MA 160
Section 2.3

Interpreting the Derivative Dr Katirane

1. Suppose it starts snowing at 12 noon. Interpret each of the following, where H(t) is the height of snow in inches as a function of time t in hours after 12 noon.

(a) $H(6)=4$ At 6 PM there is 4 inches of snow on the ground.	(b) $H'(6)=1.5$ At 6pm the amount of snow on the ground is accumulating at a rate of 1.5 inches per hour.
(c) $H(20)=13$ at 8AM next day, there is 13 inches of snow on the ground	(d) $H'(20)=0$ At 8 AM Next day, It has stopped snowing.
(e) $H'(23)<0$ At 11 AM Next day, snow is melting	

2. Let $S(t)$ be a child's distance from home as a function of time. Is $S'(t)$ positive, negative or zero if:

(a) The child is at home. $S'(t)=0$	(b) The child is going to school. $S'(t)$ is positive
(c) The child is at school. $S'(t)=0$	(d) The child is coming home. $S'(t)$ is Negative

3. Let $h(t)$ be a person's height in inches at age t years. Write a sentence, using appropriate units, explaining the meaning of each of the following.

(a) $h(12)=56$ This 12 year old person has a height of 56 inches

(b) $h'(12)=2.5$ This 12 year old person is growing at a rate of 2.5 inches per year.

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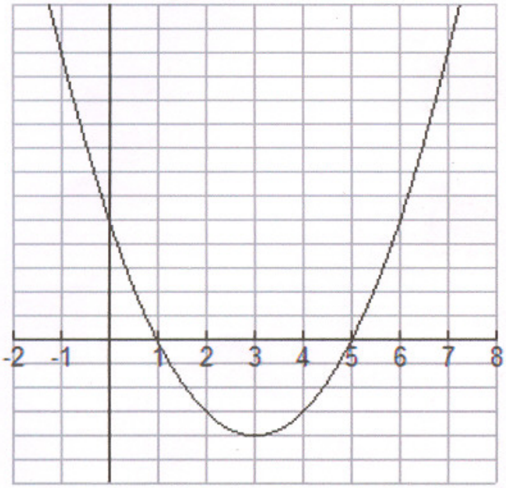
4. The graph of the function $f(x)$ is shown. For each value x = a in the chart below, indicate whether $f(a)$ is positive, negative, or zero, and whether $f'(a)$ is positive negative or zero.



a	$f(a)$	$f'(a)$
-1	positive	negative
0	Positive	Negative
1	Zero	Negative
3	Negative	Zero
5	Zero	Positive
6	Positive	Positive

5. (Based on p. 111/ #53) The table shows the estimated percentage P of the population of Brazil that are mobile-phone subscribers. (End of year estimates are given.)

Year	1997	1999	2001	2003	2005	2007
P	2.7	8.8	16.3	25.6	46.3	63.1

(a) Estimate the instantaneous rate of of growth in 2003 by taking the average of the two average rates of change of P from 2003 to 2005 and from 2001 to 2003. What are the units?

from 2003 to 2005

$$\text{Avg Rate} = \frac{46.3-25.6}{2005-2003} = 10.35$$

from 2001 to 2003

$$\text{Avg Rate} = \frac{25.6-16.3}{2003-2001} = 4.65$$

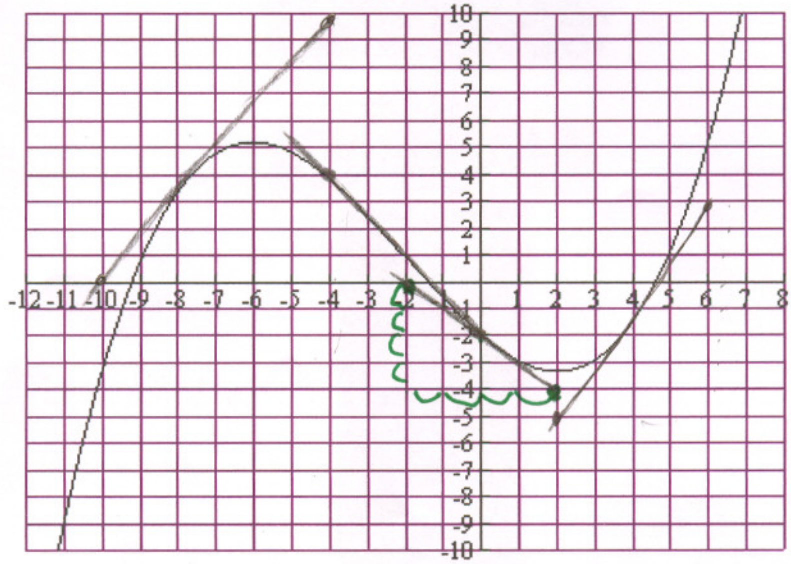
$$\text{Avg of Above two} = \frac{10.35+4.65}{2} = 7.5 \ \frac{\text{Percent of Population}}{\text{year}}$$

(b) Estimate the instantaneous rate of of growth in 2003 by finding the average rate of change of P from 2001 to 2005.

$$= \frac{46.3-16.3}{2005-2001} = 7.5 \ \frac{\text{Percent of Population that are Mobile Phone Subscribers}}{\text{year}}$$

I. Graphing the Derivative

1. The graph of the function f is shown below.

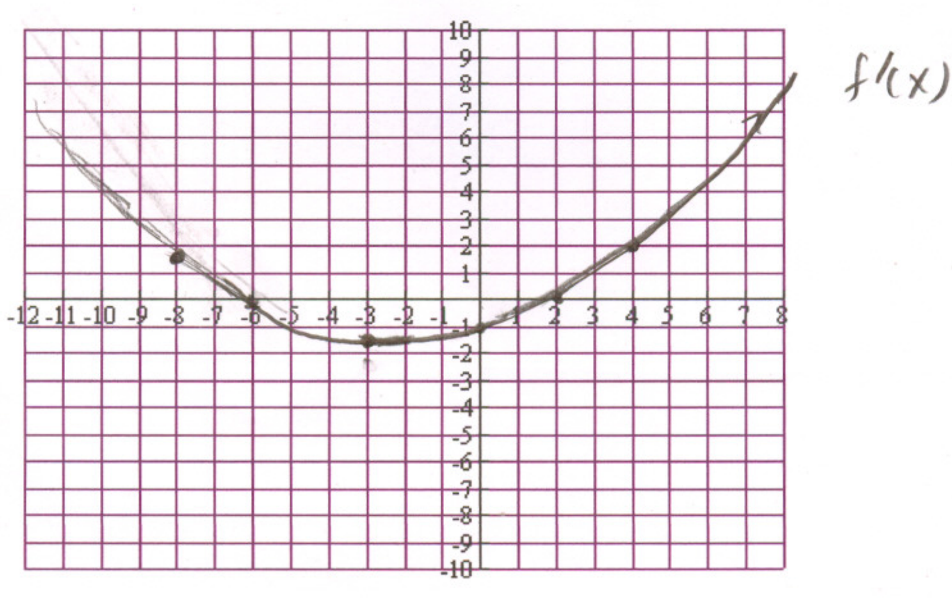


(a) Draw tangent lines and graphically estimate

(i) $f'(-8) = \frac{10}{6} = \frac{5}{3}$ (ii) $f'(-6) = 0$ (iii) $f'(-3) = \frac{-6}{4} = \frac{-3}{2}$

(iv) $f'(0) = \frac{-4}{4} = -1$ (v) $f'(2) = 0$ (vi) $f'(4) = \frac{8}{4} = 2$

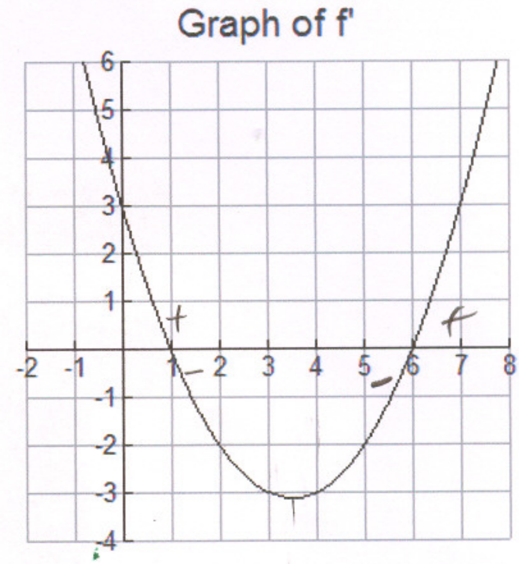
(b) Sketch a possible graph for $f'(x)$.



II. Using the Graph of the Derivative

The graph shown is the graph of f', the ***derivative*** of a function f. Note that the graph of f is not shown.

If the function f is defined for all x, use this graph to answer the following questions.



1. On what interval(s) is the function f increasing?

$(-\infty,1) \cup (6,\infty)$

2. On what interval(s) is the function f decreasing?

$(1,6)$

3. At what value(s) of x, if any, does f have a local maximum?

$x=1$

4. At what value(s) of x, if any, does f have a local minimum?

$x=6$

Suppose it is also known that f goes through the point (0,0). Based on all of the above information, sketch a possible graph of the function f.