MA 160 The Derivative of the Natural Logarithm Function
Section 4.5

1. Consider the function $f(x) = \ln x$. So far, we have not found a formula for the derivative of this function. To find such a formula, rewrite $f(x) = \ln x$ in exponential form as follows:

$$f(x) = \ln x$$
$$e^{f(x)} = e^{\ln x}$$
$$e^{f(x)} = x$$

Now differentiate $e^{f(x)} = x$, remembering to use the Chain Rule. Solve for $f'(x)$ and write your answer so that it is in terms of x only (not $f(x)$).

2. Use the formula that you developed above to differentiate each of the following functions and simplify your answer.

(a) $y = x(\ln x)$	(b) $y = \dfrac{\ln x}{x^2}$	(c) $y = (\ln x)^5$

3. Write the equation of the tangent line to the function $y = 5\ln x$ when x = 1 on the curve.

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4. Use the Chain Rule to extend the formula from #1 above to find the derivative of $f(x) = \ln[g(x)]$.

5. Use the formula from #4 to differentiate the following functions.

(a) $y = \ln(5x+2)$	(b) $y = \ln(x^3+2)$

6. Write the equation of the tangent line to $y = \ln(4x+e^2)$ at x = 0 on the curve. Use exact values, not approximations, in your answer.

7. For what value or values of x does $y = \dfrac{\ln x}{x^2}$ have a horizontal tangent line? Use exact values, not approximations, in your answer.

Selected answers:

2. (a) $1+\ln x$ (b) $\dfrac{1-2\ln x}{x^3}$ (c) $\dfrac{5(\ln x)^4}{x}$ 3. $y = 5x-5$

5. (a) $\dfrac{5}{5x+2}$ (b) $\dfrac{3x^2}{x^3+2}$ 6. $y = \dfrac{4}{e^2}x+2$ 7. $x = \sqrt{e}$