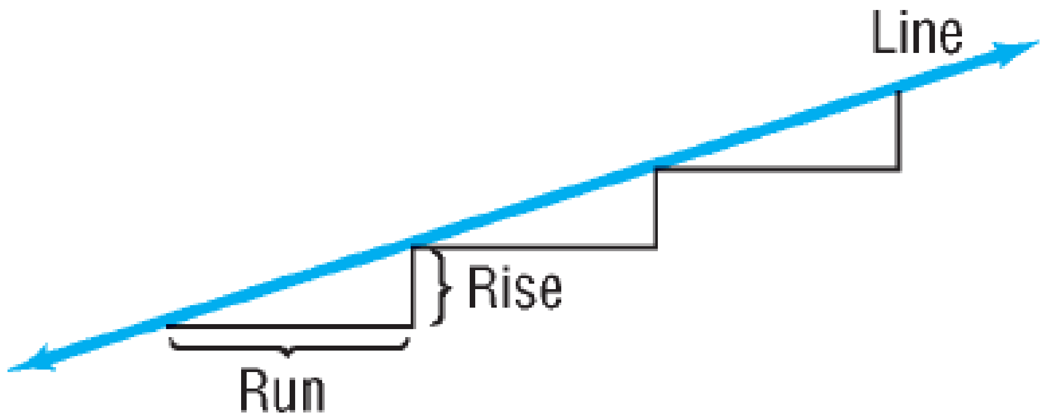
Section 1.4

Lines

OBJECTIVE 1

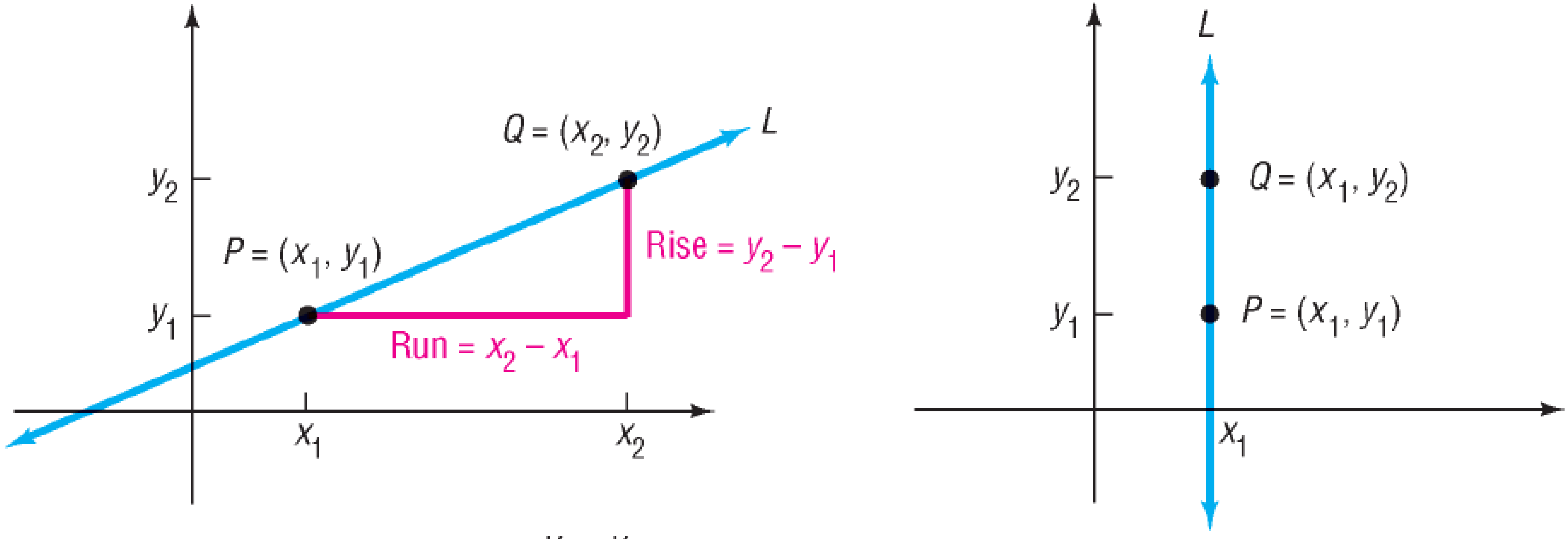
1 Calculate and Interpret the Slope of a Line



Let $P = (x_1, y_1)$ and $Q = (x_2, y_2)$ be two distinct points. If $x_1 \neq x_2$, the **slope *m*** of the nonvertical line L containing P and Q is defined by the formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}, \qquad x_1 \neq x_2 \qquad \textbf{(1)}$$

If $x_1 = x_2$, L is a **vertical line** and the slope m of L is **undefined** (since this results in division by 0).

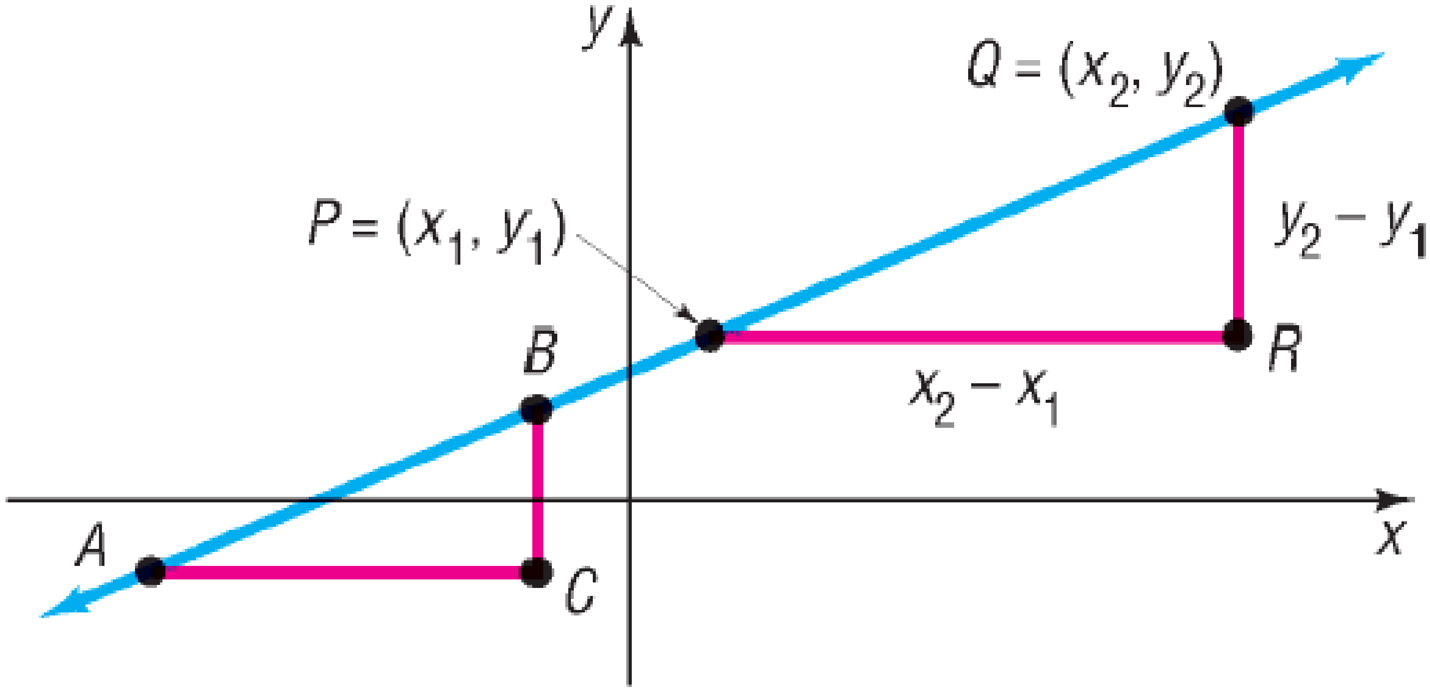


(a) Slope of L is $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ **(b)** Slope is undefined; L is vertical

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{Rise}}{\text{Run}}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{Change in } y}{\text{Change in } x} = \frac{\Delta y}{\Delta x}$$

Any two distinct points on the line can be used to compute the slope of the line.



The slope of a line may be computed from $P = (x_1, y_1)$ to $Q = (x_2, y_2)$ or from Q to P because

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2}$$

EXAMPLE

Finding and Interpreting the Slope of a Line Containing Two Points

Find the slope of the line containing the points (-1, 4) and (2, -3).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

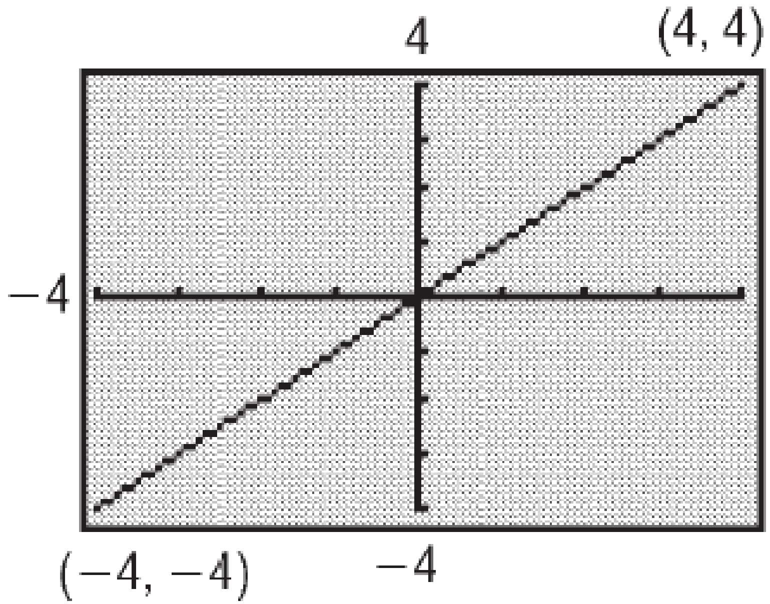
$$m = \frac{y_1 - y_2}{x_1 - x_2}$$

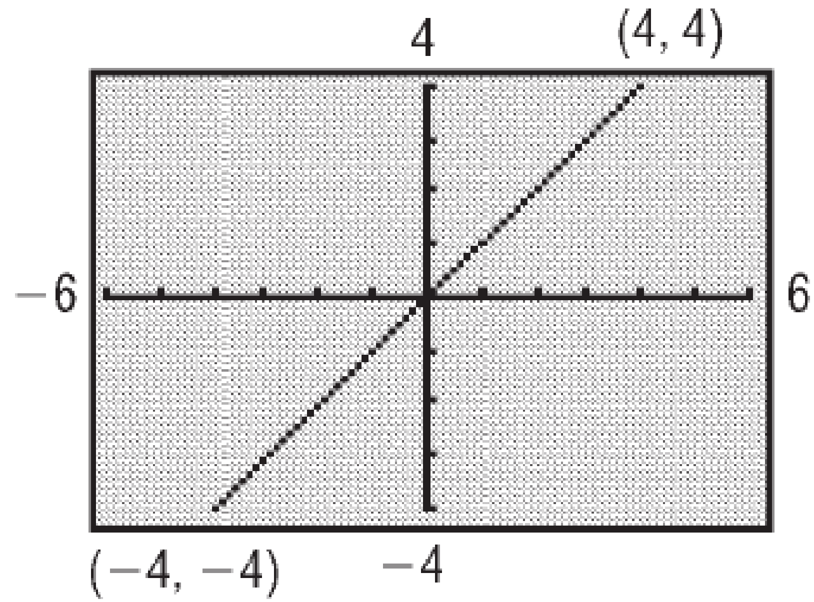
$$m = \frac{-3-4}{2+1} = -\frac{7}{3}$$

$$m = \frac{4+3}{-1-2} = -\frac{7}{3}$$

The average rate of change of *y* with respect to *x* is $-\frac{7}{3}$

Square Screens





Adjust the selections for *X*min, *X*max, *Y*min, and *Y*max by setting the ratio of *x* to *y* at 3:2.

Seeing the Concept

On the same square screen, graph the following equations:

$$Y_1 = 0$$

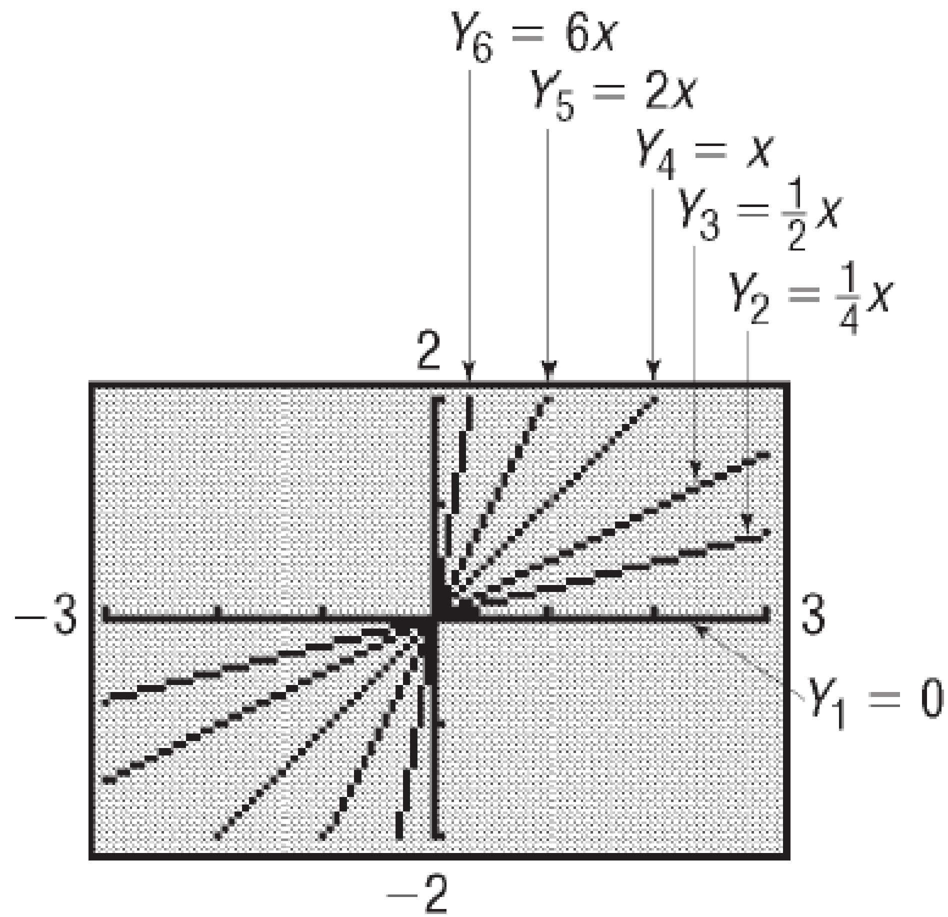
$$Y_2 = -\frac{1}{4}x$$

$$Y_3 = -\frac{1}{2}x$$

$$Y_4 = -x$$

$$Y_5 = -2x$$

$$Y_6 = -6x$$



Seeing the Concept

On the same square screen, graph the following equations:

$$Y_1 = 0$$

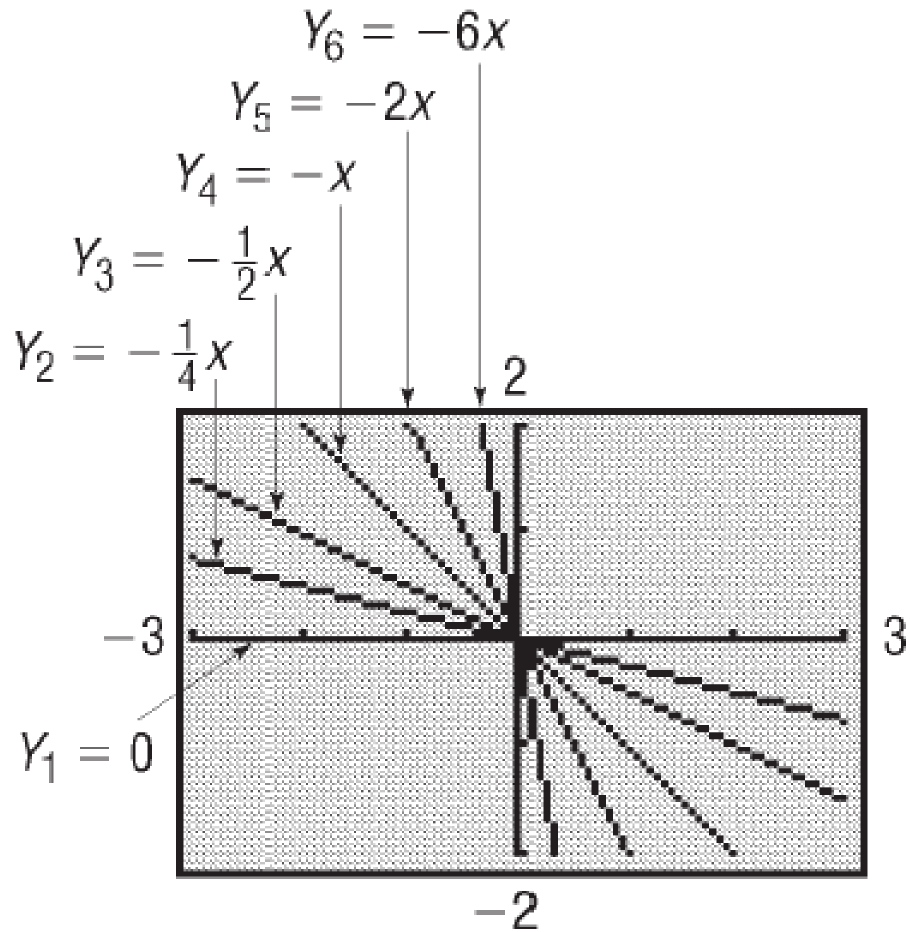
$$Y_2 = -\frac{1}{4}x$$

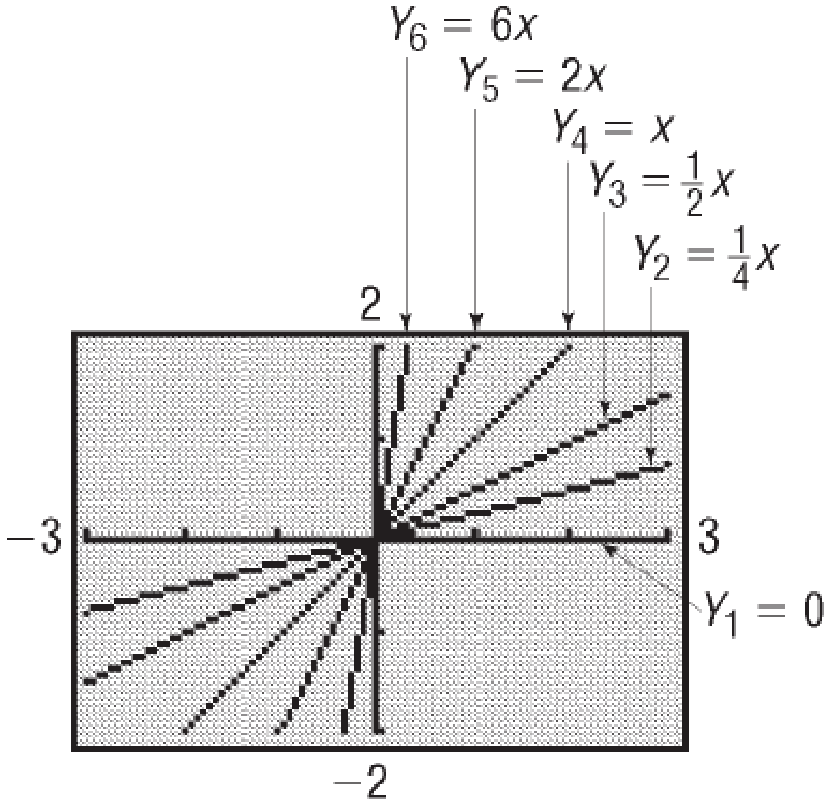
$$Y_3 = -\frac{1}{2}x$$

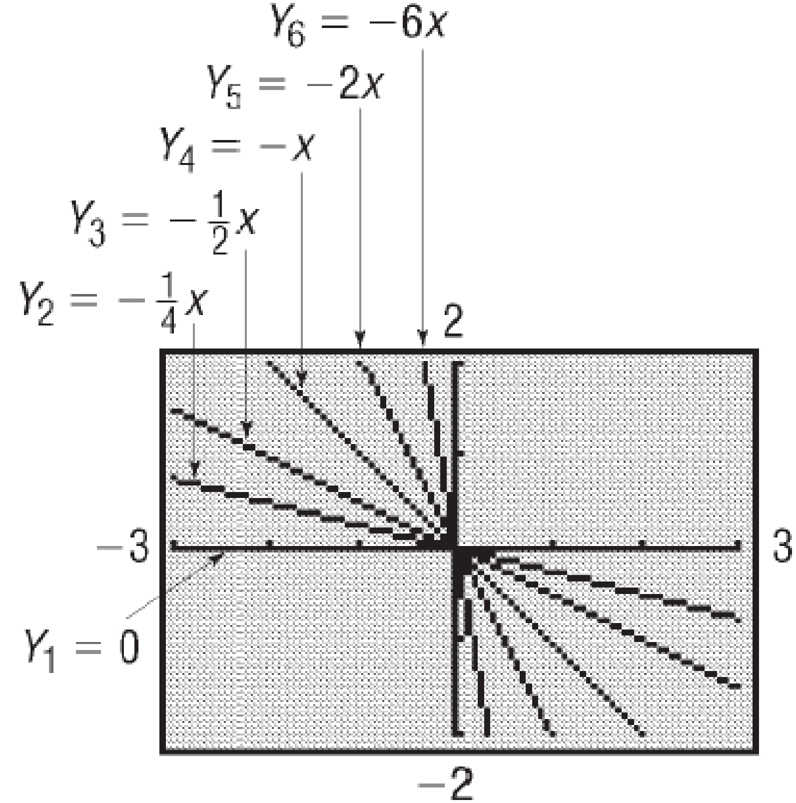
$$Y_4 = -x$$

$$Y_5 = -2x$$

$$Y_6 = -6x$$







1. When the slope of a line is positive, the line slants upward from left to right.

2. When the slope of a line is negative, the line slants downward from left to right.

3. When the slope is 0, the line is horizontal.

Figures 69 and 70 also illustrate that the closer the line is to the vertical position, the greater the magnitude of the slope.

OBJECTIVE 2

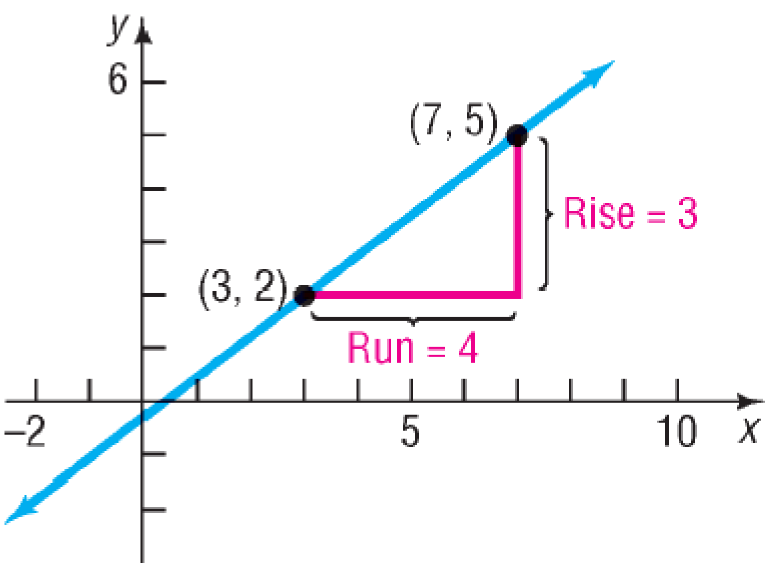
2 Graph Lines Given a Point and the Slope

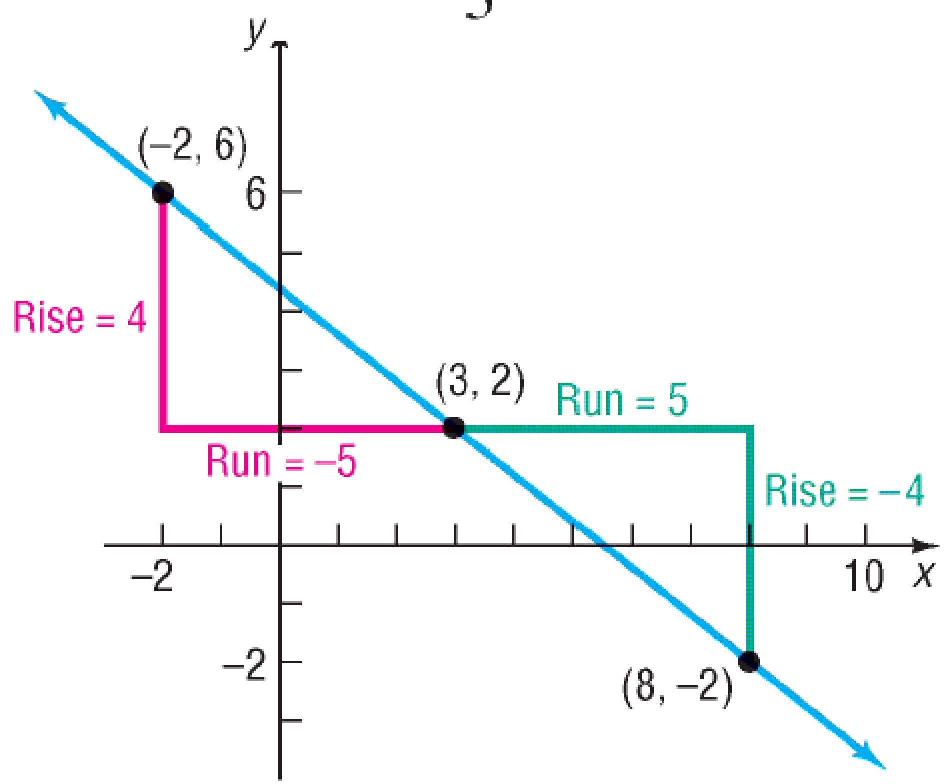
EXAMPLE

Graphing a Line Given a Point and a Slope

Draw a graph of the line that contains the point $(3, 2)$ and has a slope of:

(a) $\frac{3}{4}$ (b) $-\frac{4}{5}$



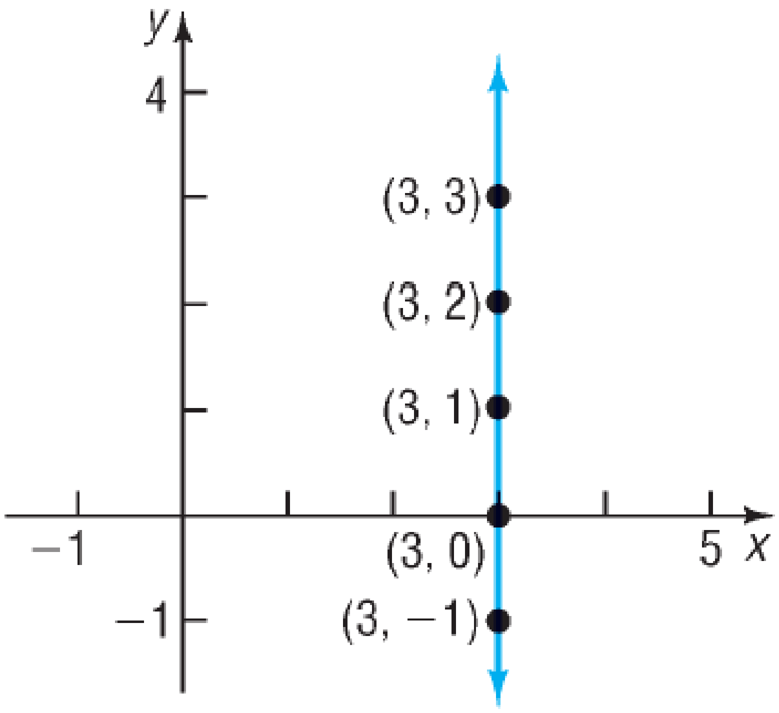


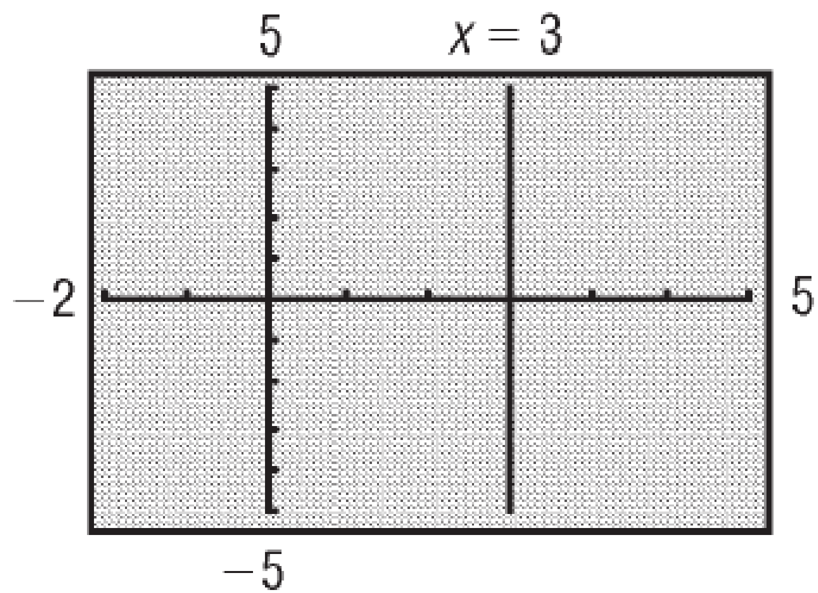
OBJECTIVE 3

3 Find the Equation of a Vertical Line

EXAMPLE **Graphing a Line**

Graph the equation: $x = 3$





Theorem

Addition Property of Inequalities

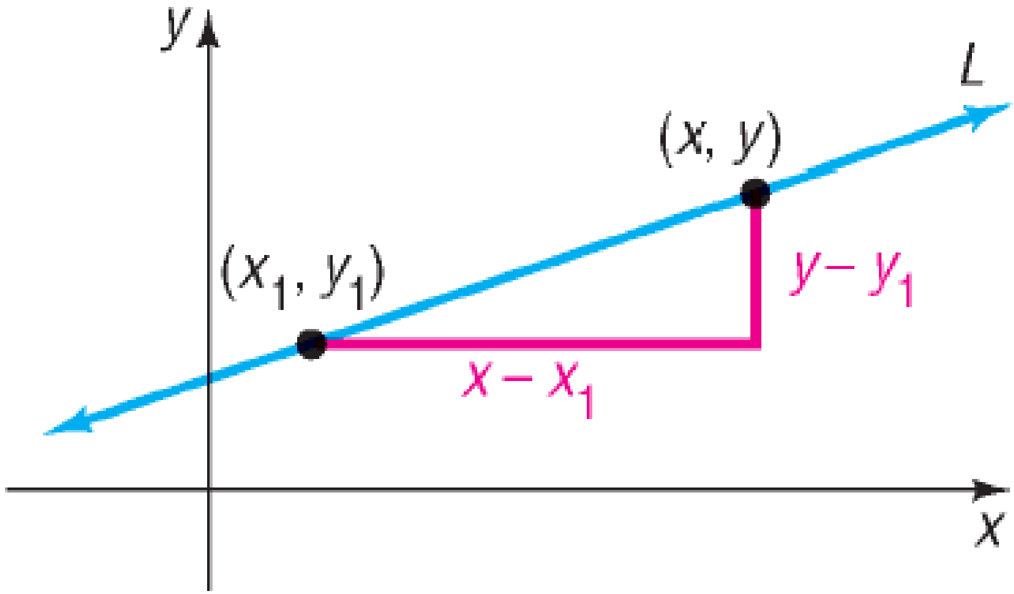
For real numbers a, b, and c,

if $a < b$, then $a + c < b + c$

if $a > b$, then $a + c > b + c$

OBJECTIVE 4

4 **Use the Point–Slope Form of a Line; Identify Horizontal Lines**



Theorem

Point–Slope Form of an Equation of a Line

An equation of a nonvertical line of slope m that contains the point (x_1, y_1) is

$$y - y_1 = m(x - x_1)$$

EXAMPLE

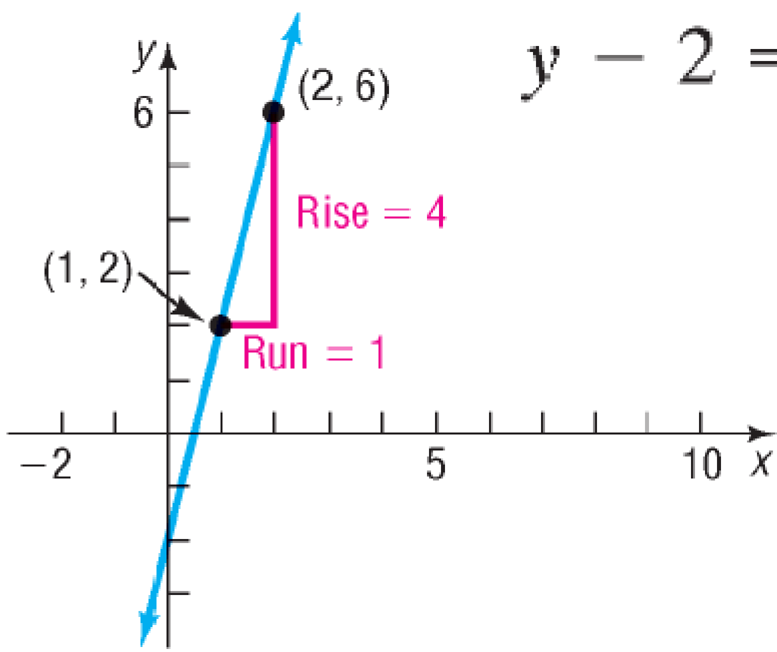
Using the Point–Slope Form of a Line

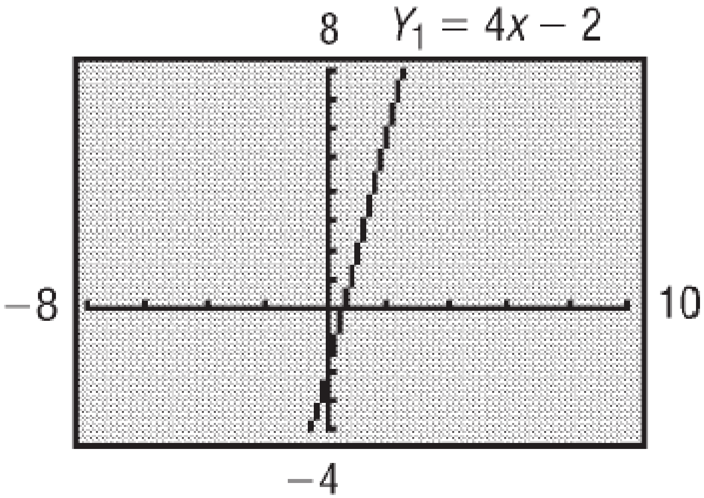
Find the equation of a line with slope 4 and containing the point (1, 2).

Solution $y - y_1 = m(x - x_1)$

$m = 4$, $x_1 = 1$, and $y_1 = 2$

$y - 2 = 4(x - 1)$ $\quad y = 4x - 2$





EXAMPLE

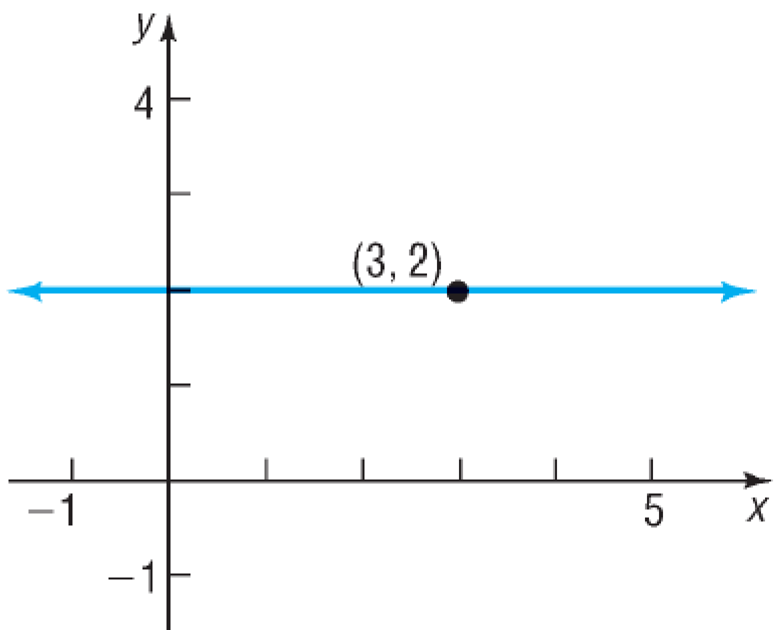
Finding the Equation of a Horizontal Line

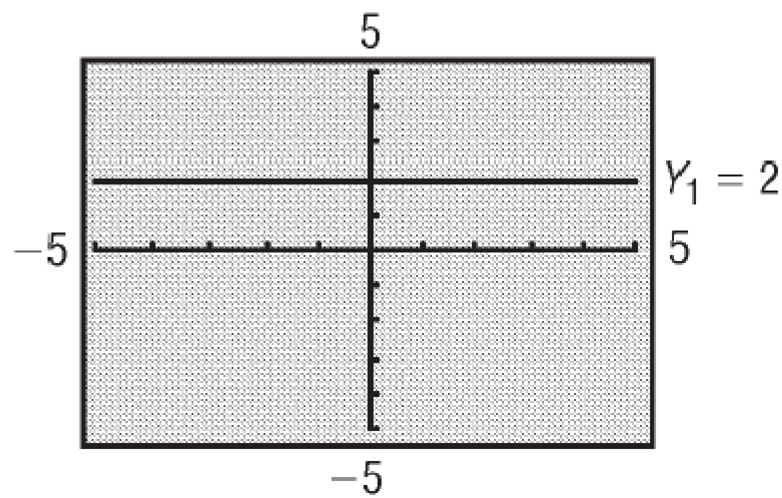
Find an equation of the horizontal line containing the point (3, 2).

Solution

$$y - y_1 = m(x - x_1)$$

$$y - 2 = 0 \cdot (x - 3) \qquad y = 2$$





Theorem

Equation of a Horizontal Line

A horizontal line is given by an equation of the form

$$y = b$$

where b is the y-intercept.

OBJECTIVE 5

5 Find the Equation of a Line Given Two Points

EXAMPLE

Finding an Equation of a Line Given Two Points

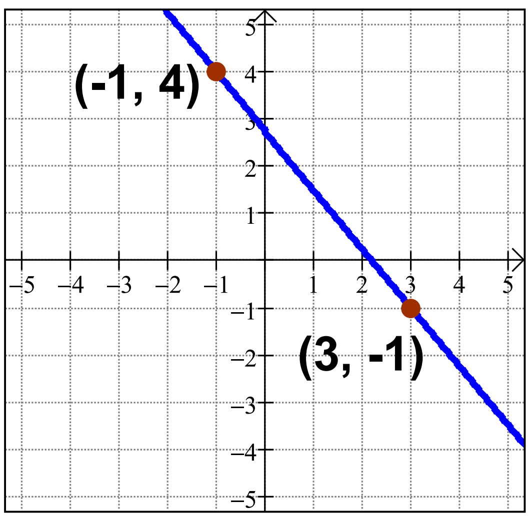
Find an equation of the line *L* containing the points (-1, 4) and (3, -1). Graph the line *L*.

Solution

$$y-4=-\frac{5}{4}(x+1)$$

or

$$y+1=-\frac{5}{4}(x-3)$$



OBJECTIVE 6

6 Write the Equation of a Line in Slope–Intercept Form

Theorem

Slope–Intercept Form of an Equation of a Line

An equation of a line L with slope m and y-intercept b is

$$y = mx + b$$

Seeing the Concept

graph the following lines on the same square screen.

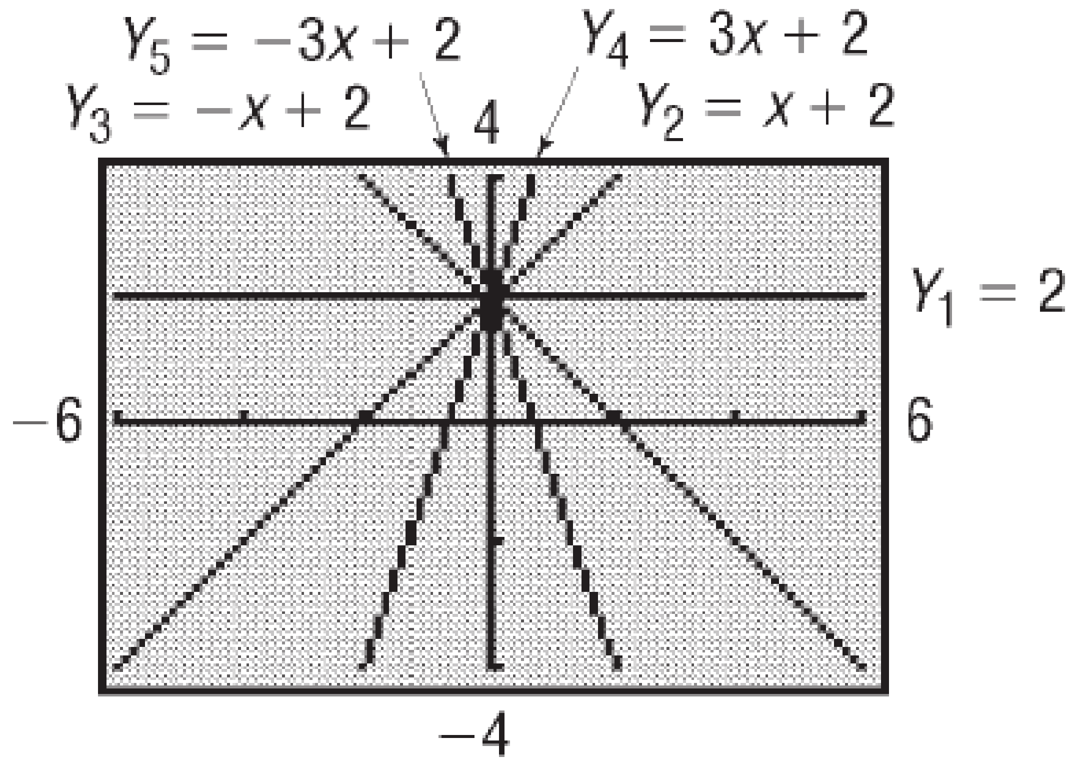
$Y_1 = 2$

$Y_2 = x + 2$

$Y_3 = -x + 2$

$Y_4 = 3x + 2$

$Y_5 = -3x + 2$



What do you conclude about the lines $y = mx + 2$?

Seeing the Concept

graph the following lines on the same square screen.

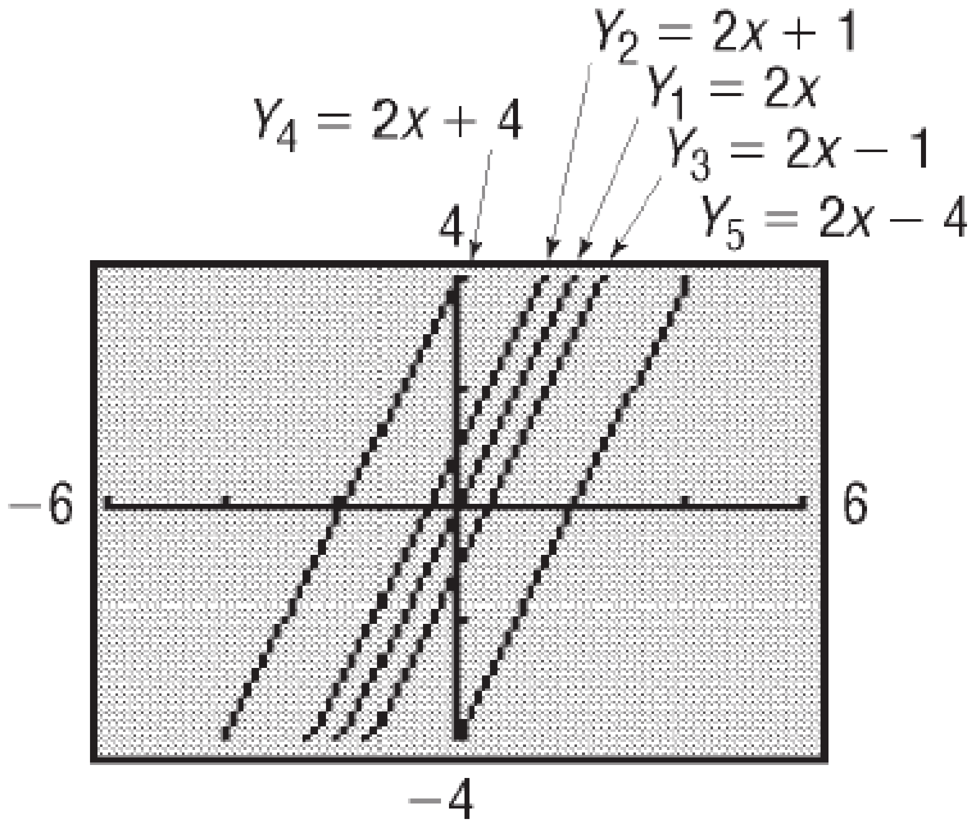
$Y_1 = 2x$

$Y_2 = 2x + 1$

$Y_3 = 2x - 1$

$Y_4 = 2x + 4$

$Y_5 = 2x - 4$



What do you conclude about the lines $y = 2x + b$?

OBJECTIVE 7

7 Identify the Slope and y-Intercept of a Line from Its Equation

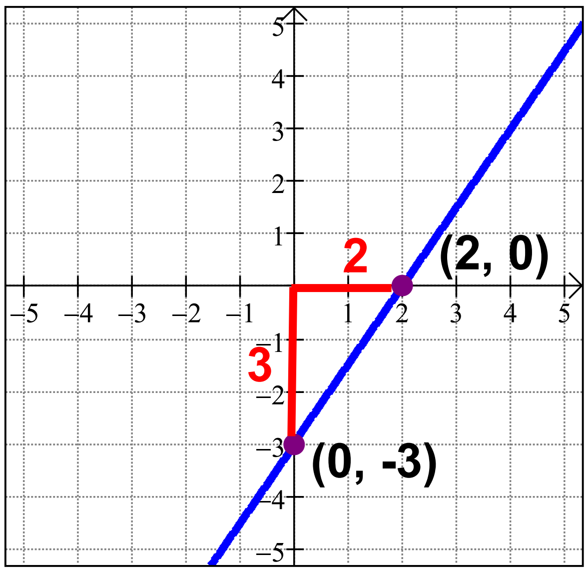
EXAMPLE Finding the Slope and y-Intercept

Find the slope m and y-intercept b of the equation $3x - 2y = 6$. Graph the equation.

Solution To obtain the slope and y-intercept, we transform the equation into its slope-intercept form by solving for y.

$$y = \frac{3}{2}x - 3$$

$$y = mx + b$$



OBJECTIVE 8

8 **Graph Lines Written in General Form Using Intercepts**

The equation of a line L is in **general form** when it is written as

$$Ax + By = C$$

where A, B, and C are real numbers and A and B are not both 0.

Procedure for Finding Intercepts

- To find the x-intercept(s), if any, of the graph of an equation, let $y = 0$ in the equation and solve for x.
- To find the y-intercept(s), if any, of the graph of an equation, let $x = 0$ in the equation and solve for y.

EXAMPLE

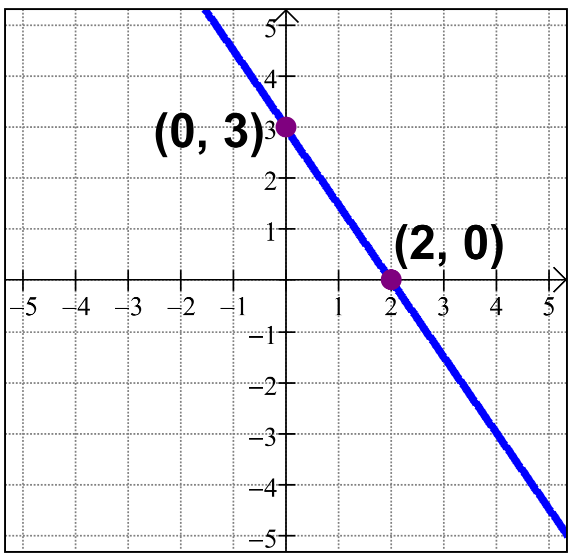
Graphing a Linear Equation Using Its Intercepts

Graph the linear equation $3x + 2y = 6$ by finding its intercepts.

Solution

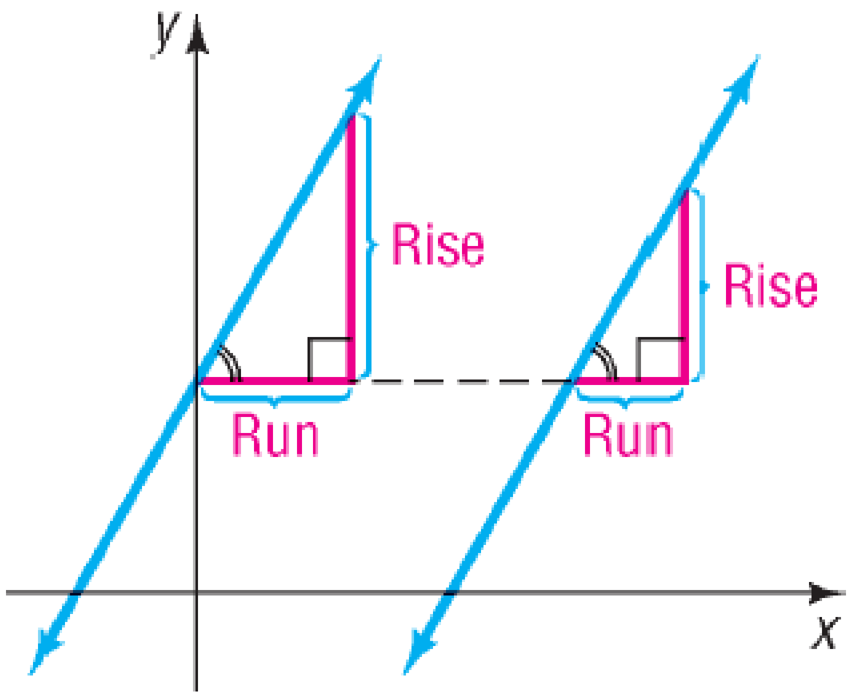
The x-intercept is at the point (2, 0)

The y-intercept is at the point (0, 3)



OBJECTIVE 9

9 **Find Equations of Parallel Lines**



Theorem

Criterion for Parallel Lines

Two nonvertical lines are parallel if and only if their slopes are equal and they have different *y*-intercepts.

EXAMPLE

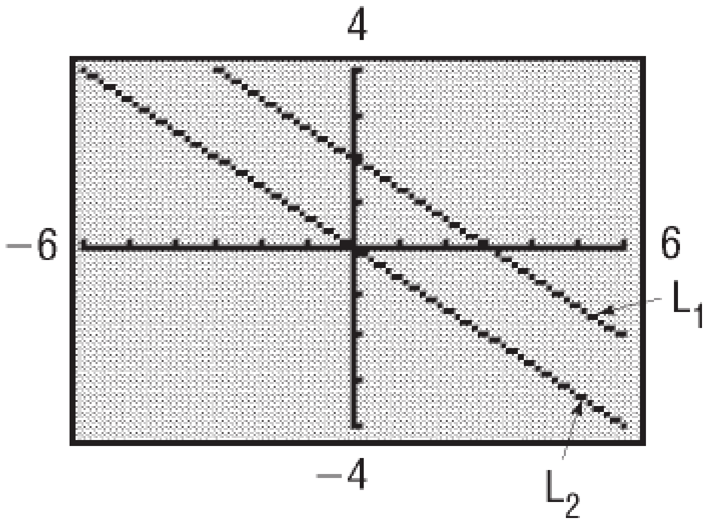
Showing That Two Lines Are Parallel

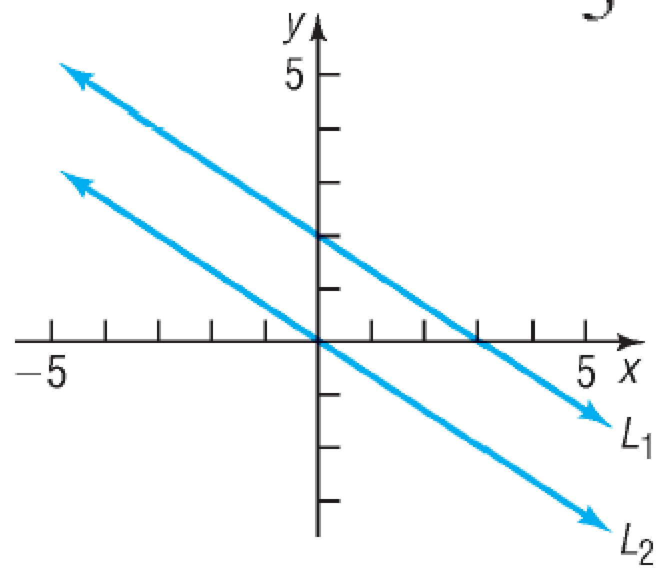
Show that the lines given by the following equations are parallel:

$$L_1: \quad 2x + 3y = 6, \qquad L_2: \quad 4x + 6y = 0$$

Solution

$$y = -\frac{2}{3}x + 2 \qquad\qquad y = -\frac{2}{3}x$$



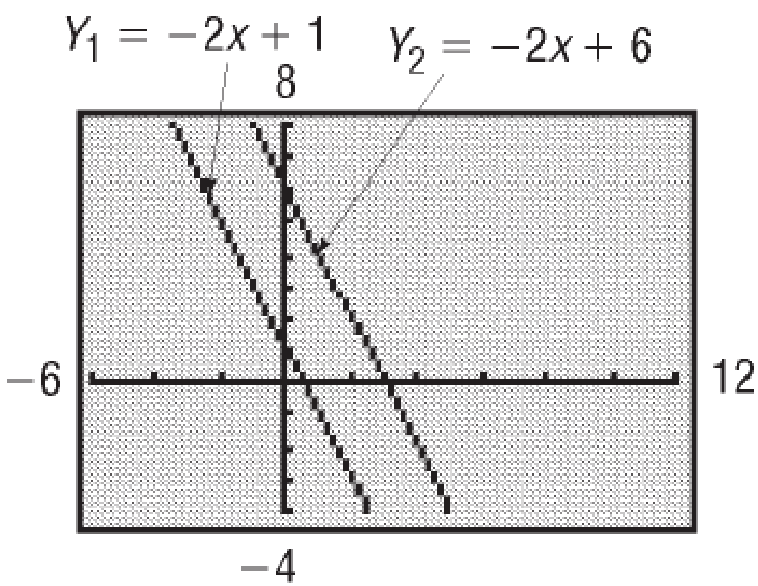


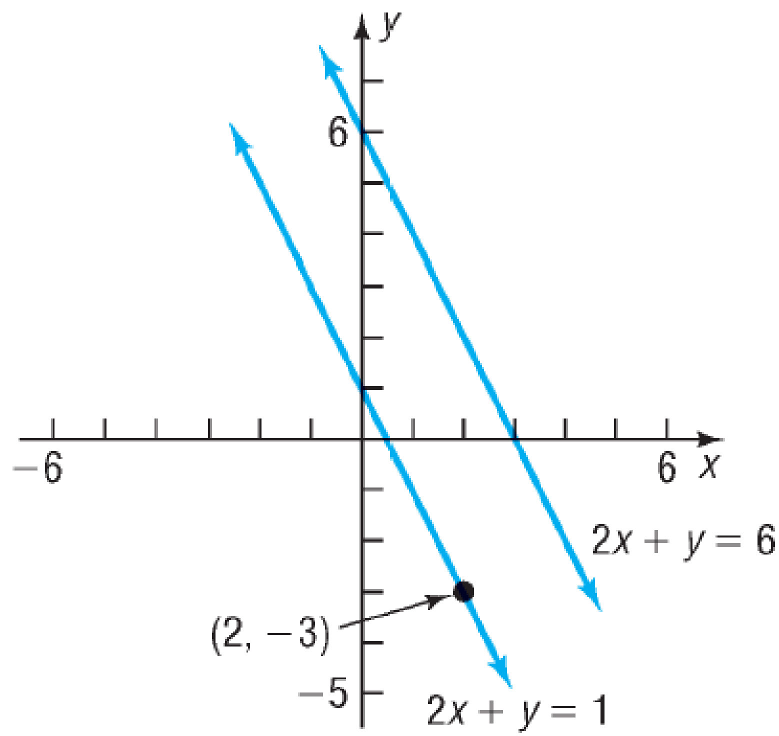
EXAMPLE

Finding a Line That Is Parallel to a Given Line

Find an equation for the line that contains the point $(2, -3)$ and is parallel to the line $2x + y = 6$.

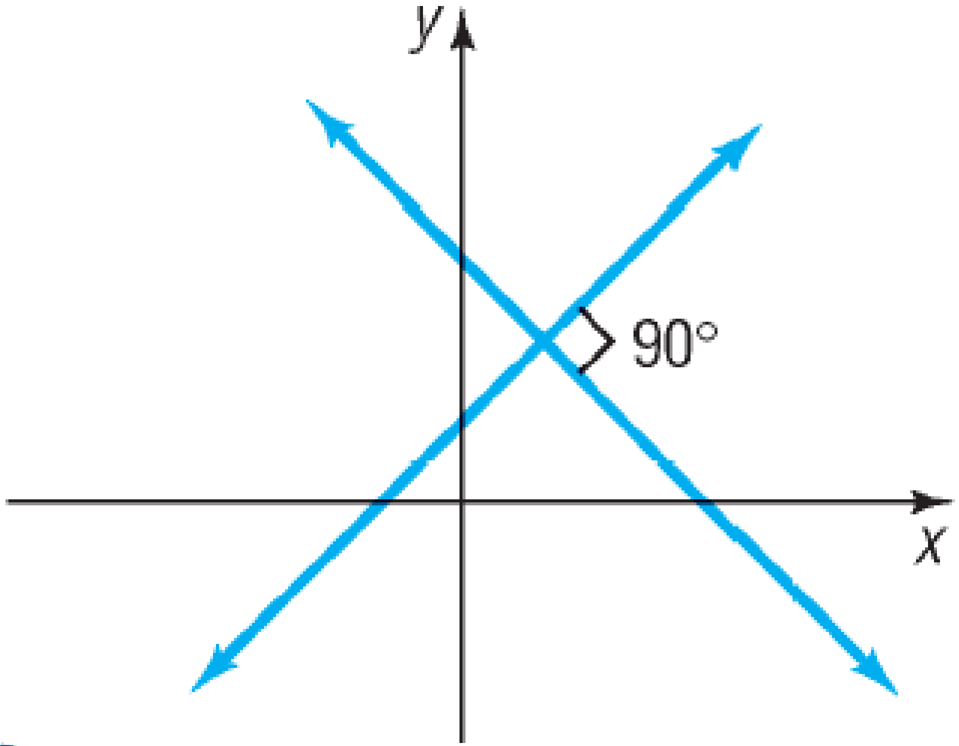
Solution





OBJECTIVE 10

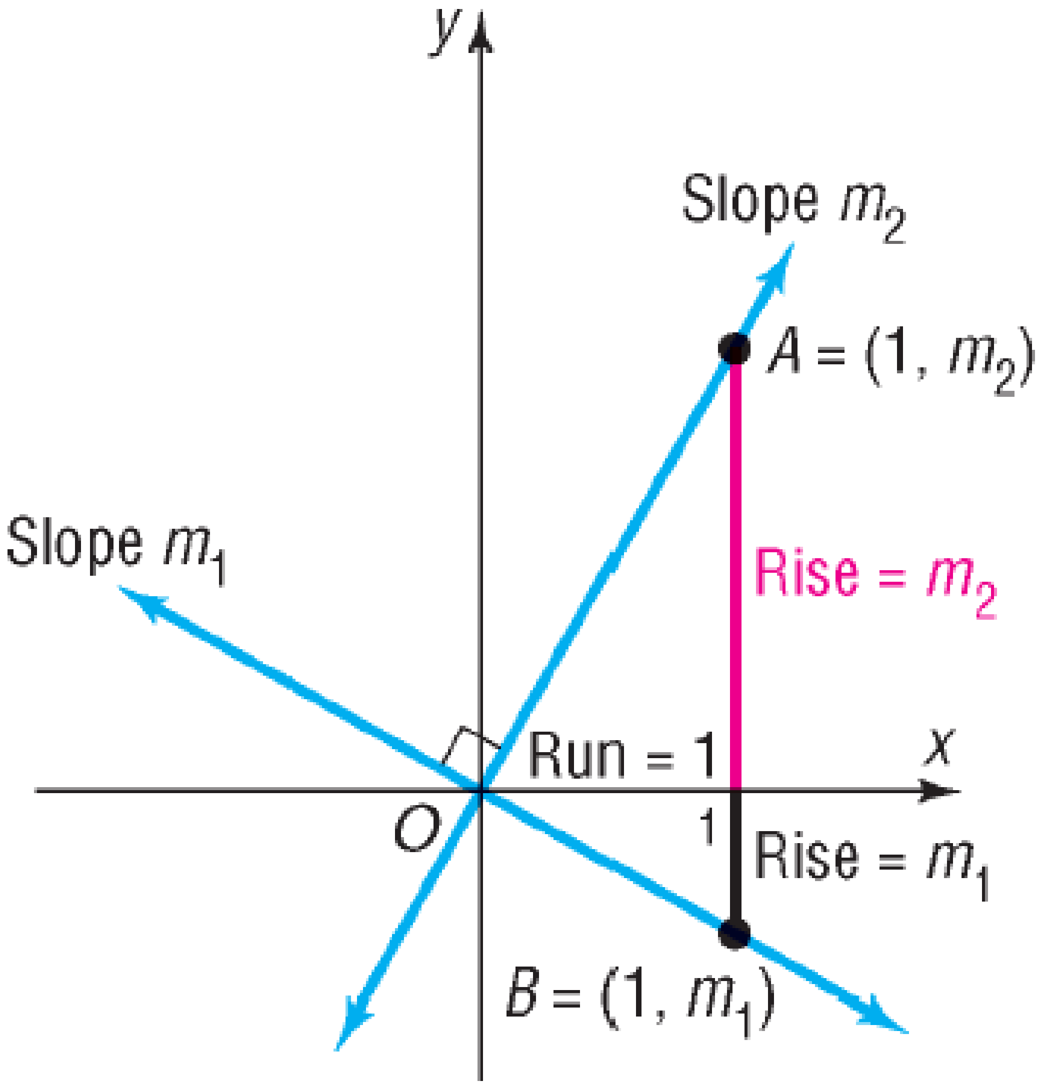
10 Find Equations of Perpendicular Lines



Theorem

Criterion for Perpendicular Lines

Two nonvertical lines are perpendicular if and only if the product of their slopes is -1.



y
x
Slope m_2
$A = (1, m_2)$
Slope m_1
Rise = m_2
Run = 1
O
1
Rise = m_1
$B = (1, m_1)$

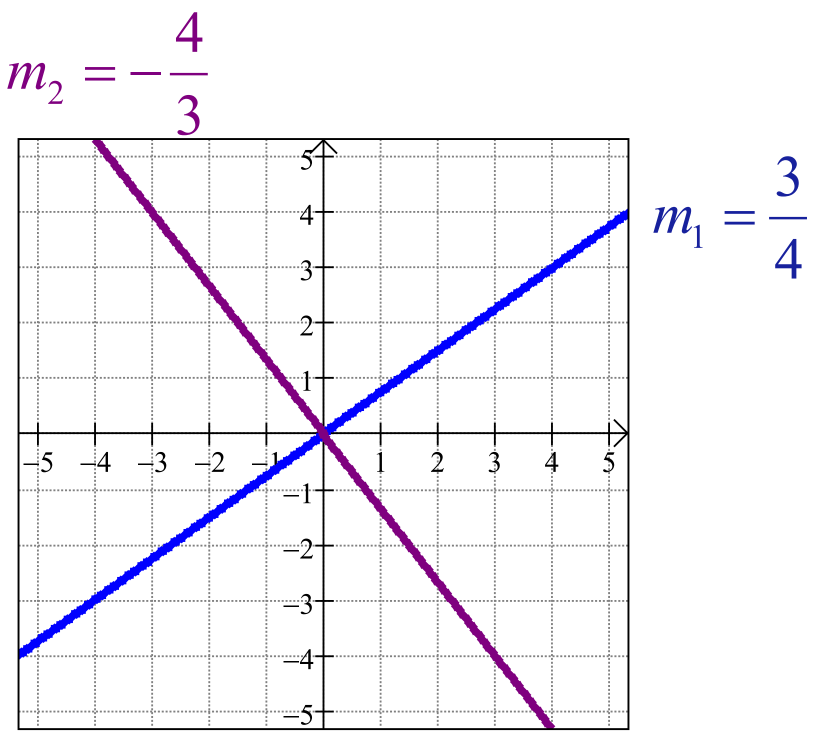
EXAMPLE

Finding the Slope of a Line Perpendicular to Another Line

Find the slope of a line perpendicular to a line with slope $\frac{3}{4}$.

Solution

$$m = -\frac{4}{3}$$



EXAMPLE

Finding the Equation of a Line Perpendicular to a Given Line

Find an equation of the line that contains the point $(1, -2)$ and is perpendicular to the line $x + 3y = 6$.
Graph the two lines.

Solution