Solutions by Dr. Katiraie

MA 160 Using Derivatives to Find Absolute Maximum and Minimum Values
Section 4.2

The Extreme-Value Theorem states that if f is continuous over a closed interval [a, b], then f must have an absolute maximum value and an absolute minimum value over [a, b].

If the hypotheses of the Extreme Value Theorem are satisfied, then the absolute maximum and absolute minimum of the function can be found by using the following procedure.

1. Find the absolute maximum and minimum values of $f(x) = \frac{x^4}{4} + \frac{4}{3}x^3 - 6x^2 + 10$ on the interval [-7, 3].

(a) Find all critical numbers of f(x) in the given interval.

$f'(x) = 4 \cdot \frac{1}{4}x^3 + 3 \cdot \frac{4}{3}x^2 - 12x = x^3 + 4x^2 - 12x$

let $f'(x) = 0 \implies x^3 + 4x^2 - 12x = 0$

factor x $\implies x(x^2 + 4x - 12) = 0$

factor again $x(x+6)(x-2) = 0 \implies$ $x = 0$, $x = -6$, $x = +2$

(b) Make a chart showing the value of the function at each critical number listed in part (a) and at the endpoints of the given interval.

	[-7, -6)	-6	(-6, 0)	0	(0, 2)	2	(2, 3]
f'	−		+		−		+
f(x)	↘		↗		↘		↗

x	f(x)
-7	-141.08
-6	-170
0	10
2	$\frac{2}{3}$
3	12.25

(c) The largest y- value in part (b) is the absolute maximum and the smallest is the absolute minimum.

The absolute maximum of f over the interval [-7, 3] is 12.25.

The absolute minimum of f over the interval [-7, 3] is -170.

OVER→

2. Use the procedure outlined in problem #1 to find the absolute maximum and minimum values of $f(x) = \frac{x^4}{4} + \frac{4}{3}x^3 - 6x^2 + 10$ on the interval [-1, 3]. Note: This is the same function as in #1 but the interval is different.

$f'(x) = x^3 + 4x^2 - 12x$

$x(x^2 + 4x - 12) = 0$

$x(x+6)(x-2) = 0$

$x = 0 \quad x = -6 \quad x = 2$

	$(-1,0)$	$(0,2)$	$(2,3]$
		$x=0$	$x=2$
$f'(x)$	+	−	+
$f(x)$	↗	↘	↗

x	f(x)
−1	2.92
0	10
2	$\frac{2}{3}$
3	12.25

$\boxed{\text{Absolute Max} = 12.25 \quad \text{Absolute Min} = \frac{2}{3}}$

3. Use the procedure outlined in problem #1 to find the absolute maximum and minimum values of $g(x) = x^{2/3}$ on the interval [-8, 1].

$g(x) = x^{\frac{2}{3}}$

$g'(x) = \frac{2}{3}x^{\frac{-1}{3}} = \frac{2}{3x^{1/3}} \Rightarrow$ critical number $x = 0$

	$[-8,0)$	$(0,1]$
$g'(x)$	−	+
$g(x)$	↘	↗

x	g(x)
−8	4
0	0
1	1

$\boxed{\text{Absolute Max} = 4 \quad \text{Absolute min} = 0}$