

Solutions  
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**Functions of Two Variables**

1. A Cost Functions with Two Inputs

A company makes two kinds of chocolate bars, plain, and with almonds. Fixed production costs are \$10,000 and it costs \$1.10 to make a plain chocolate bar and \$1.25 to make one with almonds.

- (a) Express the cost of making  $x$  plain bars and  $y$  bars with almonds as a function of two variables  $C = f(x, y)$

$$C = f(x, y) = 10000 + 1.10x + 1.25y$$

- (b) Find  $f(2000, 1000)$  and interpret it.

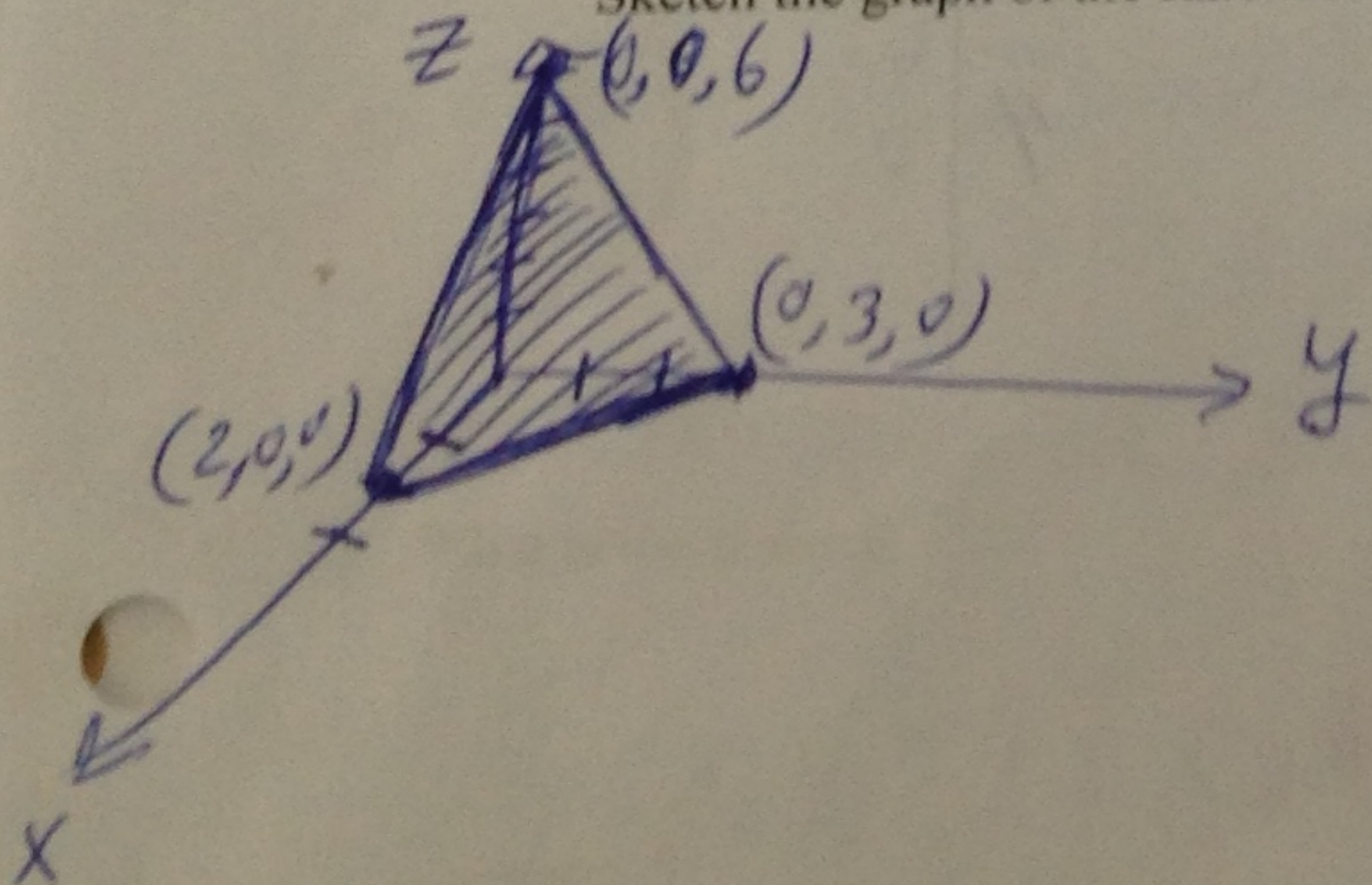
$$\begin{aligned} f(2000, 1000) &= 10000 + 1.10(2000) + 1.25(1000) \\ &= 10000 + 2200 + 1250 = 13450 \end{aligned}$$

This means that the cost of making 2000 plain chocolate bars and 1000 bars with almonds is \$13450.

- (c) What is the domain of  $f$ ?

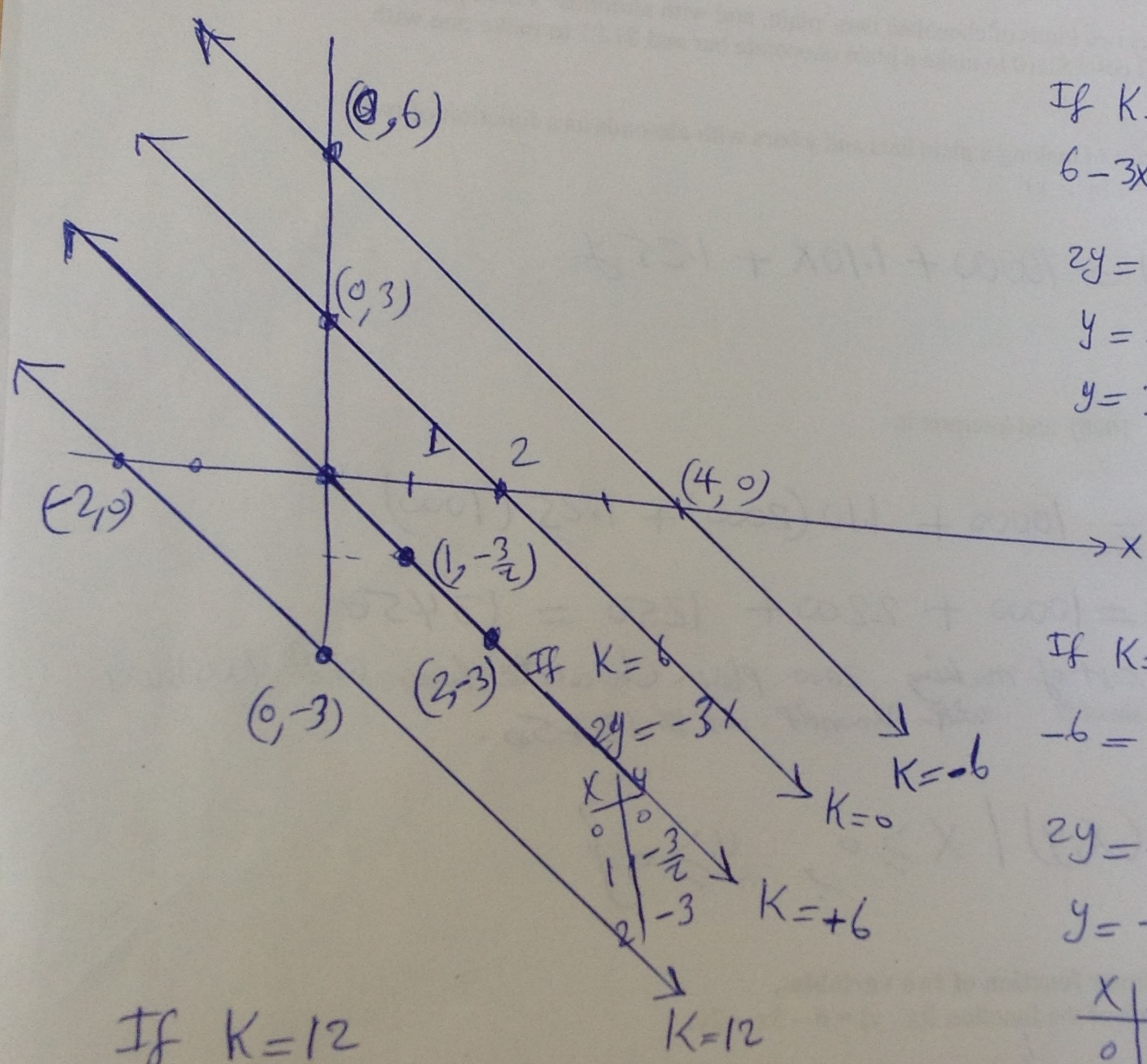
$$D = \{ (x, y) \mid x \geq 0, y \geq 0 \}$$

2. **Graphing a linear function of two variables.**  
Sketch the graph of the function  $f(x, y) = 6 - 3x - 2y$ .



$$\begin{aligned} Z &= 6 - 3X - 2y \\ 3X + 2y + Z &= 6 \\ \text{Represents a plane.} \end{aligned}$$

Also see Example 8; level curves of a linear function  
 sketch the level curves of the function  
 $f(x,y) = 6 - 3x - 2y$  for the values  
 $K = -6, 0, 6, 12$



If  $K=0$   
 $6 - 3x - 2y = 0$   
 $2y = 6 - 3x$   
 $y = \frac{6}{2} - \frac{3}{2}x$   
 $y = 3 - 1.5x$

x	y
0	3
2	0

If  $K=-6$   
 $-6 = 6 - 3x - 2y$   
 $2y = -3x + 12$   
 $y = -\frac{3}{2}x + 6$

x	y
0	6
4	0

If  $K=12$

$$6 - 3x - 2y = 12$$

$$2y = -3x + 6 - 12$$

$$2y = -3x - 6$$

x	y
0	-3
-2	0

$$y = -\frac{3}{2}x - 3$$

3. Find the values of the following function.

$$f(x, y) = 1 + 4xy - 3y^2$$

a)  $f(6, 2) = 1 + 4(6)(2) + -3(2)^2 = 1 + 48 - 12$

b)  $f(-1, 4) = 1 + 4(-1)(4) - 3(4)^2 = 49 - 12 = \boxed{37}$

c)  $f(0, -3) = 1 - 16 - 48 = \boxed{-63}$

d)  $f(x, -3) = 1 + 4(x)(-3) - 3(-3)^2 = 1 - 27 = \boxed{-26}$

$$= 1 + 4(x)(2) - 3(2)^2 = 1 + 8x - 12$$

$$= \boxed{-11 + 8x}$$

4. Joint Cost Function

A company makes three sizes of cardboard boxes: small, medium, and large. It costs \$2.50 to make a small box, \$4.00 for a medium box, and \$4.50 for a large box. Fixed costs are \$8000.

- a) Express the cost of making  $x$  small boxes,  $y$  medium boxes, and  $z$  large boxes as a function of three variables  $C = f(x, y, z)$

$$C(x, y, z) = 8000 + 2.5x + 4y + 4.5z$$

- b) Find  $f(3000, 5000, 4000)$  and interpret it.

$$= 8000 + 2.5(3000) + 4(5000) + 4.5(4000)$$

$$= \text{\$} 53500$$

It will cost the company ~~\\$~~ 53500 to make 3000 small boxes, 5000 medium boxes and 4000 large boxes.

- c) What is the domain of  $f$ ?

$$\{(x, y, z) \mid x \geq 0, y \geq 0, z \geq 0\}$$