## Section 2.2

The Graph of a Function

Average Price of Gasoline, Adjusted for Inflation.

| Year | Price | Year | Price |
| :---: | :---: | :---: | :---: |
| 1978 | 1.5207 | 1992 | 1.3460 |
| 1979 | 1.9263 | 1993 | 1.4622 |
| 1980 | 2.4197 | 1994 | 1.4166 |
| 1981 | 2.4258 | 1995 | 1.4111 |
| 1982 | 2.1136 | 1996 | 1.4804 |
| 1983 | 1.8210 | 1997 | 1.4697 |
| 1984 | 1.7732 | 1998 | 1.2685 |
| 1985 | 1.6998 | 1999 | 1.4629 |
| 1986 | 1.3043 | 2000 | 1.7469 |
| 1987 | 1.2831 | 2001 | 1.6821 |
| 1988 | 1.2674 | 2002 | 1.5339 |
| 1989 | 1.3134 | 2003 | 1.8308 |
| 1990 | 1.4160 | 2004 | 2.0047 |
| 1991 | 1.4490 |  |  |

## Price of Gasoline Adjusted

 for Inflation, 1978-2004

## OBJECTIVE 1

## Identify the Graph of a Function

## Theorem

## Vertical-line Test

A set of points in the $x y$-plane is the graph of a function if and only if every vertical line intersects the graph in at most one point.

## Identifying the Graph of a Function

Which of the following are graphs of functions?

(a) $y=x^{2}$

(b) $y=x^{3}$

## Identifying the Graph of a Function

Which of the following are graphs of functions?

(c) $x=y^{2}$

(d) $x^{2}+y^{2}=1$

## OBJECTIVE 2

2 Obtain Information from or about the Graph of a Function

## EXAMPLE

## Obtaining Information from the Graph of a Function


(a) What are $f(0), f\left(\frac{3 \pi}{2}\right)$, and $f(3 \pi)$ ?
(b) What is the domain of $f$ ?
(c) What is the range of $f$ ?
(d) List the intercepts.
(e) How often does the line $y=2$ intersect the graph?
(f) For what values of $x$ does $f(x)=-4$ ?
(g) For what values of $x$ is $f(x)>0$ ?

## Obtaining Information about the Graph of a Function

## Consider the function $f(x)=\frac{x}{x+1}$

(a) Is the point $\left(1, \frac{1}{2}\right)$ on the graph of $f$ ?
(b) If $x=2$, what is $f(x)$ ? What point is on the graph of $f$ ?
(c) If $f(x)=2$, what is $x$ ? What point is on the graph of $f$ ?

## EXAMPLE Average Cost Function

The average cost $\bar{C}$ of manufacturing $x$ computers per day is given by the function

$$
\bar{C}(x)=0.56 x^{2}-34.39 x+1212.57+\frac{20,000}{x}
$$

Determine the average cost of manufacturing:
(a) 30 computers in a day
(b) 40 computers in a day
(c) 50 computers in a day
(d) Graph the function $\bar{C}=\bar{C}(x), 0<x \leq 80$.
(e) Create a TABLE with TbIStart $=1$ and $\Delta \mathrm{Tbl}=1$.

Which value of $x$ minimizes the average cost?

## EXAMPLE Average Cost Function

$$
\bar{C}(x)=0.56 x^{2}-34.39 x+1212.57+\frac{20,000}{x}
$$

(a) 30 computers in a day $=\$ 1351.54$
(b) 40 computers in a day $=\$ 1232.97$
(c) 50 computers in a day $=\$ 1293.07$
(d) Graph the function $\bar{C}=\bar{C}(x), 0<x \leq 80$.
(e) Create a TABLE with TblStart $=1$ and $\Delta \mathrm{Tbl}=1$

| \% | F1 |  |
| :---: | :---: | :---: |
| 1 | 21179 |  |
| 2 | 11146 |  |
| 3 | 7781.1 |  |
| 4 | 6084 |  |
| 5 | 5054.6 |  |
| 6 | 4359.7 |  |
| 7 | 2856. 4 |  |


| 4 | Y1 |  |
| :---: | :---: | :---: |
| 38 | 1240.7 |  |
| 35 | 1285.9 |  |
| 40 | 1283 |  |
| 41 | 12\%317 |  |
| 42 | 1232.2 |  |
| 43 | 1254.4 |  |
| 44 | 1228.1 |  |



Which value of $x$ minimizes the average cost? $\$ 1231.74$

## Summary

## Graph of a function

The collection of points $(x, y)$ that satisfies the equation $y=f(x)$.
A collection of points is the graph of a function provided that every vertical line intersects the graph in at most one point (vertical-line test).

