Section 1.2

Graphs of Equations in Two Variables

OBJECTIVE 1

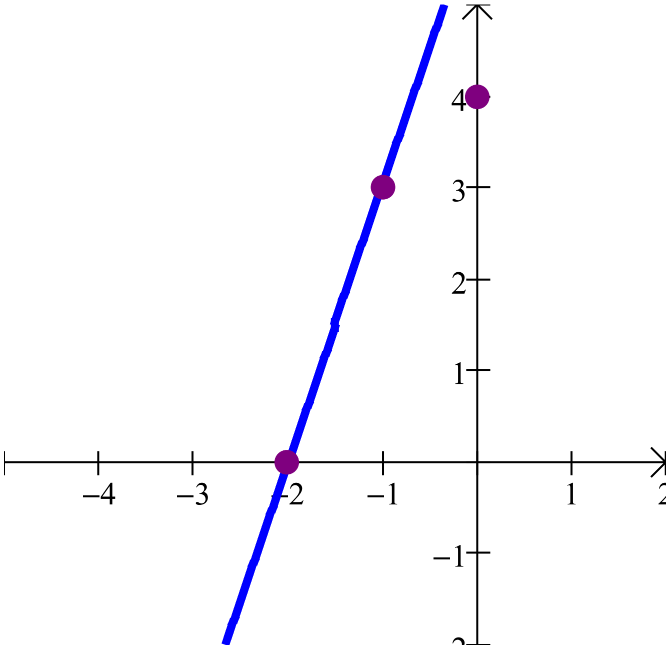
1 Graph Equations by Hand by Plotting Points

EXAMPLE

Determining Whether a Point Is on the Graph of an Equation

Determine if the following points are on the graph of the equation
$-3x + y = 6$

(a) (0, 4) (b) (2, 0) (c) (-1, 3)

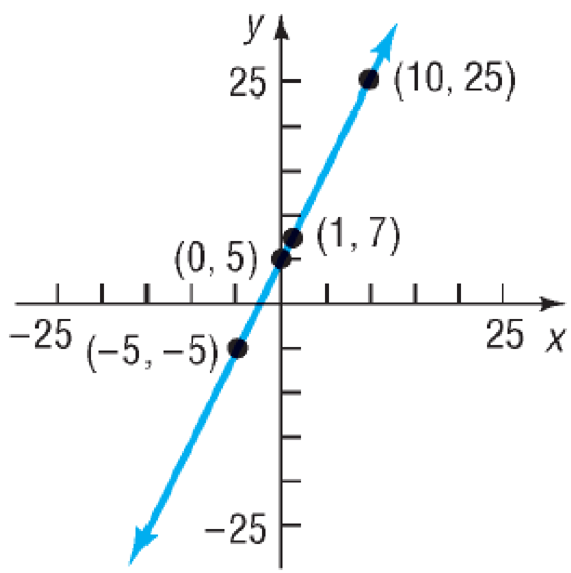


EXAMPLE

Graphing an Equation by Hand by Plotting Points

Graph the equation: $y = 2x + 5$

If	Then	Point on Graph
$x = 0$	$y = 2(0) + 5 = 5$	$(0, 5)$
$x = 1$	$y = 2(1) + 5 = 7$	$(1, 7)$
$x = -5$	$y = 2(-5) + 5 = -5$	$(-5, -5)$
$x = 10$	$y = 2(10) + 5 = 25$	$(10, 25)$

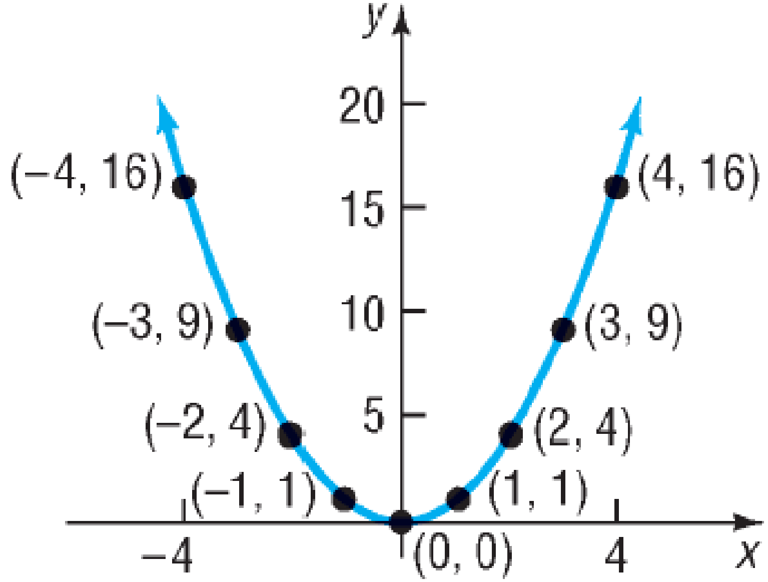


EXAMPLE

Graphing an Equation by Hand by Plotting Points

Graph the equation: $y = x^2$

x	$y = x^2$	(x, y)
−4	16	(−4, 16)
−3	9	(−3, 9)
−2	4	(−2, 4)
−1	1	(−1, 1)
0	0	(0, 0)
1	1	(1, 1)
2	4	(2, 4)
3	9	(3, 9)
4	16	(4, 16)



OBJECTIVE 2

2 Graph Equations Using a Graphing Utility

Steps for Graphing an Equation Using a Graphing Utility

STEP 1: Solve the equation for y in terms of x.

STEP 2: Get into the graphing mode of your graphing utility. The screen will usually display $Y =$, prompting you to enter the expression involving x that you found in Step 1. (Consult your manual for the correct way to enter the expression; for example, $y = x^2$ might be entered as x^2 or as x*x or as xx^Y2).

STEP 3: Select the viewing window. Without prior knowledge about the behavior of the graph of the equation, it is common to select the **standard viewing window*** initially. The viewing window is then adjusted based on the graph that appears. In this text, the standard viewing window will be

$$X\text{min} = -10 \qquad Y\text{min} = -10$$
$$X\text{max} = 10 \qquad Y\text{max} = 10$$
$$X\text{scl} = 1 \qquad Y\text{scl} = 1$$

STEP 4: Graph.

STEP 5: Adjust the viewing window until a complete graph is obtained.

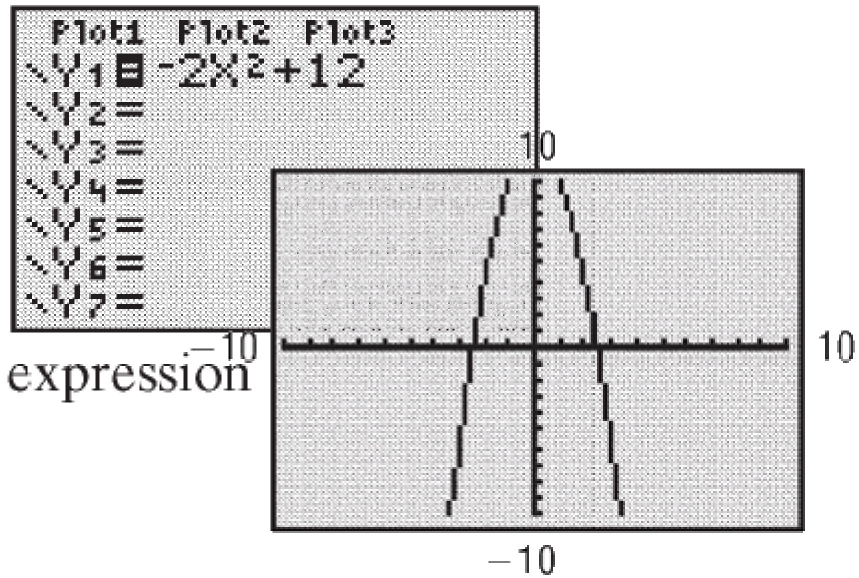
EXAMPLE

Graphing an Equation on a Graphing Utility

Graph the equation: $6x^2 + 3y = 36$

Step 1: We solve for y in terms of x.

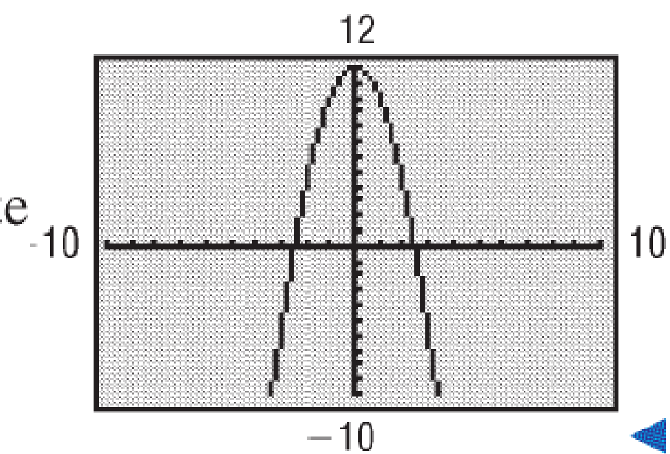
$$6x^2 + 3y = 36$$
$$3y = -6x^2 + 36$$
$$y = -2x^2 + 12$$



Step 2: From the graphing mode, enter the expression $-2x^2 + 12$ after the prompt $Y =$

Step 3: Set the viewing window to the standard viewing window.

Step 4: Graph.



Step 5: The graph of $y = -2x^2 + 12$ is not complete. The value of Ymax must be increased

OBJECTIVE 3

3 Use a Graphing Utility to Create Tables

Steps for Creating a Table of Values Using a Graphing Utility

Step 1: Solve the equation for y in terms of x.

Step 2: Enter the expression in x following the $Y =$ prompt of the graphing utility.

Step 3: Set up the table. Graphing utilities typically have two modes for creating tables. In the AUTO mode, the user determines a starting point for the table (TblStart) and ΔTbl (pronounced "delta-table"). The ΔTbl feature determines the increment for x. The ASK mode requires the user to enter values of x and then the utility determines the corresponding value of y.

Step 4: Create the table. The user can scroll within the table if the table was created in AUTO mode.

EXAMPLE

Creating a Table Using a Graphing Utility

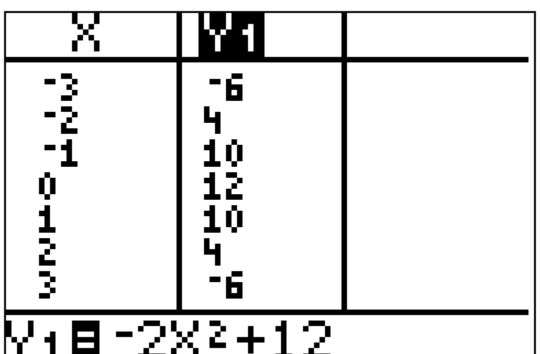
Create a table that displays the points on the graph of $6x^2 + 3y = 36$ for $x = -3, -2, -1, 0, 1, 2$, and 3.

Step 1: We solved the equation for y and obtained $y = -2x^2 + 12$.

Step 2: Enter the expression in x following the $Y =$ prompt.

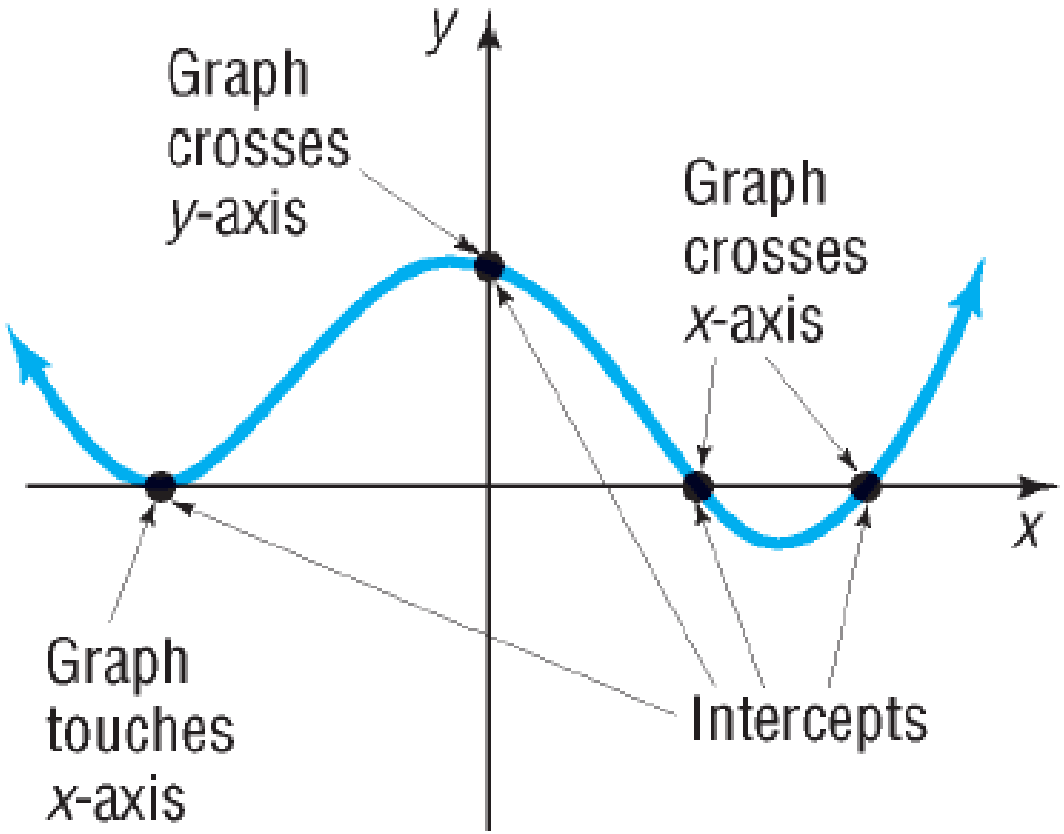
Step 3: We set up the table in the AUTO mode with TblStart $= -3$ and ΔTbl $= 1$.

Step 4: Create the table.



OBJECTIVE 4

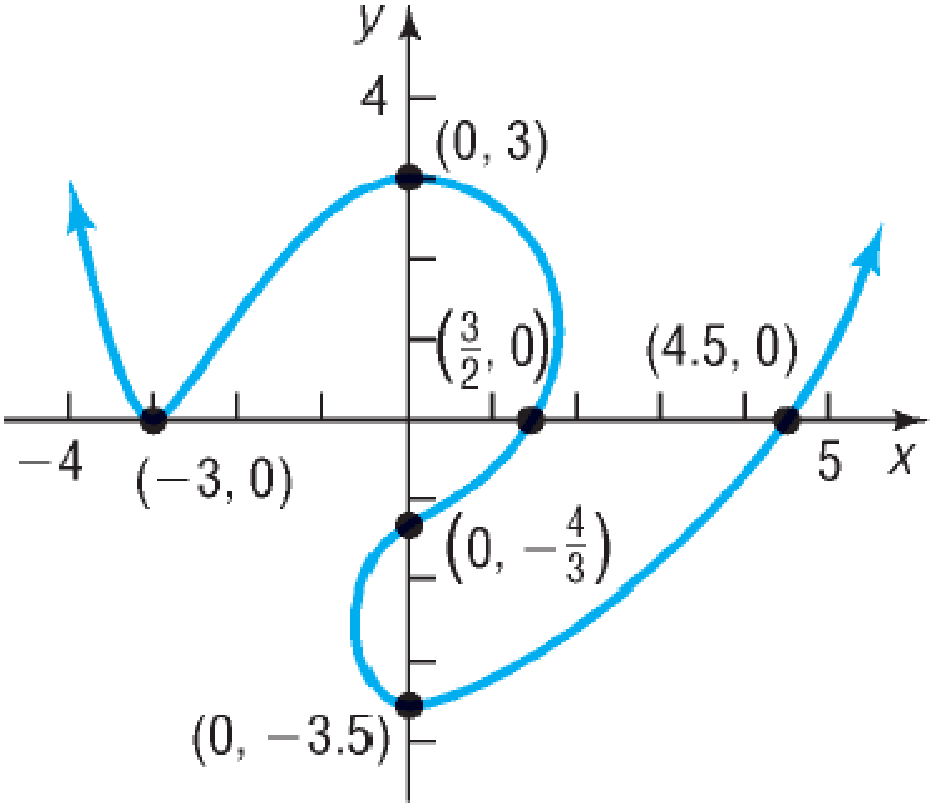
4 Find Intercepts from a Graph


y
Graph
crosses
y-axis
Graph
crosses
x-axis
x
Graph
touches
x-axis
Intercepts

EXAMPLE

Finding Intercepts from a Graph

Find the intercepts of the graph.



OBJECTIVE 5

5 Find Intercepts from an Equation

Procedure for Finding Intercepts

1. To find the x-intercept(s), if any, of the graph of an equation, let $y = 0$ in the equation and solve for x.
2. To find the y-intercept(s), if any, of the graph of an equation, let $x = 0$ in the equation and solve for y.

EXAMPLE

Finding Intercepts from an Equation

Find the x-intercept(s) and the y-intercept(s) of the graph of $y = x^2 - 4$.

X	Y1	
-3	5	
-2	0	
-1	-3	
0	-4	
1	-3	
2	0	
3	5	

$Y_1 = X^2 - 4$

OBJECTIVE 6

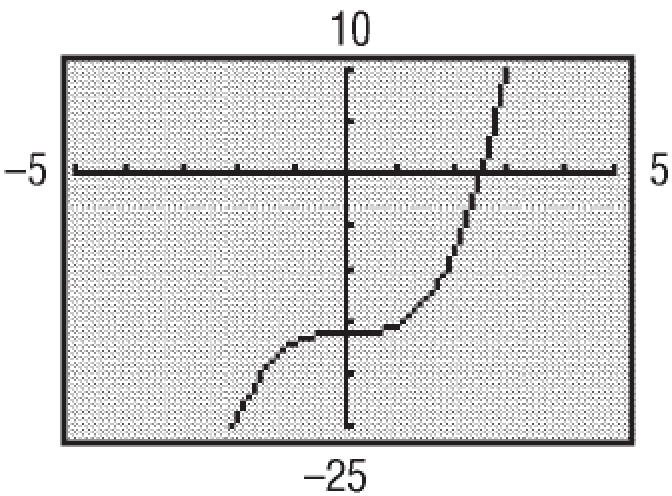
6 Use a Graphing Utility to Approximate Intercepts

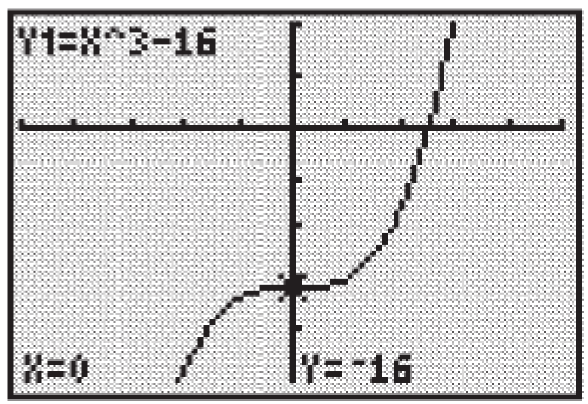
EXAMPLE

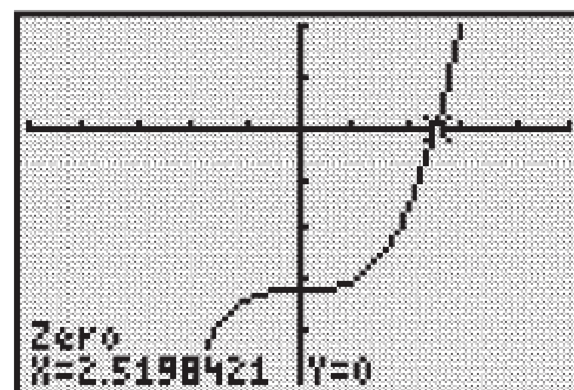
Finding Intercepts Using a Graphing Utility

Use a graphing utility to approximate the intercepts of the equation

$$y = x^3 - 16.$$



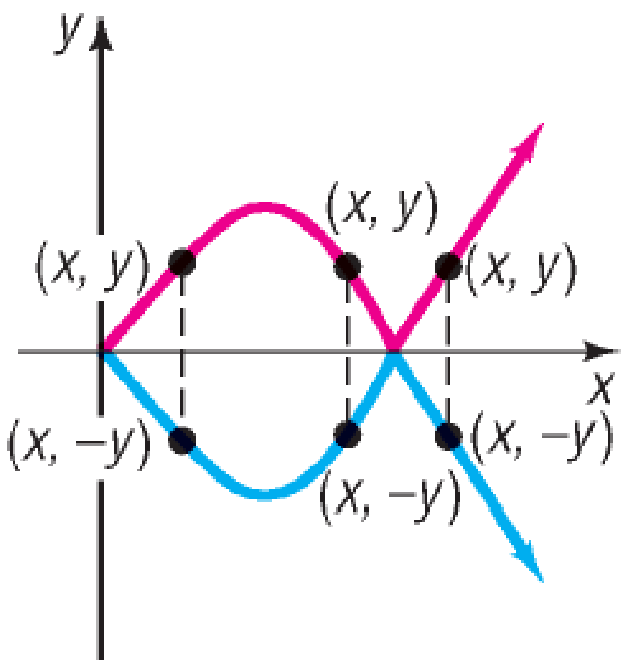




OBJECTIVE 7

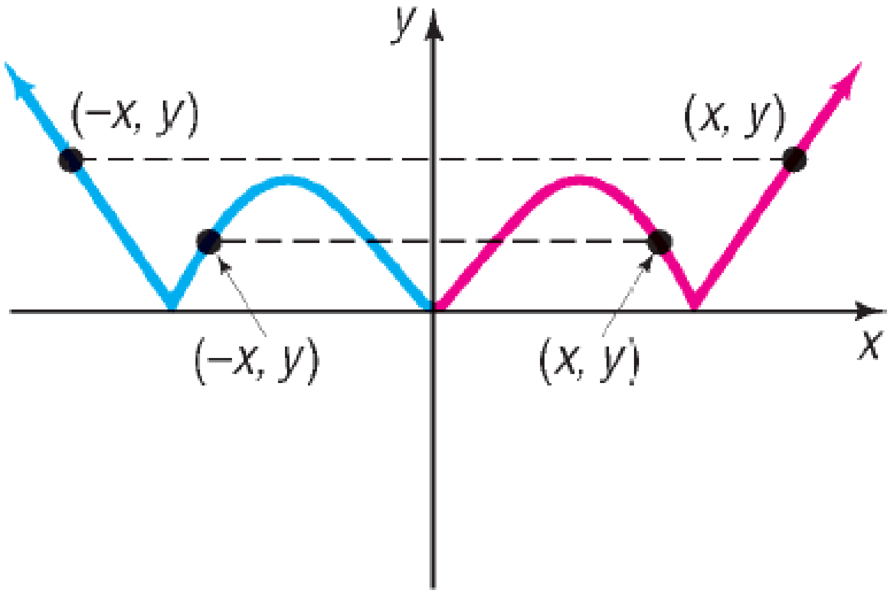
7 Test an Equation for Symmetry

A graph is said to be **symmetric with respect to the *x*-axis** if, for every point (x, y) on the graph, the point $(x, -y)$ is also on the graph.



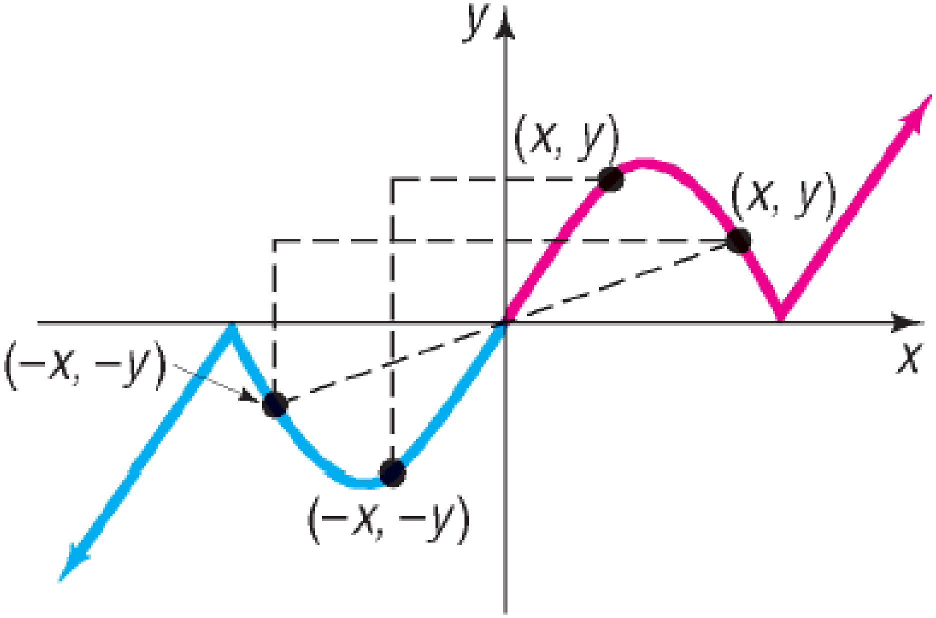
Symmetry with respect to the *x*-axis

A graph is said to be **symmetric with respect to the *y*-axis** if, for every point (x, y) on the graph, the point $(-x, y)$ is also on the graph.



Symmetry with respect to the *y*-axis

A graph is said to be **symmetric with respect to the origin** if, for every point (x, y) on the graph, the point $(-x, -y)$ is also on the graph.

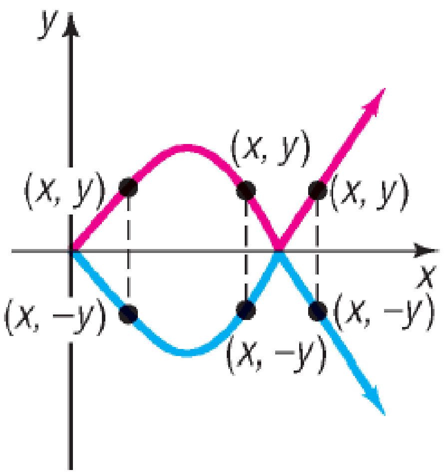


Symmetry with respect to the origin

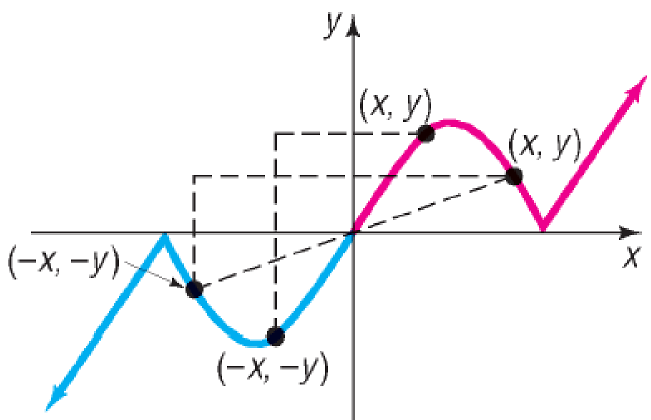
A graph is said to be **symmetric with respect to the *x*-axis** if, for every point (x, y) on the graph, the point $(x, -y)$ is also on the graph.

A graph is said to be **symmetric with respect to the *y*-axis** if, for every point (x, y) on the graph, the point $(-x, y)$ is also on the graph.

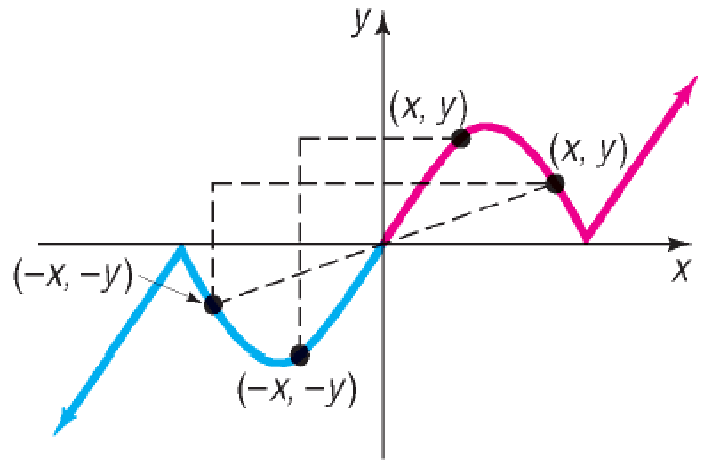
A graph is said to be **symmetric with respect to the origin** if, for every point (x, y) on the graph, the point $(-x, -y)$ is also on the graph.



Symmetry with respect to the *x*-axis



Symmetry with respect to the origin



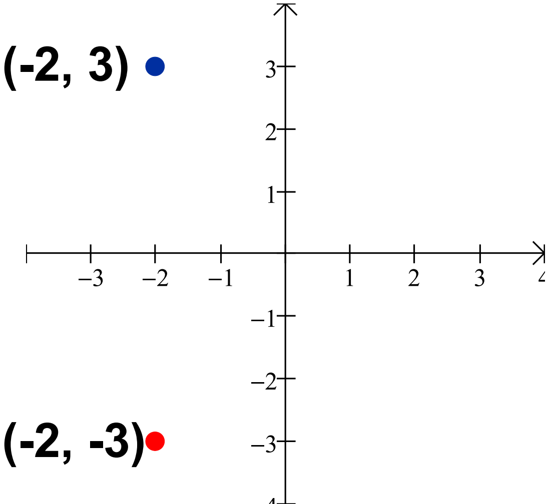
Symmetry with respect to the origin

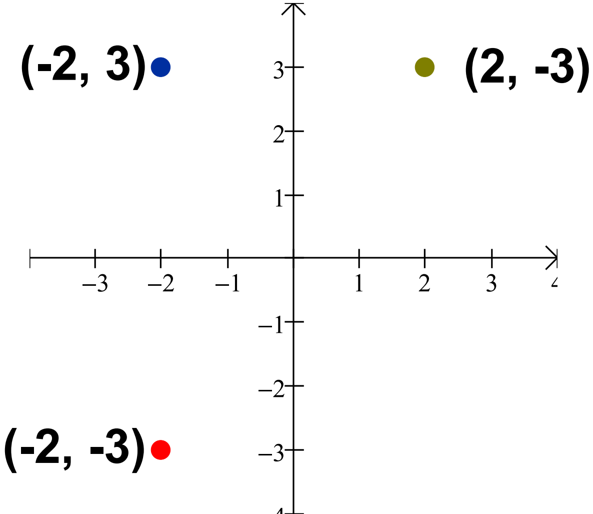
EXAMPLE Symmetric Points

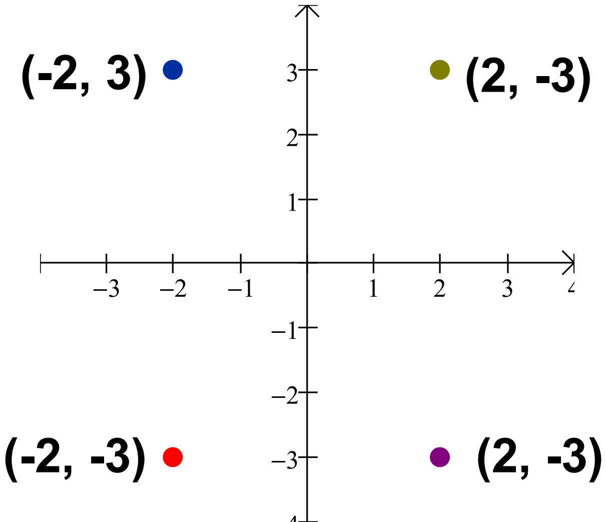
If a graph is symmetric with respect to the *x*-axis and the point (-2, 3) is on the graph, then what point is also on the graph?

If a graph is symmetric with respect to the *y*-axis and the point (-1, 3) is on the graph, then what point is also on the graph?

If a graph is symmetric with respect to the origin and the point (-1, 3) is on the graph, then what point is also on the graph?







Tests for Symmetry

To test the graph of an equation for symmetry with respect to the

x-Axis	Replace y by $-y$ in the equation. If an equivalent equation results, the graph of the equation is symmetric with respect to the x-axis.
y-Axis	Replace x by $-x$ in the equation. If an equivalent equation results, the graph of the equation is symmetric with respect to the y-axis.
Origin	Replace x by $-x$ and y by $-y$ in the equation. If an equivalent equation results, the graph of the equation is symmetric with respect to the origin.

EXAMPLE

Finding Intercepts and Testing an Equation for Symmetry

For the equation $y = \dfrac{x^2 - 9}{x^2 + 2}$ find the intercepts and test for symmetry.

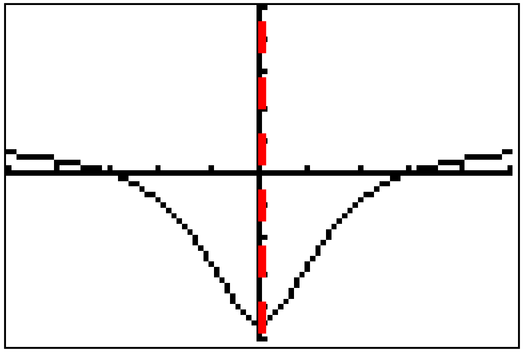
Solution

The x intercepts are -3 and 3; the y intercept is $-\dfrac{9}{2}$.

x-Axis: NO

y-Axis: YES

origin: NO



OBJECTIVE 8

8 **Know How to Graph Key Equations**

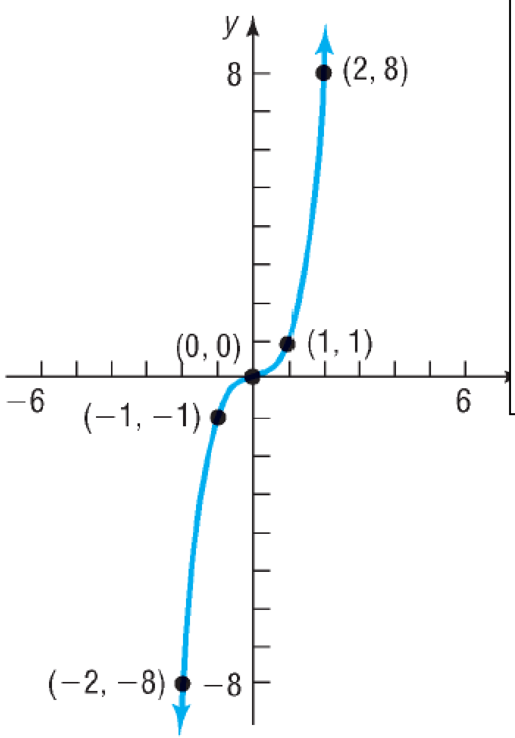
EXAMPLE

Graphing the Equation $y = x^3$ by Finding Intercepts and Checking for Symmetry

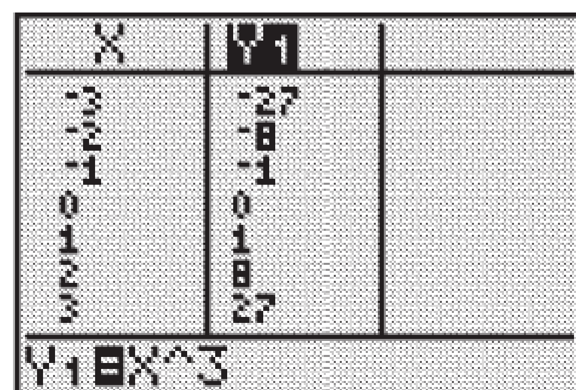
Graph the equation $y = x^3$ by hand by plotting points. Find any intercepts and check for symmetry first.

Solution

origin symmetry



x	$y = x^3$	(x, y)
0	0	(0, 0)
1	1	(1, 1)
2	8	(2, 8)
3	27	(3, 27)



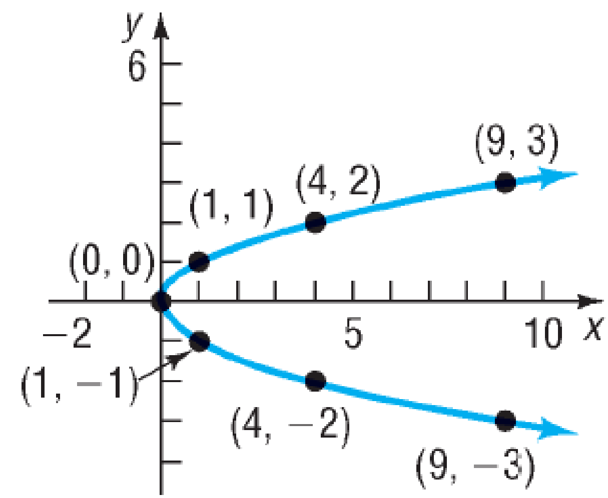
EXAMPLE Graphing the Equation $x = y^2$

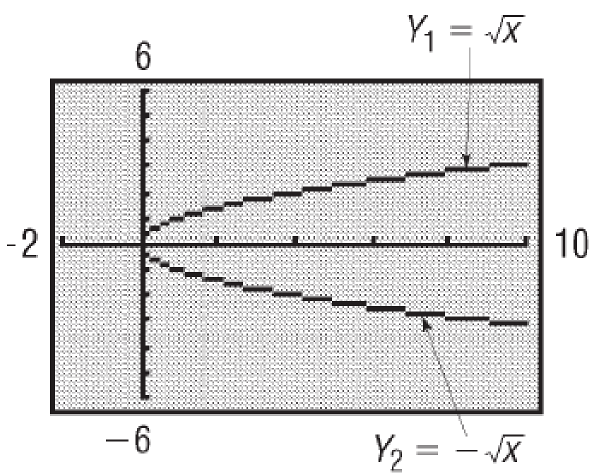
Graph the equation $x = y^2$.

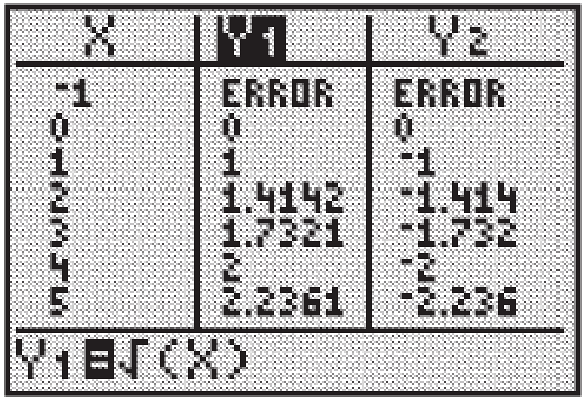
Find any intercepts and check for symmetry first.

Solution *x-Axis symmetry*

y	$x = y^2$	(x, y)
0	0	(0, 0)
1	1	(1, 1)
2	4	(4, 2)
3	9	(9, 3)







EXAMPLE Graphing the Equation $y = \frac{1}{x}$

Graph the equation $y = \frac{1}{x}$.

Find any intercepts and check for symmetry first.

Solution

origin symmetry

x	$y = \frac{1}{x}$	(x, y)
$\frac{1}{10}$	10	$\left(\frac{1}{10}, 10\right)$
$\frac{1}{3}$	3	$\left(\frac{1}{3}, 3\right)$
$\frac{1}{2}$	2	$\left(\frac{1}{2}, 2\right)$
1	1	$(1, 1)$
2	$\frac{1}{2}$	$\left(2, \frac{1}{2}\right)$
3	$\frac{1}{3}$	$\left(3, \frac{1}{3}\right)$
10	$\frac{1}{10}$	$\left(10, \frac{1}{10}\right)$