Solutions

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MA 160 Shortcuts to Finding Derivatives

Section 3.1

1. Find the derivative of each function below and write your answers with no negative exponents.

(a) $f(x) = x^3 - 5x^2 + 11x - 6$

$f'(x) = \boxed{3x^2 - 10x + 11}$

(b) $g(x) = \frac{6}{x^3} = 6x^{-3}$

$g'(x) = -18x^{-4} = \boxed{\frac{-18}{x^4}}$

(c) $y = (x+4)(x-7)$ Hint: First expand the expression.

$y = (x^2 - 7x + 4x - 28) = x^2 - 3x - 28$

$y' = \boxed{2x - 3}$

(d) $f(x) = 8e^x + \frac{x^2}{5} = 8e^x + \frac{1}{5}x^2$

$f'(x) = 8e^x + \frac{2}{5}x^1 = \boxed{8e^x + \frac{2}{5}x}$

(e) $g(w) = \frac{w^4 - 5w^2 - 3}{w^2}$ (Hint: First write the expression as three separate fractions.) $= \frac{w^4}{w^2} - \frac{5w^2}{w^2} -$

$g(w) = w^2 - 5 - 3w^{-2}$

$g'(w) = 2w - 0 + 6w^{-3} = \boxed{2w + \frac{6}{w^3}}$

(f) $y = (2x^2)^3$ (Hint: First simplify using properties of exponents.)

$y = 8x^6$

Recall $(2x^2)^3 = 2^3(x^2)^3 = 8x^6$

$\boxed{y' = 48x^5}$

2. If $f(x) = 4\sqrt{x} - \frac{2}{\sqrt{x}}$, find $\Rightarrow f(x) = 4x^{\frac{1}{2}} - 2x^{-\frac{1}{2}}$

(a) $f'(x)$

$f'(x) = 2x^{-\frac{1}{2}} + 1x^{-\frac{3}{2}}$

$\boxed{f'(x) = \frac{2}{\sqrt{x}} + \frac{1}{\sqrt{x^3}}}$

(b) $f'(4) = \frac{2}{\sqrt{4}} + \frac{1}{\sqrt{2^3}} = \boxed{\frac{9}{8}}$

OVER→

3. Write the equation of the tangent line to $f(x) = x^4 - 3x^3 + 7x - 8$ when x = 2 on the curve.

$y' = 4x^3 - 9x^2 + 7$

at $x = 2 \Rightarrow y' = 4(2)^3 - 9(2)^2 + 7 = 32 - 36 + 7 = 3$

at $x = 2$ $y = 2^4 - 3(2)^3 + 7(2) - 8 = 16 - 24 + 14 - 8 = -2$

$y - y_1 = m(x - x_1) \Rightarrow y - -2 = 3(x - +2)$

$y + 2 = 3x - 6 \Rightarrow \boxed{y = 3x - 8}$

4. The equation of motion of a moving object is $s = 2t^2 + t^{3/2}$, where a is measured in feet and t is the time in seconds. Find each of the following and use appropriate units in your answers.

(a) The velocity after 4 seconds

$s'(t) = 4t + \frac{3}{2}t^{\frac{1}{2}}$

$s'(4) = 4(4) + \frac{3}{2}(4)^{\frac{1}{2}} = 16 + 3$

$= \boxed{19 \text{ ft/sec}}$

(b) The acceleration after 4 seconds

$s''(t) = 4 + \frac{3}{4}t^{-\frac{1}{2}}$

$s''(4) = 4 + \frac{3}{4}(4)\wedge(-1 \div 2)$

$= \boxed{4.375 \text{ ft/sec}^2}$

5. The average price for a major league baseball game x years after 1990 can be modeled by $p(x) = 9.41 - 0.19x + 0.09x^2$.

(a) Use the model to find the instantaneous rate of change of the average ticket price in 2007.

$t = 2007 - 1990 = 17$

$P'(x) = -0.19 + 2 * 0.09x = -0.19 + 0.18(17) = \$2.87/\text{year}$

(b) In a sentence, explain the meaning of your answer to part (a). Use appropriate units.

Please see below

Answers

1 (a) $3x^2 - 10x + 11$ (b) $-\frac{18}{x^4}$ (c) $2x - 3$ (d) $8e^x + \frac{2}{5}x$ (e) $2w + \frac{6}{w^3}$ (f) $48x^5$

2 (a) $\frac{2}{\sqrt{x}} + \frac{1}{\sqrt{x^3}}$ (b) $\frac{9}{8}$ 3. $y = 3x - 8$ 4 (a) 19 ft/sec (b) $4.375\, ft/\sec^2$

5(a) \$2.87/year

(b) In 2007, the average ticket price of a major league baseball ticket was increasing at a rate of \$2.87 per year.