(5x + 2)$</p> $y' = \frac{1}{5x+2} \cdot 5 = \frac{5}{5x+2}$	<p>(b) $y = \ln(x^3 + 2)$</p> $y' = \frac{1}{x^3+2} \cdot (3x^2) = \frac{3x^2}{x^3+2}$
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Selected answers:

2. (a) $1 + \ln x$ (b) $\frac{1 - 2 \ln x}{x^3}$ (c) $\frac{5(\ln x)^4}{x}$

3. $y = 11x - 8$

5. (a) $\frac{5}{5x+2}$ (b) $\frac{3x^2}{x^3+2}$