MA 160 Exponential Decay Solutions By Dr. KATIRAIE

Section 3.6

There are many situations in which the amount of a substance is ***decaying*** (that is, ***decreasing***) at a rate proportional to the amount present. In this case, the differential equation describing the situation is still $A'(t) = kA(t)$, but k will be negative since a function that is decreasing must have a negative derivative.

1. (p. 201 #21) The radioactive substance Bismuth-210 has a half-life of 5.0 days.

(a) A sample originally has a mass of 800 mg. Find a formula for the mass remaining after t days.

$A(t) = Ce^{kt}$

$400 = 800e^{5k}$

$\frac{1}{2} = e^{5k} \Rightarrow \ln(1/2) = \cancel{\ln} e^{5k}$

$\Rightarrow 5k = \ln(\frac{1}{2}) \Rightarrow \boxed{k = \frac{\ln(1/2)}{5}}$

$\boxed{A(t) = 800e^{\frac{\ln(1/2)}{5}t}}$

(b) Find the mass remaining after 30 days.

$A(30) = 800e^{\frac{\ln(1/2)}{5}(30)} = \boxed{12.5 \text{ mg}}$

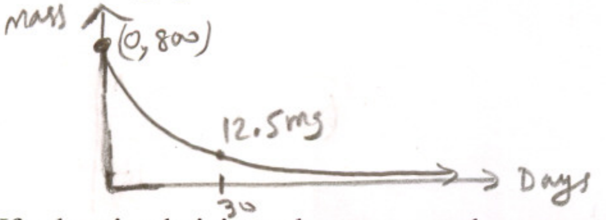
(c) When is the mass reduced to 10 mg?

$10 = 800e^{\frac{\ln(1/2)}{5}t}$

$\frac{10}{800} = e^{\frac{\ln(1/2)}{5}t} \Rightarrow \ln(\frac{10}{800}) = \cancel{\ln} e^{\frac{\ln(1/2)}{5}t}$

$t = \frac{\ln(\frac{10}{800})}{\left(\frac{\ln(1/2)}{5}\right)} = \boxed{31.61 \text{ Days}}$

(d) Sketch the graph of the mass function.



2. If a drug is administered to a person, the amount of the drug in the person's body decays at a rate proportional to the amount of the drug present. Suppose that 10 mg of a drug are given to a patient and that 8 mg of the drug are left in the body after 3 hours.

(0, 10 mg)

(3, 8mg)

(a) Find an exponential function of the type $A(t) = Ce^{kt}$ that models this situation.

$A(t) = 10e^{kt}$

$8 = 10e^{k(3)} \Rightarrow 0.8 = e^{3k} \Rightarrow \ln(0.8) = 3k$

$k = \frac{\ln(0.8)}{3} \Rightarrow \boxed{A(t) = 10e^{\frac{\ln(0.8)}{3}t}}$

(b) How many mg of the drug will remain in the person's body after 11 hours?

$A(11) = 10e^{\frac{\ln(0.8)}{3}(11)} = \boxed{4.41 \text{ mg}}$

3. **Carbon-14 dating** is a way of determining the age of certain archeological artifacts of a biological origin.

Cosmic rays enter the earth's atmosphere in large numbers every day. These rays convert nitrogen to carbon-14, ^{14}C, which is a radioactive substance with a half-life of about 5,730 years. The ^{14}C atoms that cosmic rays create combine with oxygen to form carbon dioxide, which plants absorb naturally and incorporate into plant fibers by photosynthesis. Animals and people eat plants and take in ^{14}C as well. The ratio of normal carbon to ^{14}C in the air and in all living things at any given time is nearly constant. The ^{14}C atoms are always decaying, but they are being replaced by new ^{14}C atoms at a constant rate. All living plants and animals have the same percentage.

As soon as a living organism dies, it stops taking in new carbon. The ratio of normal carbon to ^{14}C at the moment of death is the same as every other living thing, but the ^{14}C decays and is not replaced. The ^{14}C decays with its half-life of 5,730 years, while the amount of normal carbon remains constant in the sample. By looking at the ratio of normal carbon to ^{14}C in the sample and comparing it to the ratio in a living organism, it is possible to determine the age of a formerly living thing fairly precisely.

Example: (p. 201 #26) In 1991 two mountain hikers discovered a human body frozen in ice. It was soon discovered that the man, dubbed "the iceman," had been in the ice for quite some time. Tissue samples indicated that his body had 57.67% of the ^{14}C that is present in a living person. How long ago did the iceman die?

(a) The first step is to find the decay constant for ^{14}C.

$$A(t) = Ce^{Kt}$$

$$\frac{1}{2} = 1e^{K(5730)} \Rightarrow \ln(1/2) = \ln e^{5730K} \Rightarrow K = \frac{\ln(1/2)}{5730}$$

$$A(t) = 1e^{\frac{\ln(0.5)}{5730}t}$$

$$0.5767 = 1e^{\frac{\ln(0.5)t}{5730}} \Rightarrow t = \frac{\ln(0.5767)}{[(\ln 0.5)/5730]} = 4550.23 \text{ years}$$

(b) Now we can use the decay constant for carbon-14 and the fact that the amount of ^{14}C in the fossil is 57.67% compared to a living person.

$$0.5767 = 1e^{\frac{\ln(0.5)t}{5730}}$$

$$\ln\left(\frac{0.5767}{1}\right) = \ln e^{\frac{\ln(0.5)t}{5730}}$$

$$\ln(0.5767) = \frac{\ln(0.5)t}{5730}$$

$$t = \frac{\ln(0.5767)}{[\ln(0.5)/5730]} = \boxed{4550.23 \text{ years}}$$