MA 160 Dr. Katiraie **USING THE PRODUCT AND QUOTIENT RULES**
Section 3.3

Solutions by Dr. Katiraie

1. Use the product rule to find each derivative.

(a) $f(x) = (3x-1)(2x+5)$ $f'(x) = 3(2x+5) + 2(3x-1)$ $= 6x + 15 + 6x - 2$ $= \boxed{12x + 13}$	(c) $f(x) = (x^3+5)(3x^2-7)$ $f'(x) = 3x^2(3x^2-7) + 6x(x^3+5)$ $= 9x^4 - 21x^2 + 6x^4 + 30x$ $= \boxed{15x^4 - 21x^2 + 30x}$
(b) $f(x) = x^3 e^x$ $f'(x) = \boxed{3x^2 e^x + x^3 e^x}$	(d) $f(x) = (2\sqrt{x}-3)(x^3 - 5e^x)$ Note: Just use the product rule; do not simplify your answer. $f'(x) = x^{-\frac{1}{2}}(x^3 - 5e^x) + (3x^2 - 5e^x)(2\sqrt{x} - 3)$ $= \boxed{\dfrac{x^3 - 5e^x}{\sqrt{x}} + (3x^2 - 5e^x)(2\sqrt{x} - 3)}$

2. If f is a differentiable function of x and $g(x) = \sqrt[3]{x}\, f(x)$,

(a) Find an expression for the derivative of $g(x)$ in terms of $f(x)$ *and* $f'(x)$.

$$g'(x) = \frac{1}{3} x^{-\frac{2}{3}} f(x) + x^{1/3} f'(x)$$

(b) If it is known that $f(8) = 12$ *and* $f'(8) = 5$, find $g'(8)$.

$$g'(8) = \frac{1}{3}(8)^{-\frac{2}{3}} f(8) + (8)^{1/3}(f'(8))$$

$$= \frac{1}{3} \frac{1}{\sqrt[3]{8^2}}(12) + \sqrt[3]{8}\,(5) = \boxed{11}$$

$$\boxed{g'(8) = 11}$$

OVER →

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

3. Differentiate each quotient and simplify your results.

(a) $f(x)=\dfrac{3x+1}{3x-1}$ $f'(x)=\dfrac{3(3x-1)-3(3x+1)}{(3x-1)^2}$ $f'(x)=\dfrac{\cancel{9x}-3-\cancel{9x}-3}{(3x-1)^2}=\boxed{\dfrac{-6}{(3x-1)^2}}$	(c). $f(x)=\dfrac{x^2+8x+7}{\sqrt{x}}$ $f'(x)=\dfrac{(2x+8)\sqrt{x}-\frac{1}{2}x^{-\frac{1}{2}}(x^2+8x+7)}{(\sqrt{x})^2}$ $=\dfrac{2x^{3/2}+8\sqrt{x}-\frac{1}{2}x^{3/2}-4x^{\frac{1}{2}}-\frac{7}{2}x^{-\frac{1}{2}}}{x}$ $=\dfrac{1.5x^{3/2}+4\sqrt{x}-3.5x^{-1/2}}{x}=$
(b) $f(x)=\dfrac{x}{x^2+1}$ $f'(x)=\dfrac{1(x^2+1)-2x(x)}{(x^2+1)^2}$ $=\dfrac{x^2+1-2x^2}{(x^2+1)^2}=\boxed{\dfrac{-x^2+1}{(x^2+1)^2}}$	$=\boxed{\frac{3}{2}x^{1/2}+\frac{4}{\sqrt{x}}-\frac{7}{2x^{3/2}}}$ (d) $f(x)=\dfrac{x^2+x-2}{x^2+5}$ $f'(x)=\dfrac{(2x+1)(x^2+5)-2x(x^2+x-2)}{(x^2+5)^2}$ $=\dfrac{\cancel{2x^3}+10x+1x^2+5-\cancel{2x^3}-2x^2+4x}{(x^2+5)^2}$ $=\boxed{\dfrac{-x^2+14x+5}{(x^2+5)^2}}$

4. Find the equation of the tangent line to the function $f(x)=\dfrac{x^2+1}{x-1}$ when x = 3 on the curve.

$f'(x)=\dfrac{2x(x-1)-1(x^2+1)}{(x-1)^2}$ at $x=3 \Rightarrow f'(3)=\dfrac{2(3)(3-1)-1(3^2+1)}{(3-1)^2}=\dfrac{2}{4}$

$\therefore m=\frac{1}{2}$

& $f(3)=\dfrac{3^2+1}{3-1}=\dfrac{10}{2}=5$

So, equation of tangent line should be $y-5=\frac{1}{2}(x-3)$

$y=\frac{1}{2}x-\frac{3}{2}+5 \Rightarrow \boxed{y=\frac{1}{2}x+\frac{7}{2}}$

Answers

1. (a) $(3x-1)2+(2x+5)3=12x+13$ (b) $x^3e^x+e^x3x^2=x^2e^x(x+3)$ (c) $(x^3+5)(6x)+(3x^2-7)(3x^2)$ $=15x^4-21x^2+30x=3x(5x^3-7x+10)$ (d) $(2\sqrt{x}-3)(3x^2-5e^x)+\dfrac{x^3-5e^x}{\sqrt{x}}$	2. (a) $x^{1/3}f'(x)+\dfrac{f(x)}{3x^{2/3}}$ (b) 11 3. (a) $-\dfrac{6}{(3x-1)^2}$ (b) $\dfrac{1-x^2}{(x^2+1)^2}$	3. (c) $\dfrac{3}{2}x^{1/2}+\dfrac{4}{x^{1/2}}-\dfrac{7}{2x^{3/2}}$ (d) $\dfrac{-x^2+14x+5}{(x^2+5)^2}$ 4. $y=\dfrac{1}{2}x+\dfrac{7}{2}$