Section 2.3

Properties of Functions

OBJECTIVE 1

Determine Even and Odd Functions from a Graph

A function f is **even** if, for every number x in its domain, the number $-x$ is also in the domain and

$$\boxed{f(-x) = f(x)}$$

For an **even** function, for every point (x, y) on the graph, the point ($-x$, y) is also on the graph.

A function f is **odd** if, for every number x in its domain, the number $-x$ is also in the domain and

$$\boxed{f(-x) = -f(x)}$$

So for an **odd** function, for every point (x, y) on the graph, the point ($-x$, $-y$) is also on the graph.

Theorem

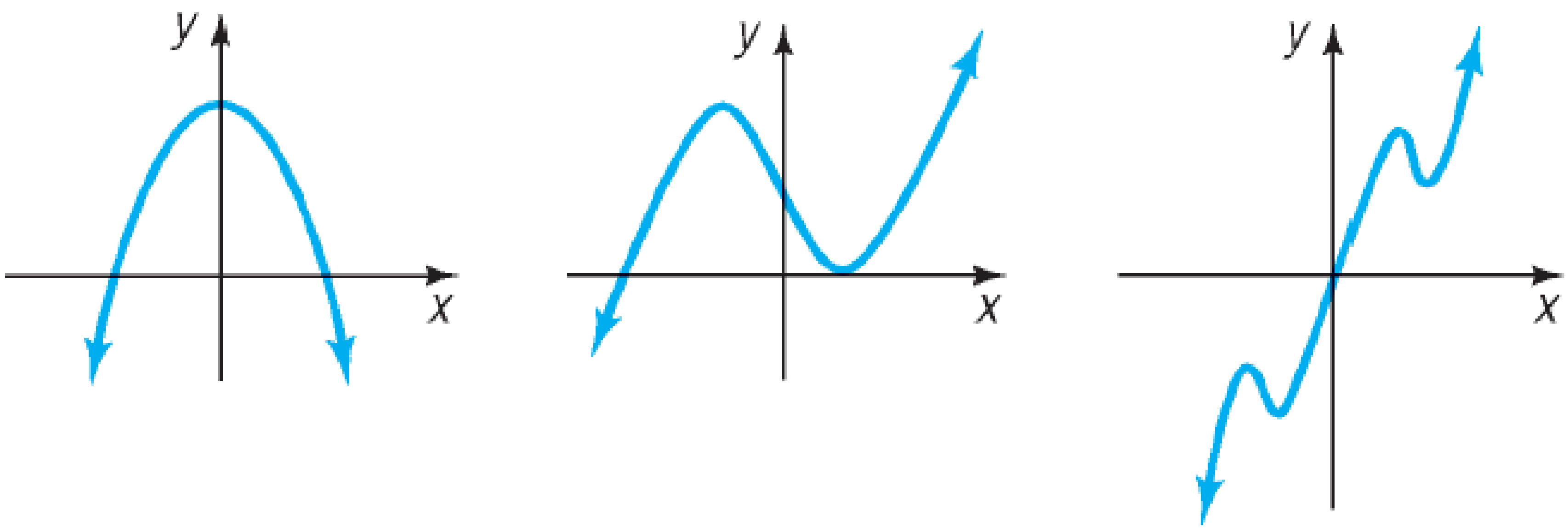
A function is even if and only if its graph is symmetric with respect to the y-axis.

A function is odd if and only if its graph is symmetric with respect to the origin.

EXAMPLE

Determining Even and Odd Functions from the Graph

Determine whether each graph given is an even function, an odd function, or a function that is neither even nor odd.



OBJECTIVE 2

2 Identify Even and Odd Functions from the Equation

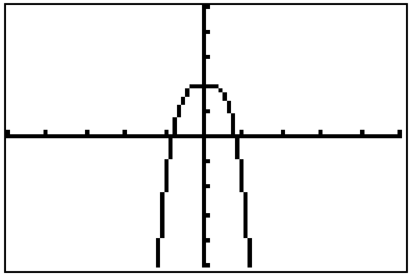
EXAMPLE

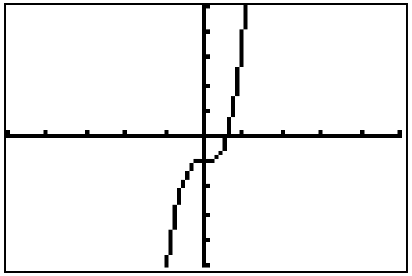
Identifying Even and Odd Functions

Use a graphing utility to conjecture whether each of the following functions is even, odd, or neither. Verify the conjecture algebraically. Then state whether the graph is symmetric with respect to the y-axis or with respect to the origin.

$$f(x) = -3x^4 - x^2 + 2$$

$$f(-x) = -3x^4 - x^2 + 2$$



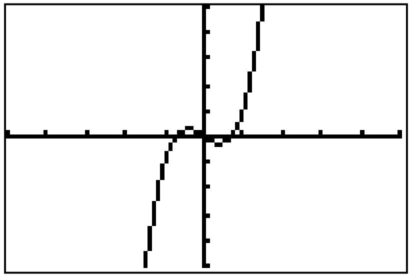


$$g(x) = 5x^3 - 1$$

$$g(-x) = -5x^3 - 1$$

$$h(x) = 2x^3 - x$$

$$h(-x) = -2x^3 + x$$



OBJECTIVE 3

3 Use a Graph to Determine Where a Function Is Increasing, Decreasing, or Constant

INCREASING

DECREASING

CONSTANT

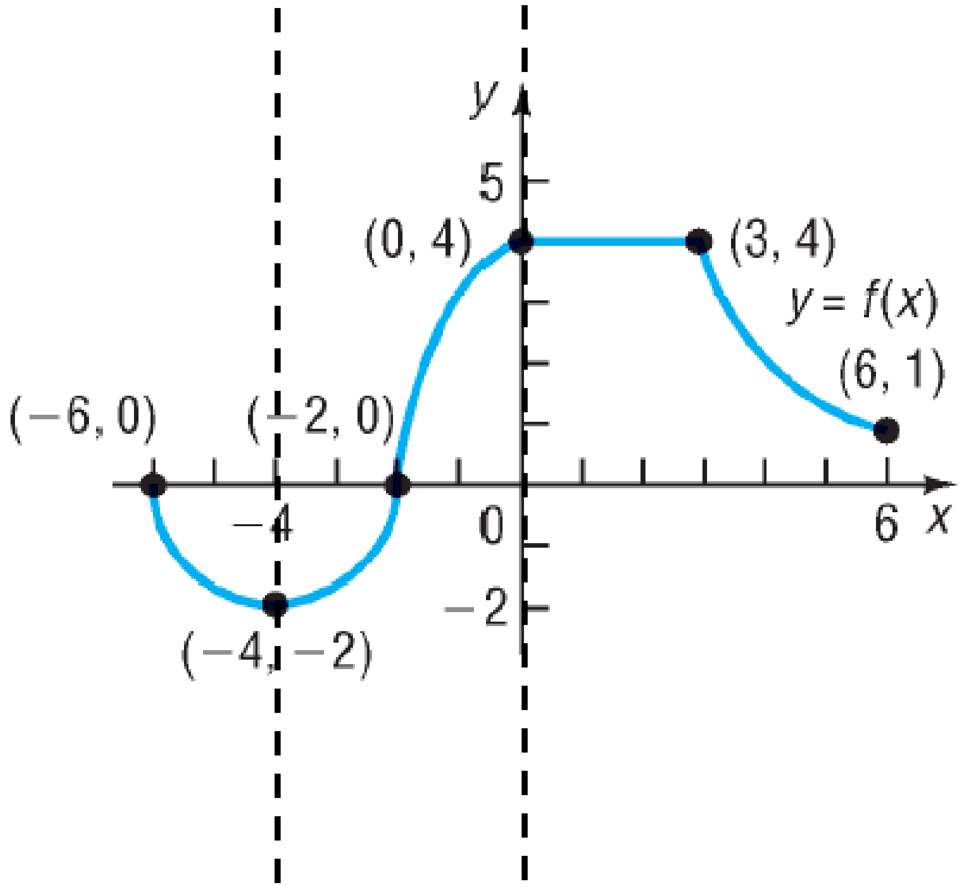
EXAMPLE

Determining Where a Function Is Increasing, Decreasing, or Constant from Its Graph

Where is the function increasing?

$$-4 < x < 0$$

$$(-4, 0)$$



EXAMPLE

Determining Where a Function Is Increasing, Decreasing, or Constant from Its Graph

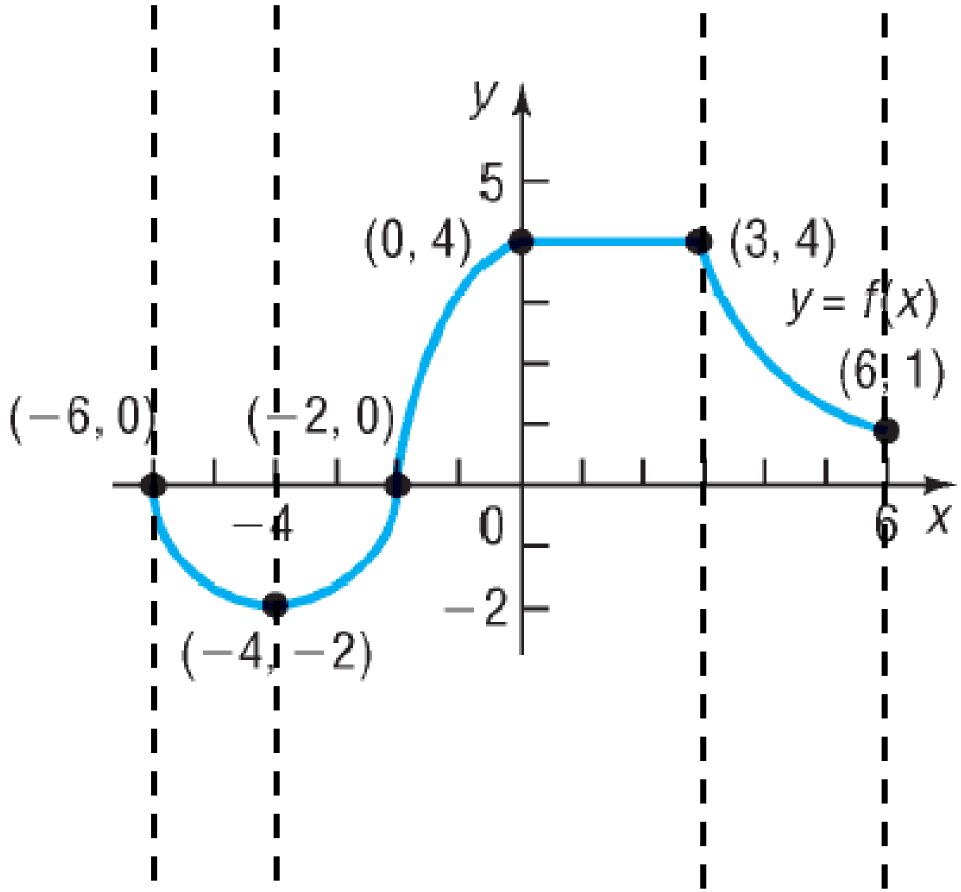
Where is the function decreasing?

$$-6 < x < -4$$

$$(-6, -4)$$

$$3 < x < 6$$

$$(3, 6)$$



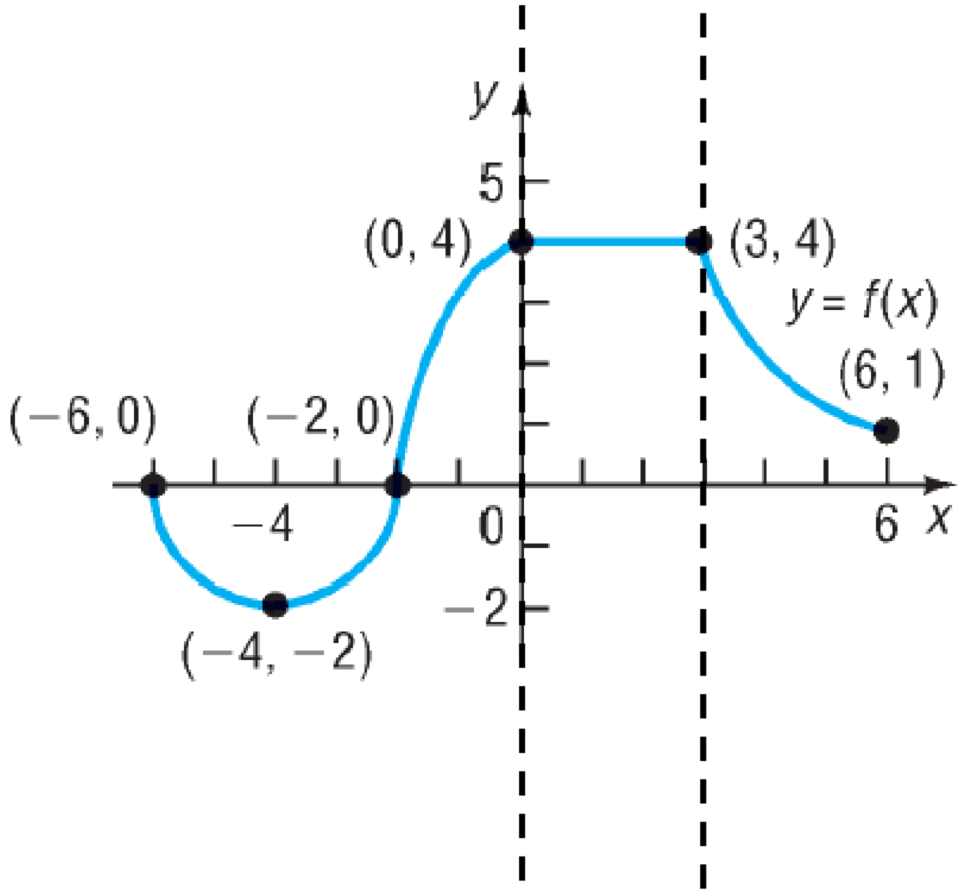
EXAMPLE

Determining Where a Function Is Increasing, Decreasing, or Constant from Its Graph

Where is the function constant?

$$0 < x < 3$$

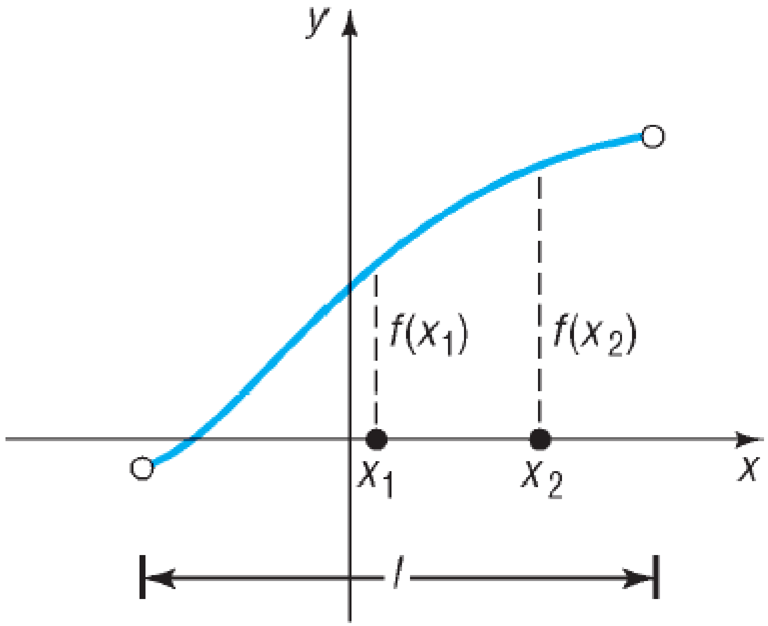
$$(0, 3)$$



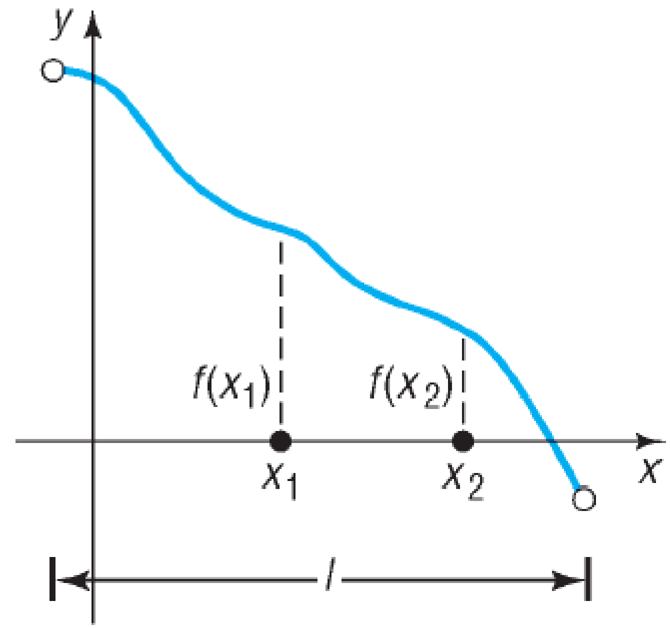
A function f is **increasing** on an open interval I if, for any choice of x_1 and x_2 in I, with $x_1 < x_2$, we have $f(x_1) < f(x_2)$.

A function f is **decreasing** on an open interval I if, for any choice of x_1 and x_2 in I, with $x_1 < x_2$, we have $f(x_1) > f(x_2)$.

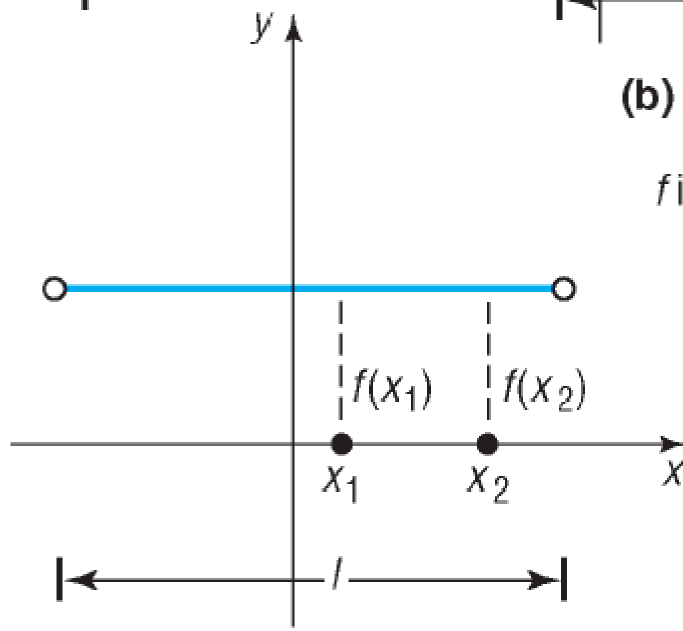
A function f is **constant** on an interval I if, for all choices of x in I, the values $f(x)$ are equal.



(a) For $x_1 < x_2$ in I,
$f(x_1) < f(x_2)$;
f is increasing on I



(b) For $x_1 < x_2$ in I,
$f(x_1) > f(x_2)$;
f is decreasing on I



(c) For all x in I, the values of
f are equal; f is constant on I

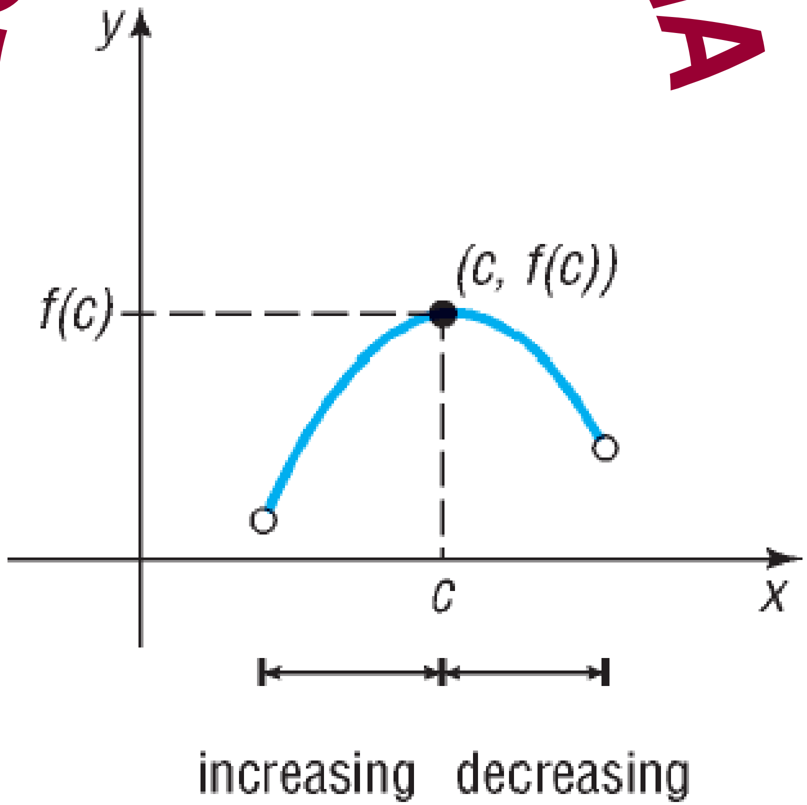
OBJECTIVE 4

4 **Use a Graph to Locate Local Maxima and Local Minima**

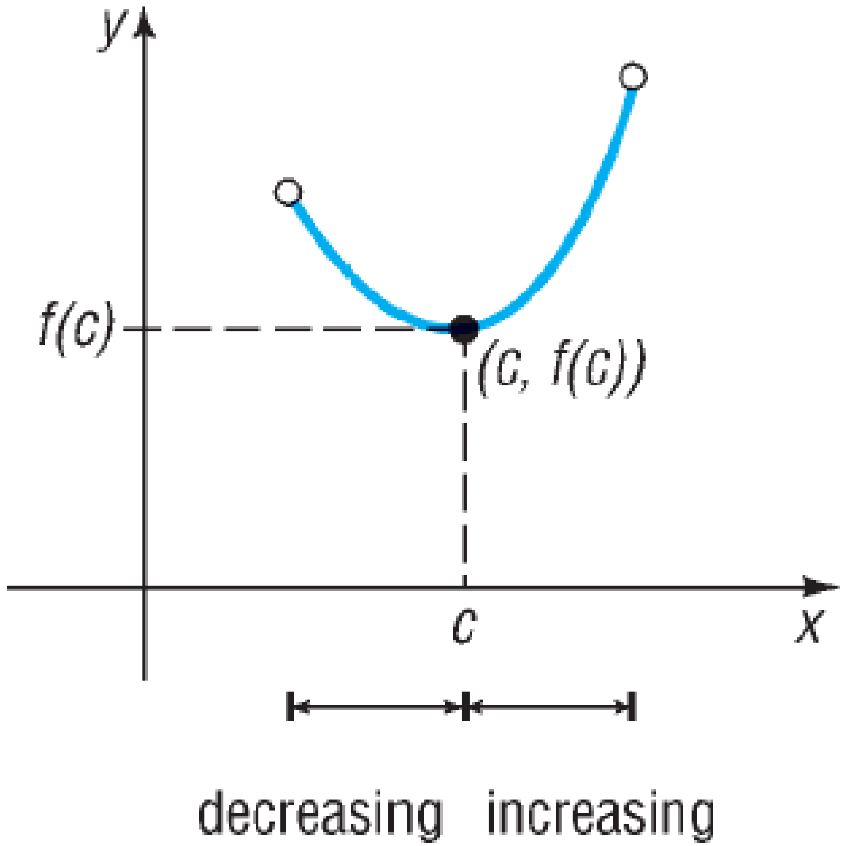
LOCAL MAXIMA

LOCAL MINIMA

LOCAL MAXIMA



The local maximum is $f(c)$ and occurs at $x = c$.



The local minimum is $f(c)$ and occurs at $x = c$.

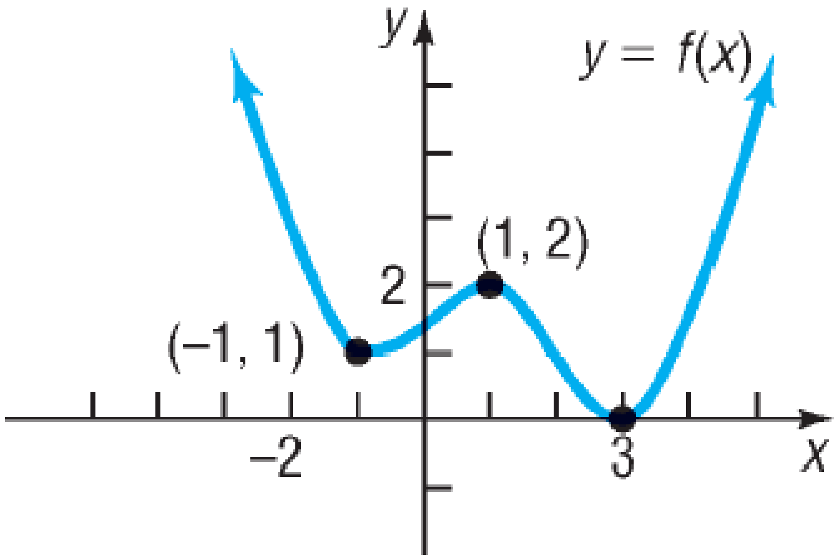
LOCAL MINIMA

A function f has a **local maximum** at c if there is an open interval I containing c so that, for all $x \neq c$ in I, $f(x) \leq f(c)$. We call $f(c)$ a **local maximum of *f***.

A function f has a **local minimum** at c if there is an open interval I containing c so that, for all $x \neq c$ in I, $f(x) \geq f(c)$. We call $f(c)$ a **local minimum of *f***.

EXAMPLE

Finding Local Maxima and Local Minima from the Graph of a Function and Determining Where the Function Is Increasing, Decreasing, or Constant

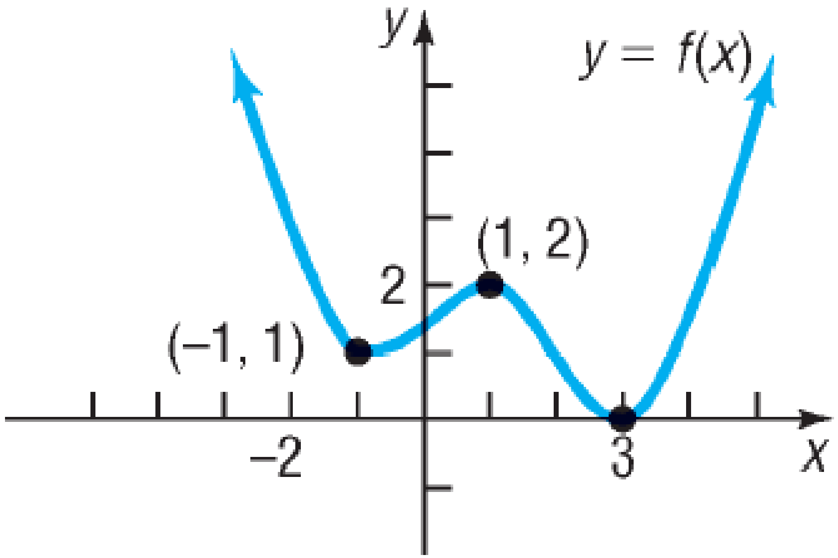


(a) At what number(s), if any, does f have a local maximum?

(b) What are the local maxima?

EXAMPLE

Finding Local Maxima and Local Minima from the Graph of a Function and Determining Where the Function Is Increasing, Decreasing, or Constant

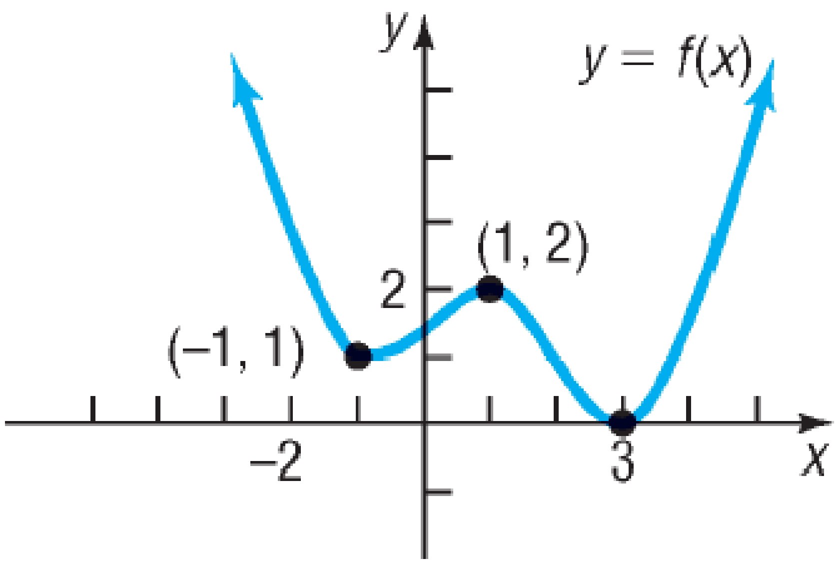


(c) At what number(s), if any, does f have a local minimum?

(d) What are the local minima?

EXAMPLE

Finding Local Maxima and Local Minima from the Graph of a Function and Determining Where the Function Is Increasing, Decreasing, or Constant



(e) List the intervals on which f is increasing. List the intervals on which f is decreasing.

OBJECTIVE 5

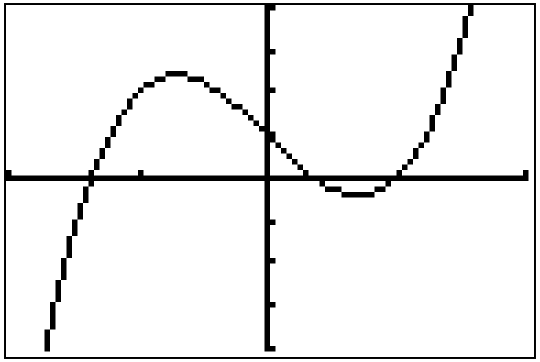
5 **Use a Graphing Utility to Approximate Local Maxima and Local Minima and to Determine Where a Function Is Increasing or Decreasing**

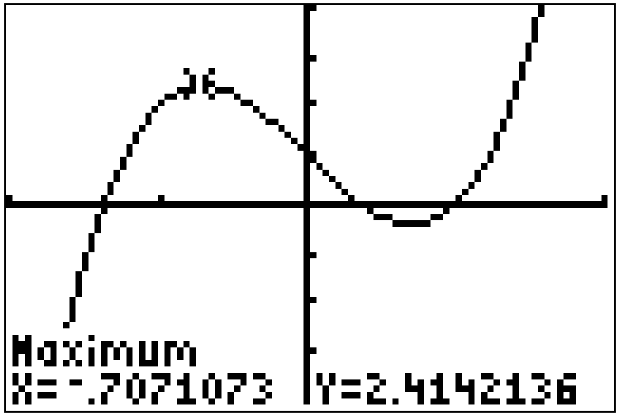
EXAMPLE

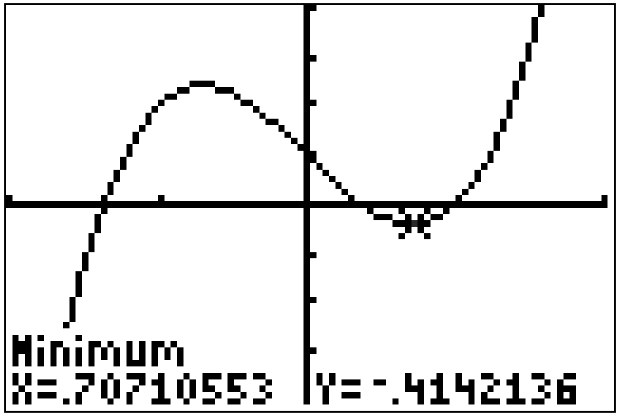
Using a Graphing Utility to Approximate Local Maxima and Minima and to Determine Where a Function Is Increasing or Decreasing

Use a graphing utility to graph $f(x) = 2x^3 - 3x + 1$ for $-2 < x < 2$.

Approximate where f has any local maxima or local minima.





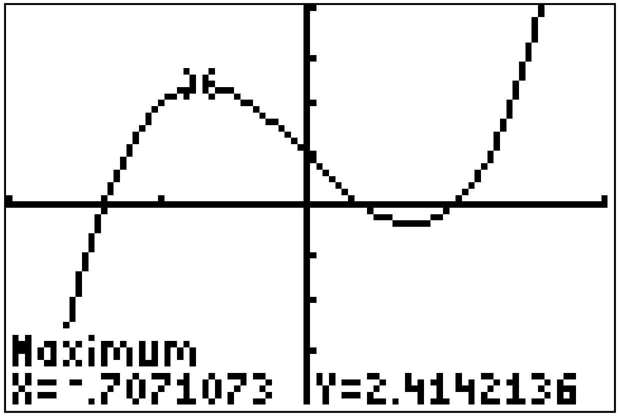


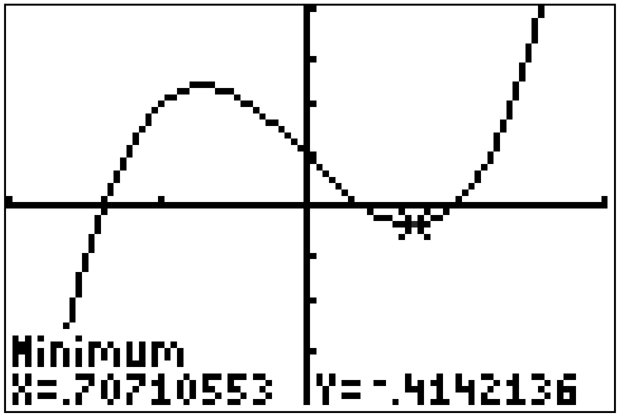
EXAMPLE

Using a Graphing Utility to Approximate Local Maxima and Minima and to Determine Where a Function Is Increasing or Decreasing

Use a graphing utility to graph $f(x) = 2x^3 - 3x + 1$ for $-2 < x < 2$.

Determine where f is increasing and where it is decreasing.





OBJECTIVE 6

6 **Find the Average Rate of Change of a Function**

If c is in the domain of a function $y = f(x)$, the **average rate of change of f from c to x** is defined as

$$\boxed{\text{Average rate of change} = \frac{\Delta y}{\Delta x} = \frac{f(x) - f(c)}{x - c}, \qquad x \neq c \qquad \textbf{(1)}}$$

In calculus, this expression is called the **difference quotient** of f at c.

EXAMPLE

Finding the Average Rate of Change

Find the average rate of change of $f(x) = \frac{1}{2}x^2$:

From 0 to 1

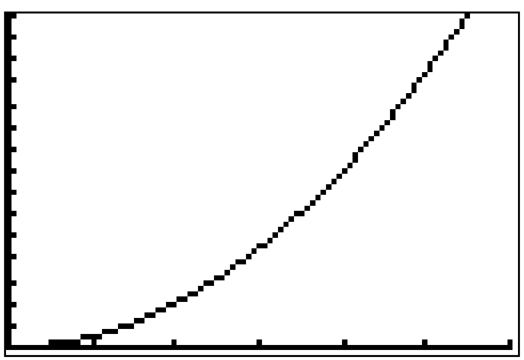
$$\frac{\Delta y}{\Delta x} = \frac{1}{2}$$

From 0 to 3

$$\frac{\Delta y}{\Delta x} = \frac{3}{2}$$

From 0 to 5

$$\frac{\Delta y}{\Delta x} = \frac{5}{2}$$



$$\boxed{\frac{\Delta y}{\Delta x} = \frac{f(x) - f(c)}{x - c}}$$

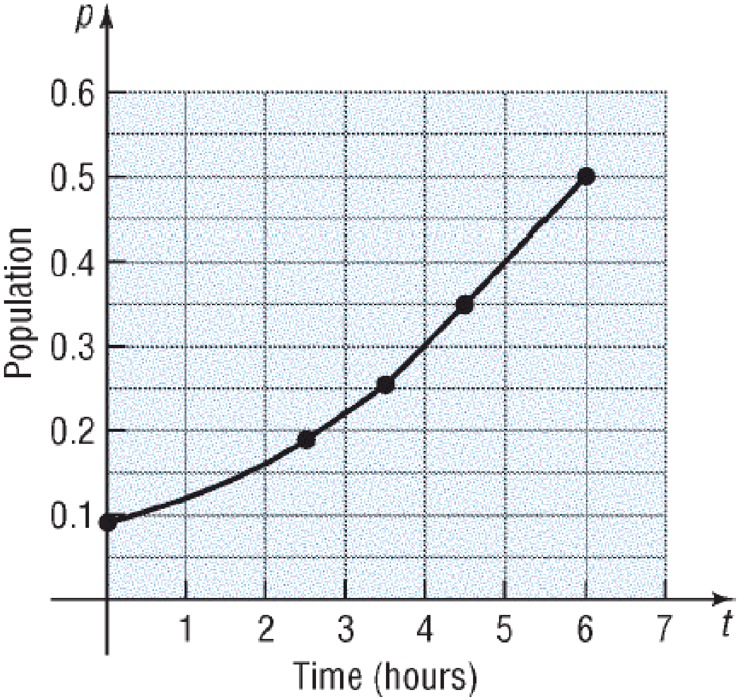
EXAMPLE

Average Rate of Change of a Population

A strain of E-coli Beu 397-recA441 is placed into a nutrient broth at 30° Celsius and allowed to grow. The data shown in Table 9 are collected. The population is measured in grams and the time in hours. Since population P depends on time t and each input corresponds to exactly one output, we can say that population is a function of time, so $P(t)$ represents the population at time t. For example, $P(2.5) = 0.18$.

Table 9

Time (hours), x	Population (grams), y
0	0.09
2.5	0.18
3.5	0.26
4.5	0.35
6	0.50



EXAMPLE

Average Rate of Change of a Population

(a) Find the average rate of change of the population from 0 to 2.5 hours.
(b) Find the average rate of change of the population from 4.5 to 6 hours.
(c) What is happening to the average rate of change as time passes?

Table 9

Time (hours), x	Population (grams), y
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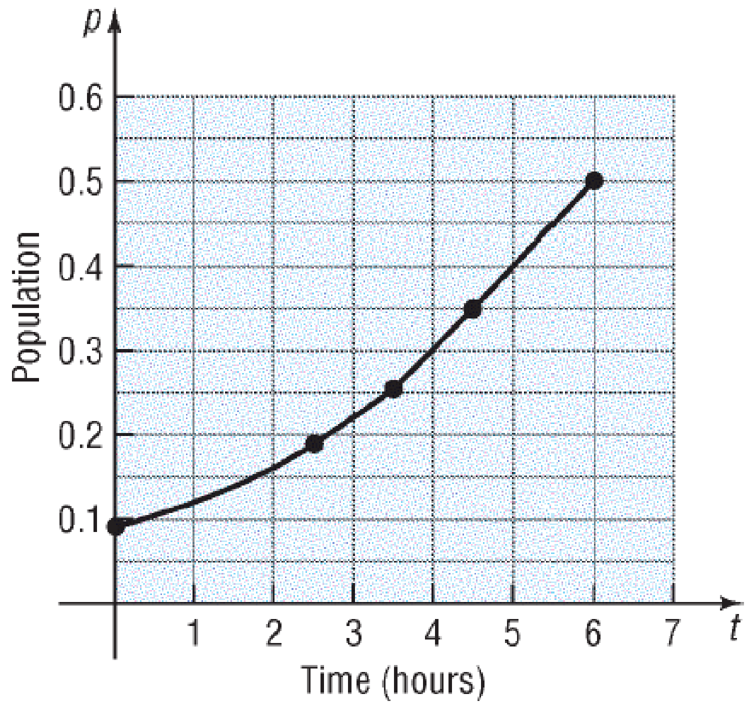
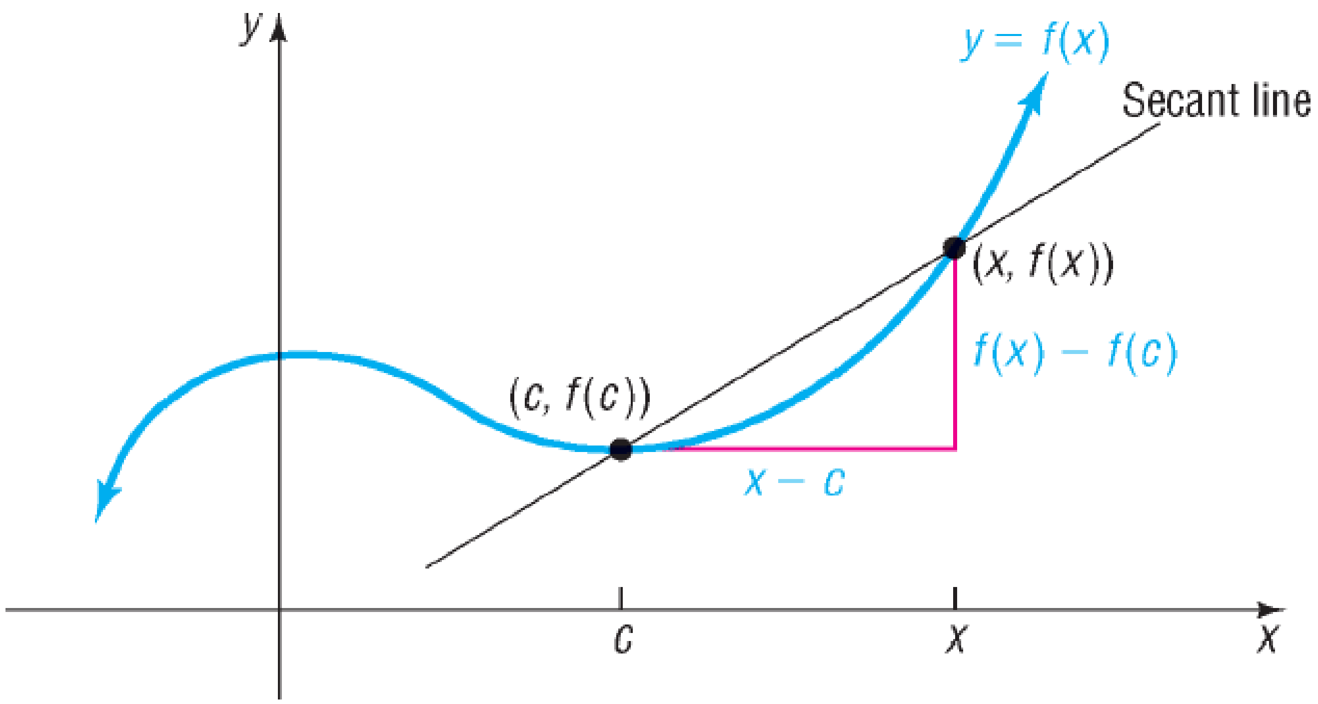


The Secant Line

$$m_{\text{sec}} = \frac{f(x) - f(c)}{x - c}$$



Theorem

Slope of the Secant Line

The average rate of change of a function equals the slope of the secant line containing two points on its graph.

EXAMPLE

Finding the Equation of a Secant Line

Suppose that $g(x) = -2x^2 + 4x - 3$.

(a) Find the average rate of change of g from -2 to x.

(b) Use the result of part (a) to find the average rate of change of g from -2 to 1. Interpret this result.

(c) Find an equation of the secant line containing $(-2, g(-2))$ and $(1, g(1))$.

(d) Using a graphing utility, draw the graph of g and the secant line obtained in part (c) on the same screen.