MA 160
Section 5.1

Cost and Area

Solutions by Dr. Katiraie

I. Estimating Cost from Marginal Cost

1. The table below shows the marginal cost for a company's product at various production levels.

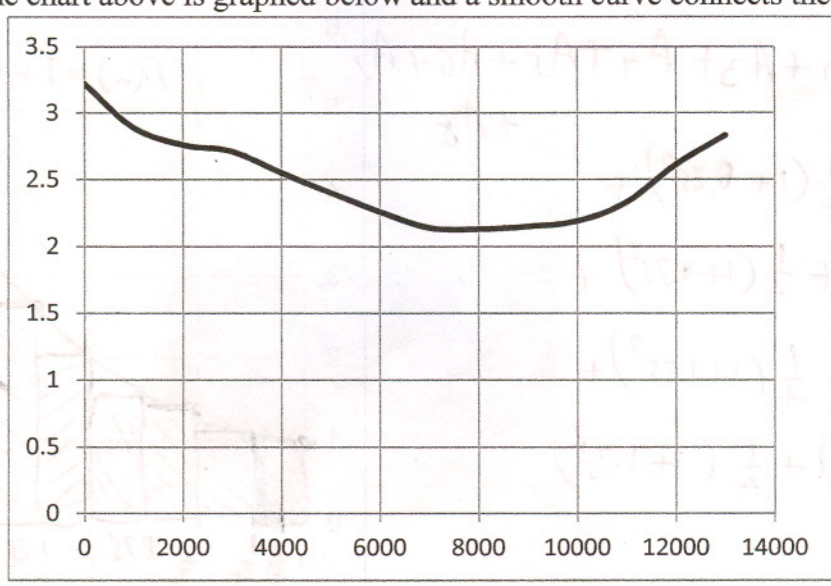
Units	Marginal Cost ($/unit)	Units	Marginal Cost ($/unit)
0	3.22	7000	2.14
1000	2.89	8000	2.13
2000	2.76	9000	2.15
3000	2.71	10,000	2.19
4000	2.55	11,000	2.33
5000	2.40	12,000	2.61
6000	2.26	13,000	2.83

We are going to estimate the cost (not including fixed costs) to produce the first 12,000 batches of 2000 units using the initial marginal cost for each batch.

Cost $= 2000(3.22 + 2.76 + 2.55 + 2.26 + 2.13 + 2.19)$

Cost $= \boxed{\$30220}$

2. The data in the chart above is graphed below and a smooth curve connects the data points.



Use 6 Approx. Rectangles
Midpoint Rule

We will use this graph to visualize the cost calculations made in #1 above.

Cost $= 2000(2.89 + 2.71 + 2.40 + 2.14 + 2.15 + 2.33) = \$\,29240$

II. **Estimating the Area under a Curve**

3.

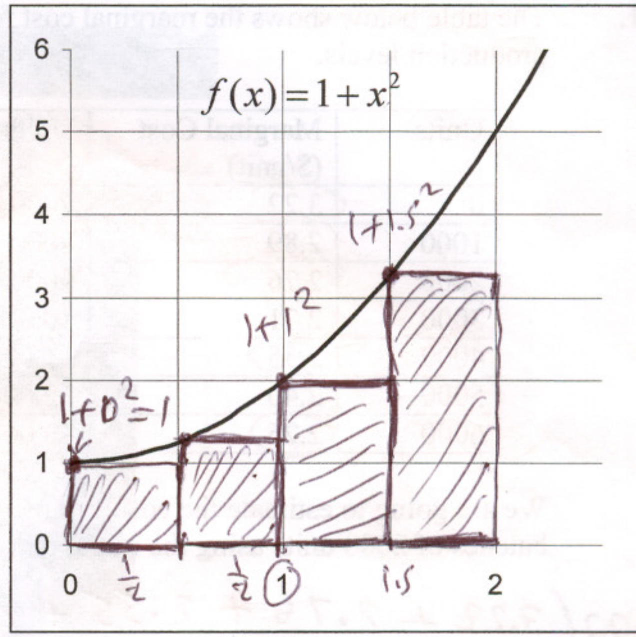
(a) Use 4 approximating rectangles and left endpoints to estimate the area under the graph of $f(x) = 1 + x^2$ on the interval [0, 2].

$A_{total} = A_1 + A_2 + A_3 + A_4$

$= \frac{1}{2}(1) + \frac{1}{2}(1+0.5^2) + \frac{1}{2}(1+1^2)$

$+ \frac{1}{2}(1+1.5^2)$

$= \boxed{3.75}$



(b) Use 8 approximating rectangles and left endpoints to estimate the area under the graph of $f(x) = 1 + x^2$ on the interval [0, 2].

$A_{Total} = A_1 + A_2 + A_3 + A_4 + A_5 + A_6 + A_7$
$+ A_8$

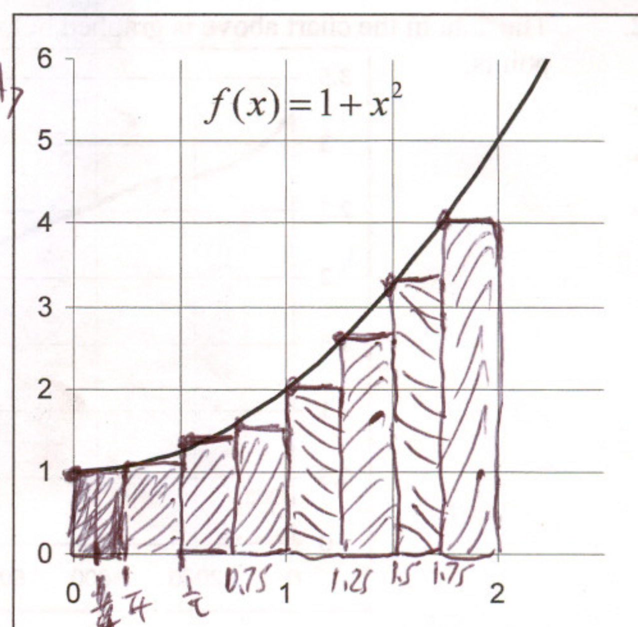
$= \frac{1}{4}(1+0^2) + \frac{1}{4}(1+0.25^2) +$

$+ \frac{1}{4}(1+0.5^2) + \frac{1}{4}(1+0.75^2) +$

$+ \frac{1}{4}(1+1^2) + \frac{1}{4}(1+1.25^2) +$

$+ \frac{1}{4}(1+1.5^2) + \frac{1}{4}(1+1.75^2)$

$= \boxed{4.1875}$



(c) Later on in this chapter, you will learn how to determine the exact value of the area under the graph of $f(x) = 1 + x^2$ on the interval [0, 2]. It is 14/3 or about 4.67. Which of the estimates in (a) and (b) is the better estimate? Why do you think this is the case?

Why do you think the estimates in (a) and (b) are less than the actual value of the area?

(d) Estimates can also be calculated using approximating rectangles and right endpoints or midpoints. If we used right endpoints for the graph above, how do you think the estimate would compare to the actual value of the area?

(e) We usually get a better approximation using midpoints instead of left or right endpoints. To see this, use 4 approximating rectangles and midpoints to estimate the area under the graph of $f(x) = 1 + x^2$ on the interval [0, 2]. How does this estimate compare to the actual value?

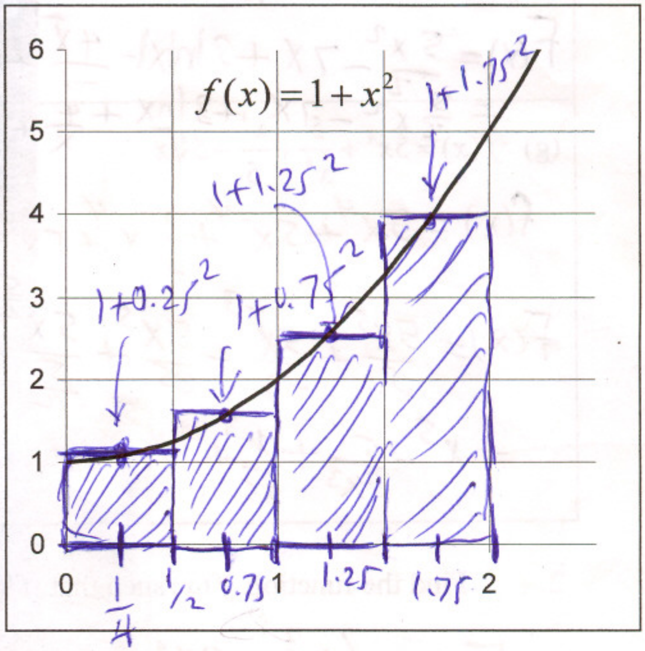
$A_{total} =$

$A_1 + A_2 + A_3 + A_4$

$= \frac{1}{2}(1+0.25^2) + \frac{1}{2}(1+0.75^2) +$

$\frac{1}{2}(1+1.25^2) + \frac{1}{2}(1+1.75^2)$

$= \boxed{4.625}$



1. Find the antiderivative of each function. Write answers with no negative exponents.

(a) $f(x)=12x^3-8x+5$ $F(x)=\frac{12x^4}{4}-\frac{8x^2}{2}+5x+C$ $=3x^4-4x^2+5x+C$	(b) $f(x)=12\sqrt{x}$ $F(x)=\frac{12x^{1.5}}{1.5}+C=8x^{\frac{3}{2}}+C$
(c) $f(x)=\frac{12}{x^2}=12x^{-2}$ $F(x)=\frac{12x^{-1}}{-1}+C=\frac{-12}{x}+C$	(d) $f(x)=\frac{12}{\sqrt{x}}=12x^{-\frac{1}{2}}$ $F(x)=\frac{12x^{\frac{1}{2}}}{\frac{1}{2}}+C=24x^{\frac{1}{2}}+C$ $\boxed{=24\sqrt{x}+C}$
(e) $f(x)=\frac{5x^4-7x^3+3x^2-4x}{x^3}$ $f(x)=5x-7+\frac{3}{x}-4x^{-2}$ $F(x)=\frac{5x^2}{2}-7x+3\ln\lvert x\rvert-\frac{4x^{-1}}{-1}+C$ $=\frac{5}{2}x^2-7x+3\ln x+\frac{4}{x}+C$	(f) $f(x)=3e^{7x}$ $F(x)=\frac{3e^{7x}}{7}+C$
(g) $f(x)=5x^4+\frac{5}{x^4}+\frac{x^4}{5}+5\sqrt[4]{x}$ $f(x)=5x^4+5x^{-4}+\frac{1}{5}x^4+5x^{\frac{1}{4}}$ $F(x)=\frac{5x^5}{5}+\frac{5x^{-3}}{-3}+\frac{x^5}{25}+\frac{5x^{\frac{5}{4}}}{\frac{5}{4}}+C$ $=x^5-\frac{5}{3x^3}+\frac{x^5}{25}+4x^{\frac{5}{4}}+C$	(h) $f(x)=(3x+2)(x-5)$ FOIL $=3x^2-15x+2x-10$ $f(x)=3x^2-13x-10$ $F(x)=\frac{3x^3}{3}-\frac{13x^2}{2}-10x+C$ $F(x)=x^3-\frac{13}{2}x^2-10x+C$

2. Find the function $f(x)$ such that $f'(x)=6x^2+8x-5$ and $f(3)=9$.

$F(x)=\frac{6x^3}{3}+\frac{8x^2}{2}-5x+C$

$F(x)=2x^3+4x^2-5x+C$

But $F(3)=9 \Rightarrow 2(3)^3+4(3)^2-5(3)+C=9 \Rightarrow \boxed{C=-66}$

$\boxed{F(x)=2x^3+4x^2-5x-66}$

OVER→

3. Find the function $f(x)$ such that $f'(x)=\frac{4}{x}$ and $f(e)=10$.

$f(x) = 4\ln|x| + C$

But $f(e) = 10 \Rightarrow 10 = 4\ln e + C$ & $\ln e = 1$

$\therefore 10 = 4\ln e + C \Rightarrow 10 = 4 + C \Rightarrow C = 6$

Therefore $\boxed{f(x) = 4\ln|x| + 6}$

4. Evaluate each definite integral using the Fundamental Theorem of Calculus.

(a) $\int_1^3 (1+2x-4x^3)\,dx$ $x + \frac{2x^2}{2} - \frac{4x^4}{4}\Big\vert_1^3$ $= x + x^2 - x^4 \Big\vert_1^3$ $=(3+3^2-3^4)-(1+1^2-1^4)$ $\boxed{=-70}$	(b) $\int_4^9 \frac{3}{\sqrt{x}}\,dx = \int_4^9 3x^{-\frac{1}{2}}\,dx$ $= \frac{3x^{1/2}}{1/2}\Big\vert_4^9$ $= 6x^{1/2}\Big\vert_4^6$ $= 6(9)^{1/2} - 6(4)^{\frac{1}{2}}$ $= 18 - 12$ $\boxed{=6}$	(c) $\int_0^{2.6} 8.4e^{0.4t}\,dt$ $= \frac{8.4e^{0.4t}}{0.4}\Big\vert_0^{2.6}$ $= 21e^{0.4t}\Big\vert_0^{2.6}$ $= 21e^{0.4(2.6)} - 21e^{0}$ $= 21e^{1.04} - 21$ $\boxed{\cong 38.41}$

Answers

1(a) $3x^4-4x^2+5x+C$	(b) $8x^{3/2}+C$	(c) $-\frac{12}{x}+C$
(d) $24x^{1/2}+C$	(e) $\frac{5x^2}{2}-7x+3\ln\lvert x\rvert+\frac{8}{x^3}+C$	(f) $\frac{3}{7}e^{7x}+C$
(g) $x^5-\frac{5}{3x^3}+\frac{1}{25}x^5+4x^{5/4}+C$	(h) $x^3-\frac{13}{2}x^2-10x+C$	2. $f(x)=2x^3+4x^2-5x-66$
3. $f(x)=4\ln\lvert x\rvert+6$	4. (a) -70 (b) 6 (c) $21(e^{1.04}-1)\approx 38.41$	

1. Marginal revenue is the rate of change of revenue with respect to the number of units purchased, measured in dollars per unit. Suppose the marginal revenue when a company sells q units of a new product is given by $R'(q) = 26 - 0.04q$

Compute and interpret $\int_{200}^{300} R'(q)\,dq = \int_{200}^{300} (26 - 0.04q)\,dq$

$= 26q - 0.04\frac{q^2}{2}\Big|_{200}^{300} = 26q - 0.02q^2\Big|_{200}^{300}$

$= 26(300) - 0.02(300)^2 - (26 * 200 - 0.02(200)^2) = 1600$

This is the net change in revenue for $200 \leq q \leq 300$.
Our answer says that revenue increases by \$1600 when production is increased from 200 units to 300 units.

2. An object moves along a line so that its velocity at time t is $v(t) = t^2 - t - 6$ (measured in meters per second).

Find the displacement of the object during the time period $1 \leq t \leq 4$

$\int_1^4 (t^2 - t - 6)\,dt = \frac{t^3}{3} - \frac{t^2}{2} - 6t\Big|_1^4 = -\frac{9}{2}$

This means that the objects position at time $t = 4$ is 4.5 meters to the left of its position at the start of time period.

b) Find the distance traveled during the time period $1 \leq t \leq 4$

$t^2 - t - 6 = 0$

$(t-3)(t+2) = 0 \quad \boxed{t = 3} \quad t = -2$

Distance traveled $= \left|\int_1^3 (t^2 - t - 6)\,dt\right| + \left|\int_3^4 (t^2 - t - 6)\,dt\right|$

$= \left|\frac{t^3}{3} - \frac{t^2}{2} - 6t\Big|_1^3\right| + \left|\frac{t^3}{3} - \frac{t^2}{2} - 6t\Big|_3^4\right|$

$\cong 10.17$

Therefore, the particle has traveled 10.17 meters in the time period [1 to 4] seconds.

3. Compute the average value of $f(x) = 1 + x^2$ on the interval [-1 , 2]

<u>Recall:</u> $f_{avg} = \frac{1}{b-a}\int_a^b f(x)\,dx$

therefore, $f_{avg} = \frac{1}{2--1}\int_{-1}^{2}(1+x^2)\,dx$

$$= \frac{1}{3}\left(x + \frac{x^3}{3}\right)\Big|_{-1}^{2}$$

$$= \frac{1}{3}\left(2 + \frac{8}{3} - \left(-1 - \frac{1}{3}\right)\right) = 2$$

Therefore, the average value of $f(x)$ on the interval $[-1, 2]$ is 2.

4. The temperature (in ^{o}F) in Mexico City t hours after midnight during a day in April was modeled by the function $T(t) = -.017t^3 + 0.53t^2 - 2.9t + 65$
Find the average temperature during that day from 8 AM to 6 PM.

8AM 6PM

the indicated hours correspond to the interval $[8, 18]$

So, the average temperature was $T_{avg} = \frac{1}{18-8}\int_8^{18}(-0.017t^3 + 0.53t^2 - 2.9t + 65)\,dt$

$$= \frac{1}{10}\left[-0.017\frac{t^4}{4} + 0.53\frac{t^3}{3} - 2.9\frac{t^2}{2} + 65t\right]\Big|_8^{18}$$

$$\cong 78.4^{o}F$$

therefore, the average temperature in Mexico City during that day in April was $78.4^{o}F$.