Section 2.6

Graphing Techniques;

Transformations

OBJECTIVE 1

1 Graph Functions Using Vertical and Horizontal Shifts

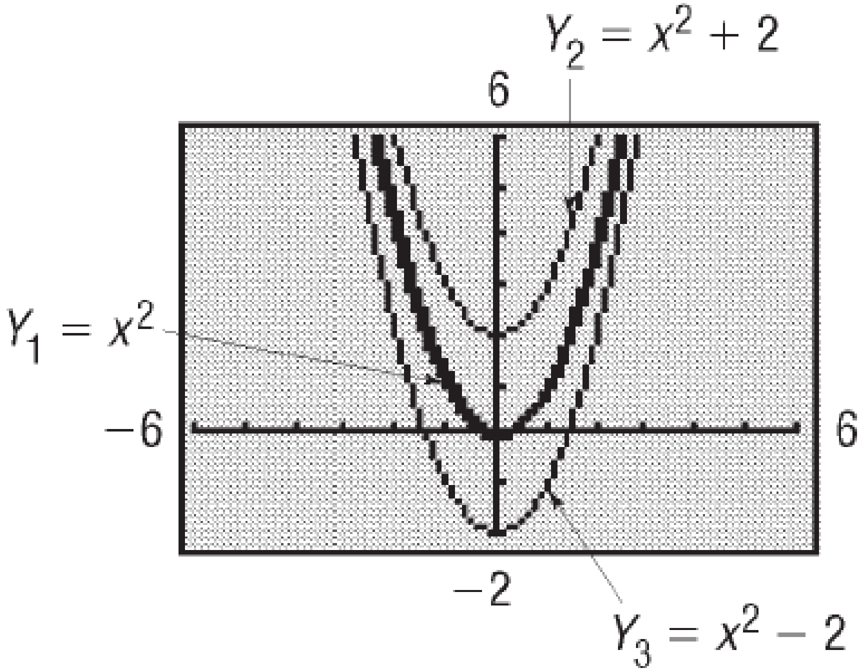
Exploration

On the same screen, graph each of the following functions:

$Y_1 = x^2$

$Y_2 = x^2 + 2$

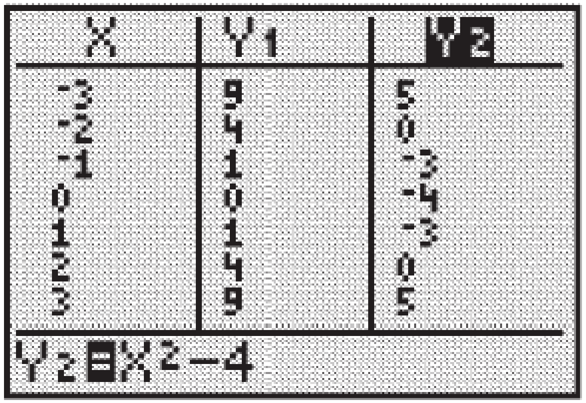
$Y_3 = x^2 - 2$

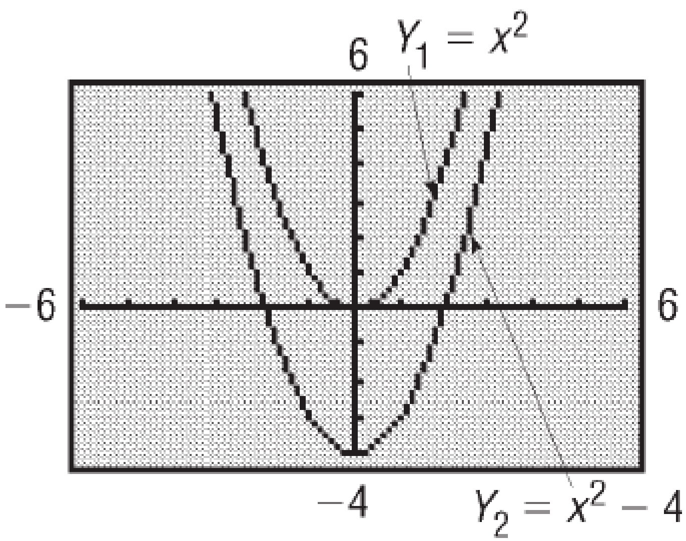


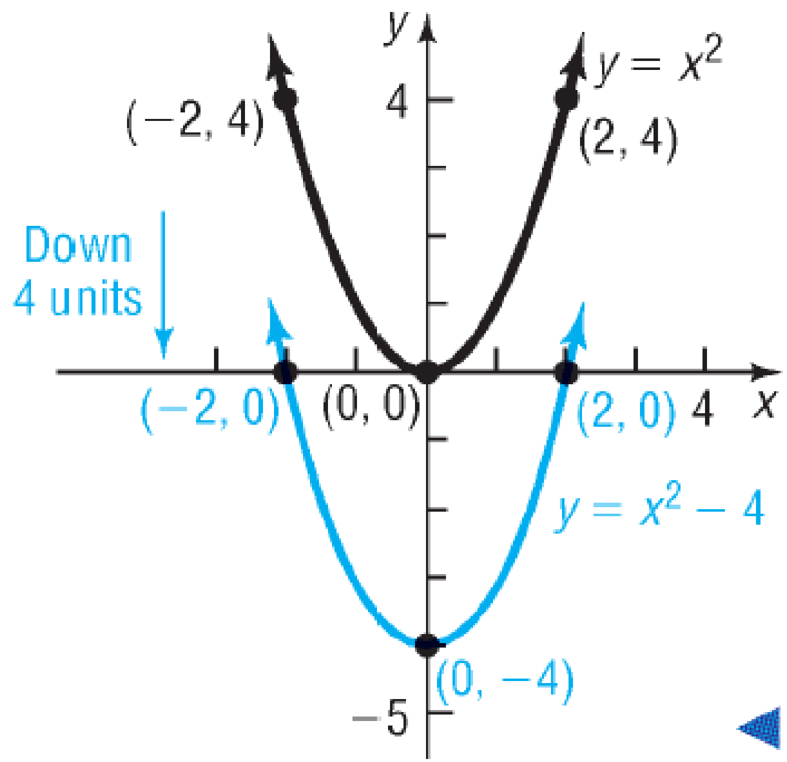
If a real number k is added to the right side of a function $y = f(x)$, the graph of the new function $y = f(x) + k$ is the graph of f **shifted vertically up** k units (if $k > 0$) or **down** $|k|$ units (if $k < 0$).

EXAMPLE Vertical Shift Down

Use the graph of $f(x) = x^2$ to obtain the graph of $h(x) = x^2 - 4$.

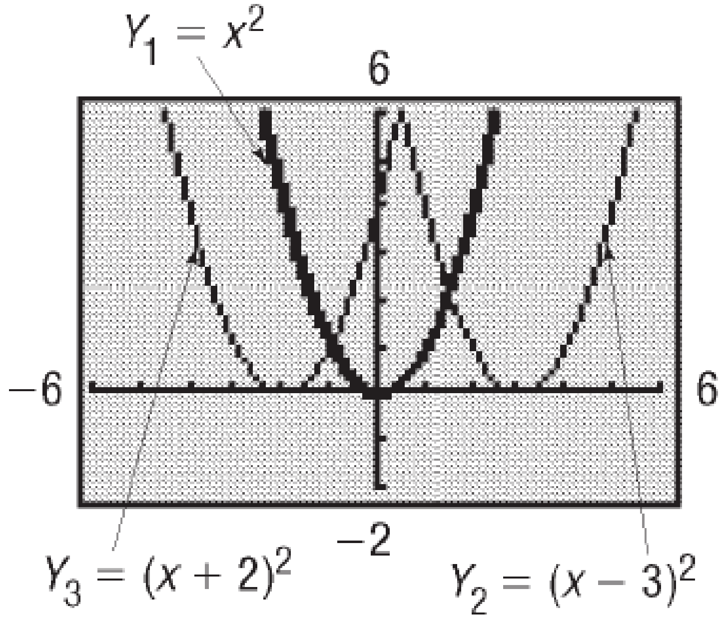






Exploration

On the same screen, graph each of the following functions:



$$Y_1 = x^2$$

$$Y_2 = (x - 3)^2$$

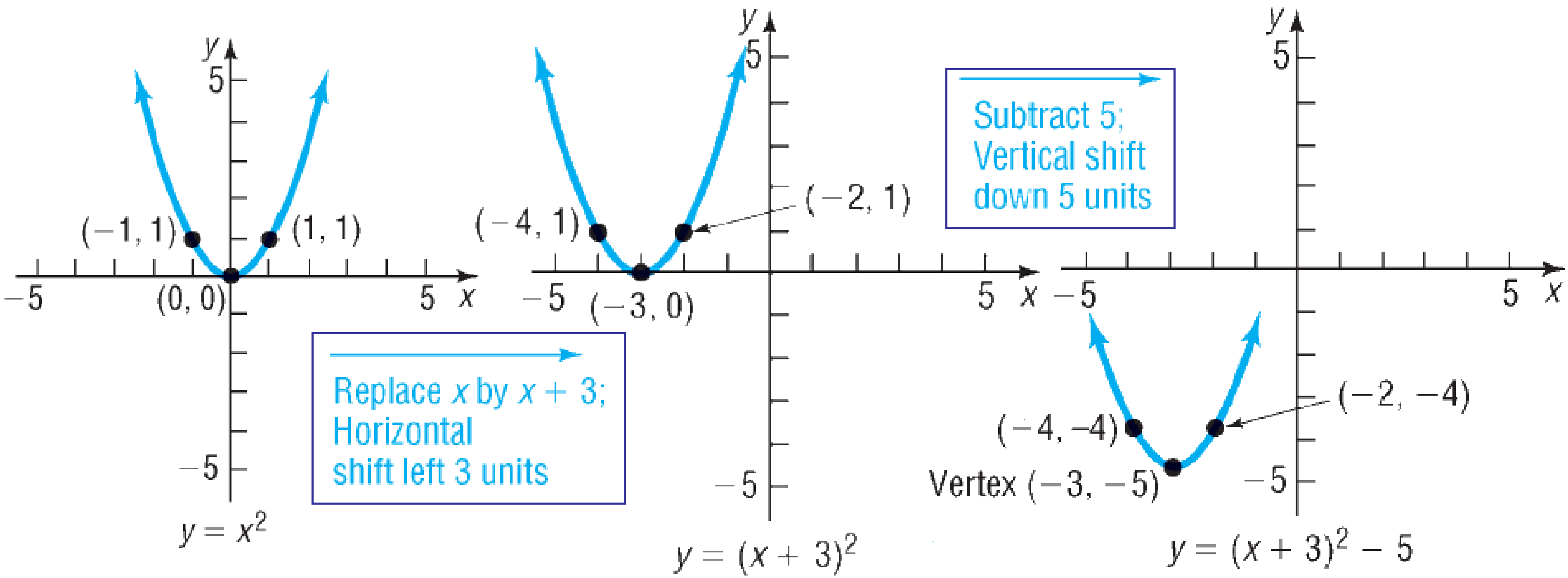
$$Y_3 = (x + 2)^2$$

If the argument x of a function f is replaced by $x - h, h > 0$, the graph of the new function $y = f(x - h)$ is the graph of f **shifted horizontally right** h units. If the argument x of a function f is repaced by $x + h, h > 0$, the graph of the new function $y = f(x + h)$ is the graph of f **shifted horizontally left** h units..

EXAMPLE

Combining Vertical and Horizontal Shifts

Graph the function $f(x) = (x + 3)^2 - 5$.

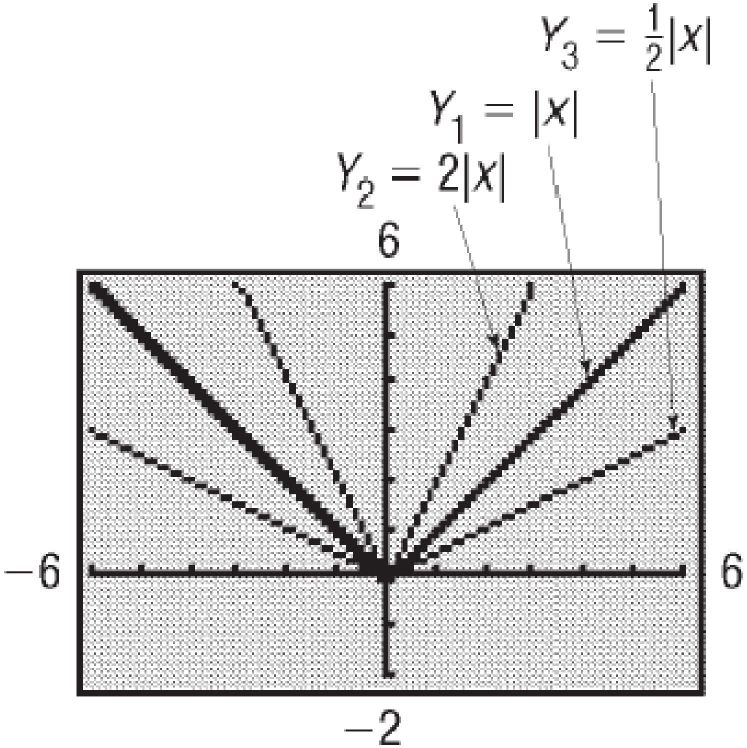


OBJECTIVE 2

2 Graph Functions Using Compressions and Stretches

Exploration

On the same screen, graph each of the following functions:



$$Y_1 = |x|$$

$$Y_2 = 2|x|$$

$$Y_3 = \frac{1}{2}|x|$$

X	Y1	Y2
-2	2	4
-1	1	2
0	0	0
1	1	2
2	2	4
3	3	6
4	4	8

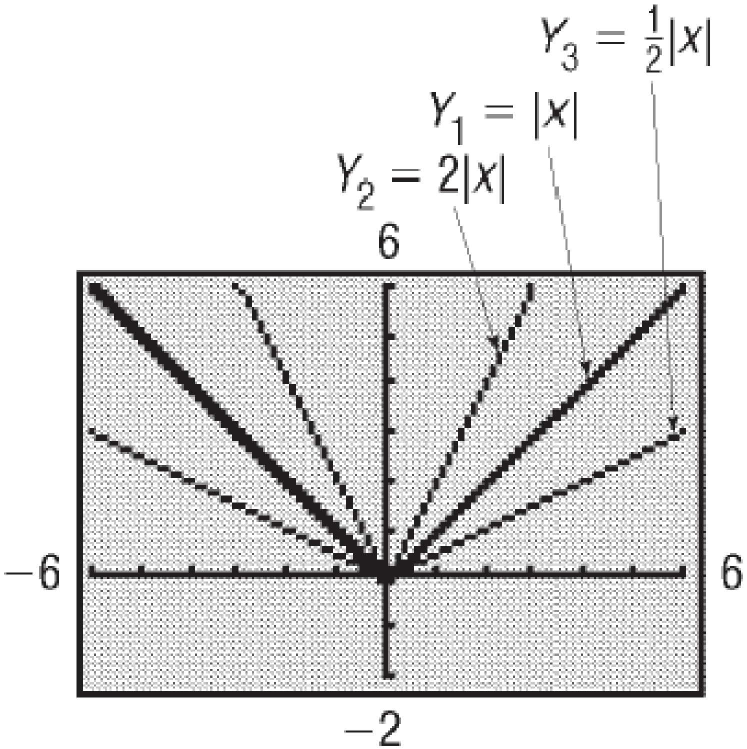
Y2=2abs(X)

X	Y1	Y3
-2	2	1
-1	1	.5
0	0	0
1	1	.5
2	2	1
3	3	1.5
4	4	2

Y3=.5abs(X)

Exploration

On the same screen, graph each of the following functions:



$$Y_1 = |x|$$

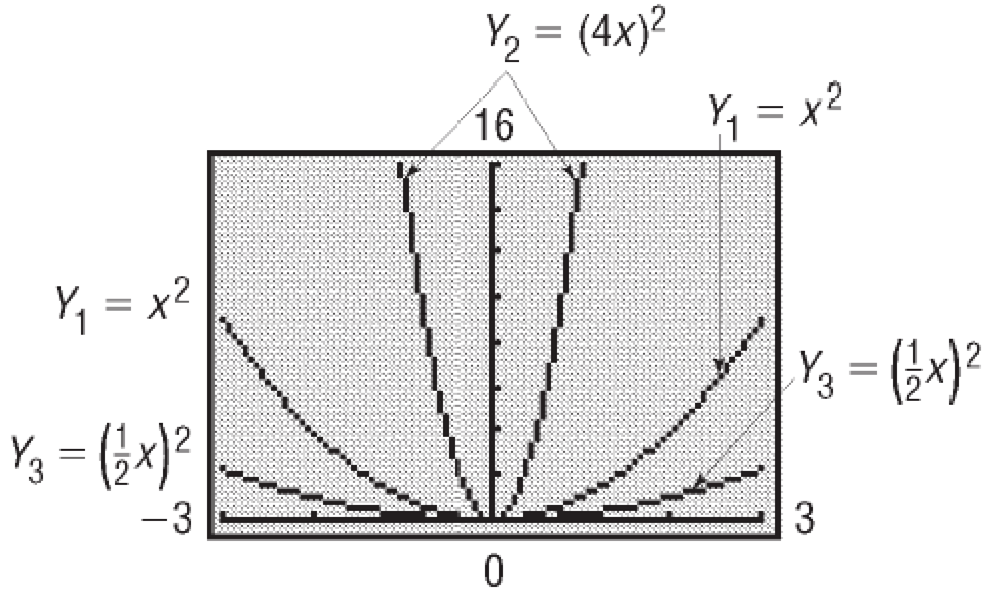
$$Y_2 = 2|x|$$

$$Y_3 = \frac{1}{2}|x|$$

When the right side of a function $y = f(x)$ is multiplied by a positive number a, the graph of the new function $y = af(x)$ is obtained by multiplying each y-coordinate on the graph of $y = f(x)$ by a. The new graph is a **vertically compressed** (if $0 < a < 1$) or a **vertically stretched** (if $a > 1$) version of the graph of $y = f(x)$.

Exploration

On the same screen, graph each of the following functions:



$$Y_1 = f(x) = x^2$$

$$Y_2 = f(4x) = (4x)^2$$

$$Y_3 = f\left(\frac{1}{2}x\right) = \left(\frac{1}{2}x\right)^2$$

X	Y1	Y2
0	0	0
.25	.0625	1
1	1	16
4	16	256
16	256	4096

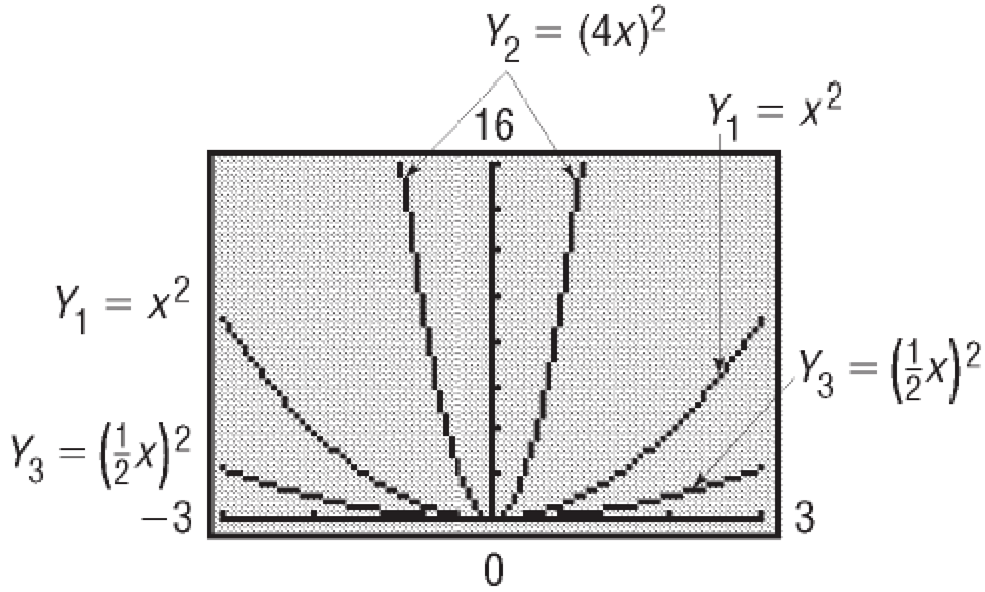
Y2=(4X)²

X	Y1	Y3
0	0	0
.5	.25	.0625
1	1	.25
2	4	1
4	16	4
8	64	16
16	256	64

Y3=(X/2)²

Exploration

On the same screen, graph each of the following functions:



$$Y_1 = f(x) = x^2$$

$$Y_2 = f(4x) = (4x)^2$$

$$Y_3 = f\left(\frac{1}{2}x\right) = \left(\frac{1}{2}x\right)^2$$

If the argument x of a function $y = f(x)$ is multiplied by a positive number a, the graph of the new function $y = f(ax)$ is obtained by multiplying each x-coordinate of $y = f(x)$ by $\frac{1}{a}$. A **horizontal compression** results if $a > 1$, and a **horizontal stretch** occurs if $0 < a < 1$.

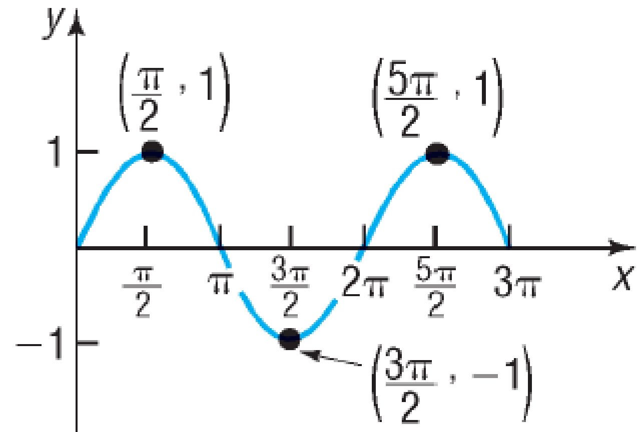
EXAMPLE

Graphing Using Stretches and Compressions

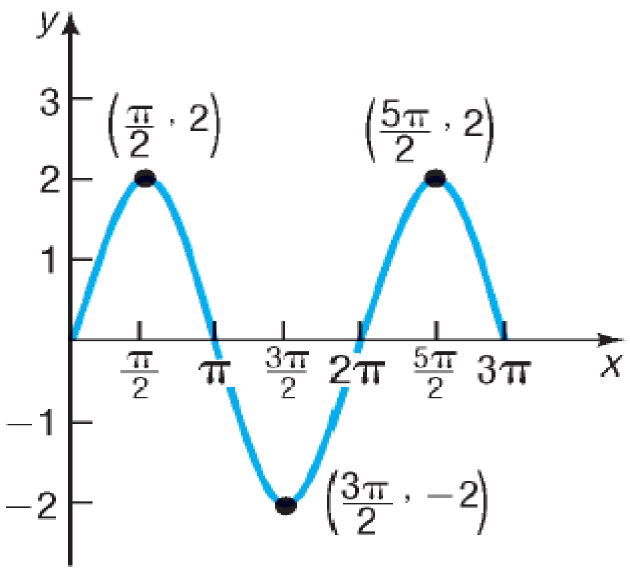
The graph of $y = f(x)$ is given.

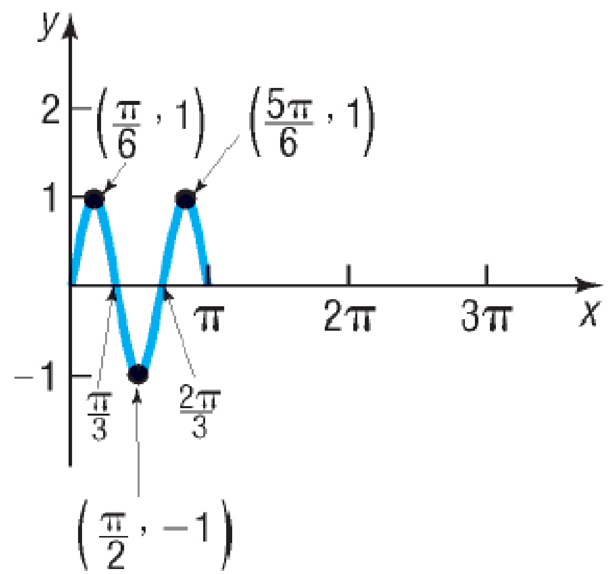
Use this graph to find the graphs of

(a) $y = 2f(x)$ (b) $y = f(3x)$



$y = f(x)$





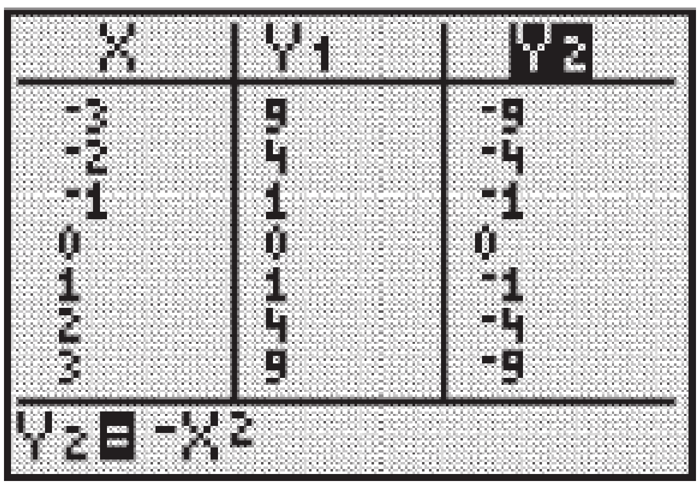
OBJECTIVE 3

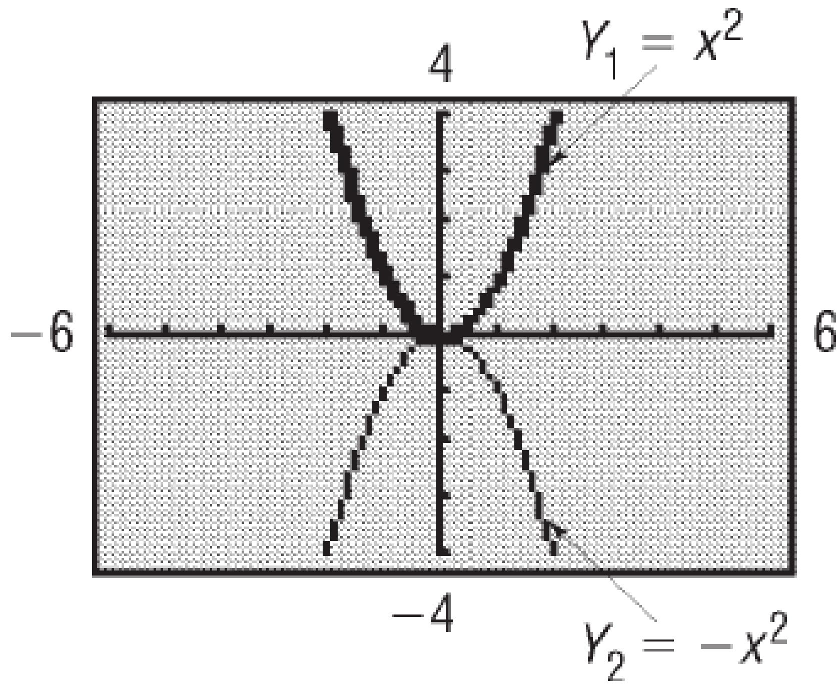
3 **Graph Functions Using Reflections about the *x*-Axis or *y*-Axis**

Exploration

Reflection about the x-axis:

(a) Graph $Y_1 = x^2$, followed by $Y_2 = -x^2$.



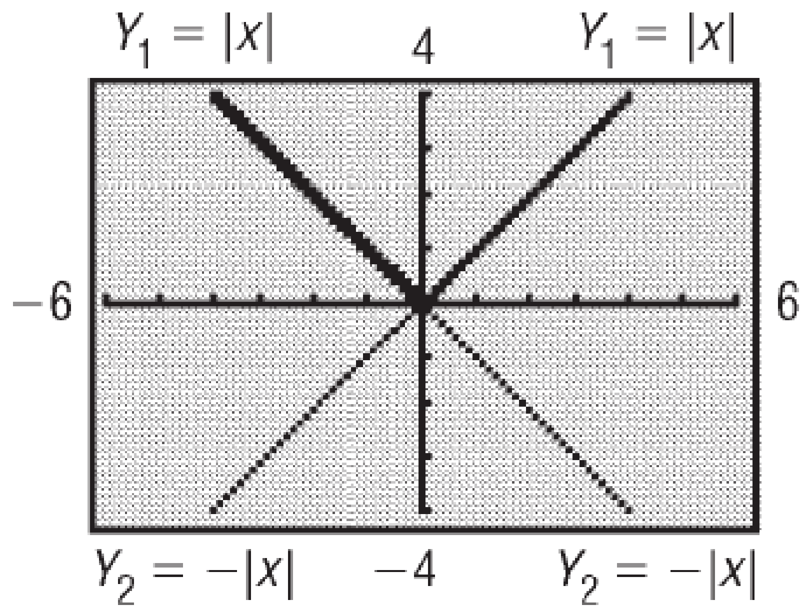


Exploration

(b) Graph $Y_1 = |x|$, followed by $Y_2 = -|x|$.

X	Y_1	Y_2
-3	3	-3
-2	2	-2
-1	1	-1
0	0	0
1	1	-1
2	2	-2
3	3	-3

$Y_2 = -abs(X)$

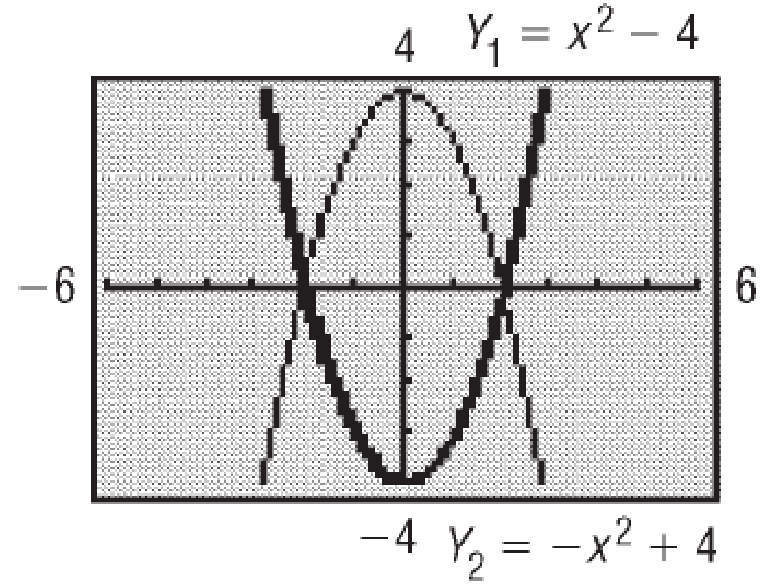


Exploration

(c) Graph $Y_1 = x^2 - 4$, followed by $Y_2 = -(x^2 - 4) = -x^2 + 4$.

X	Y1	Y2
-3	5	-5
-2	0	0
-1	-3	3
0	-4	4
1	-3	3
2	0	0
3	5	-5

Y2 = -X²+4



When the right side of the function $y = f(x)$ is multiplied by -1, the graph of the new function $y = -f(x)$ is the **reflection about the x-axis** of the graph of the function $y = f(x)$.

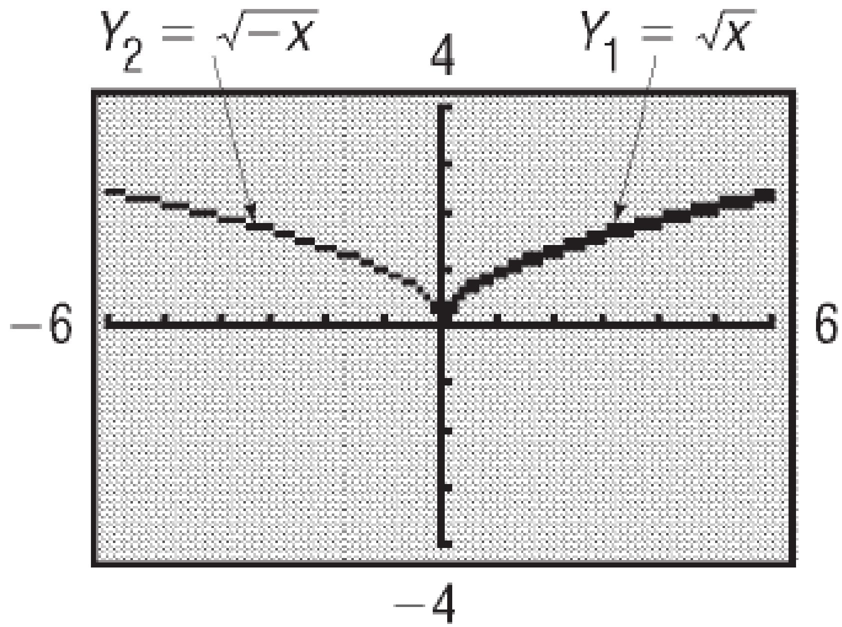
Exploration

Reflection about the y-axis:

(a) Graph $Y_1 = \sqrt{x}$, followed by $Y_2 = \sqrt{-x}$.

X	Y1	Y2
-3	ERROR	1.7321
-2	ERROR	1.4142
-1	ERROR	1
0	0	0
1	1	ERROR
2	1.4142	ERROR
3	1.7321	ERROR

Y2=√(-X)

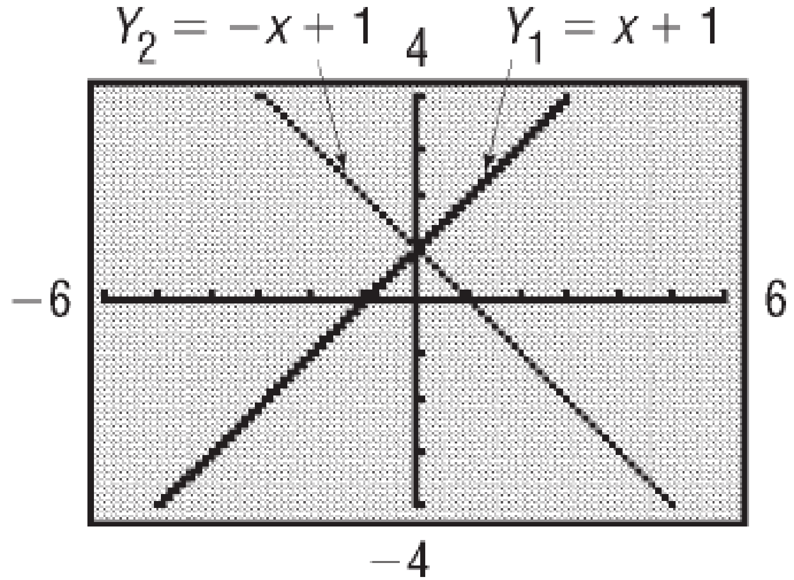


Exploration

(b) Graph $Y_1 = x + 1$, followed by $Y_2 = -x + 1$.

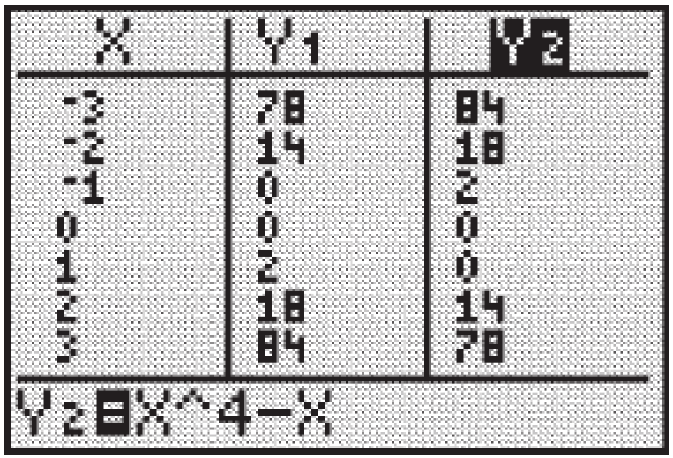
X	Y1	Y2
-3	-2	4
-2	-1	3
-1	0	2
0	1	1
1	2	0
2	3	-1
3	4	-2

Y2 = -X+1



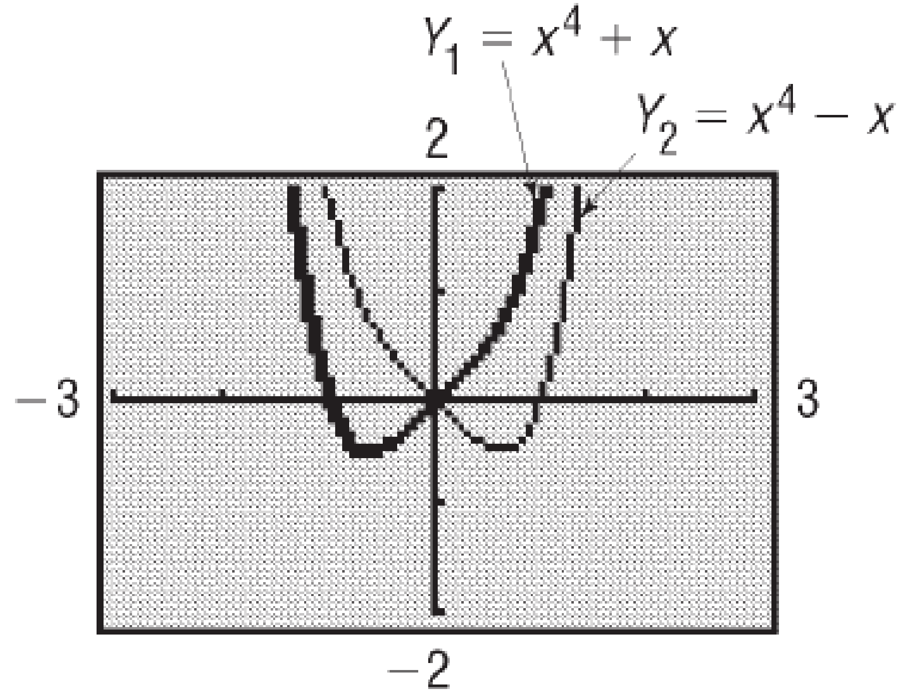
Exploration

(c) Graph $Y_1 = x^4 + x$, followed by $Y_2 = (-x)^4 + (-x) = x^4 - x$.



X	Y1	Y2
-3	78	84
-2	14	18
-1	0	2
0	0	0
1	2	0
2	18	14
3	84	78

Y2=X^4-X



When the graph of the function $y = f(x)$ is known, the graph of the new function $y = f(-x)$ is the **reflection about the *y*-axis** of the graph of the function $y = f(x)$.

Summary of Graphing Techniques

To Graph:	Draw the Graph of f and:	Functional Change to $f(x)$
Vertical shifts		
$y = f(x) + k, \quad k > 0$	Raise the graph of f by k units.	Add k to $f(x)$.
$y = f(x) - k, \quad k > 0$	Lower the graph of f by k units	Subtract k from $f(x)$.

Horizontal shifts		
$y = f(x + h), \quad h > 0$	Shift the graph of f to the left h units.	Replace x by $x + h$.
$y = f(x - h), \quad h > 0$	Shift the graph of f to the right h units.	Replace x by $x - h$.

Summary of Graphing Techniques

To Graph:	Draw the Graph of f and:	Functional Change to $f(x)$
Compressing or stretching		
$y = af(x), \quad a > 0$	Multiply each y-coordinate of $y = f(x)$ by a. Stretch the graph of f vertically if $a > 1$. Compress the graph of f vertically if $0 < a < 1$.	Multiply $f(x)$ by a.
$y = f(ax), \quad a > 0$	Multiply each x-coordinate of $y = f(x)$ by $\frac{1}{a}$. Stretch the graph of f horizontally if $0 < a < 1$. Compress the graph of f horizontally if $a > 1$.	Replace x by ax.

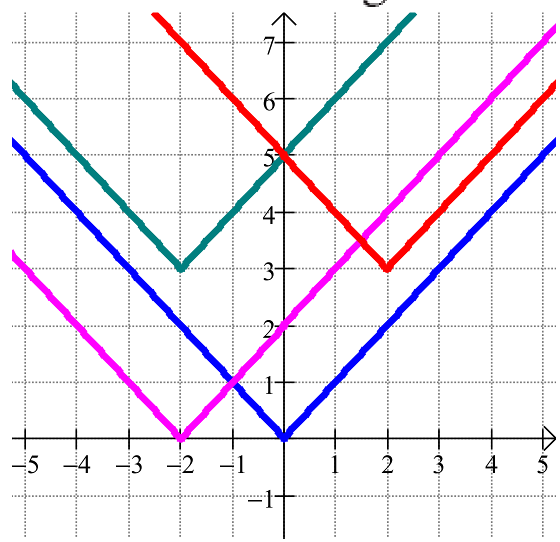
Summary of Graphing Techniques

To Graph:	Draw the Graph of f and:	Functional Change to $f(x)$
Reflection about the x-axis $y = -f(x)$	Reflect the graph of f about the x-axis.	Multiply $f(x)$ by -1.
Reflection about the y-axis $y = f(-x)$	Reflect the graph of f about the y-axis.	Replace x by $-x$.

EXAMPLE

Determining the Function Obtained from a Series of Transformations

Find the function that is finally graphed after the following three transformations are applied to the graph of $y = |x|$.



1. Shift left 2 units.

2. Shift up 3 units.

3. Reflect about the *y*-axis.

1. Shift left 2 units: Replace x by $x + 2$. $y = |x + 2|$

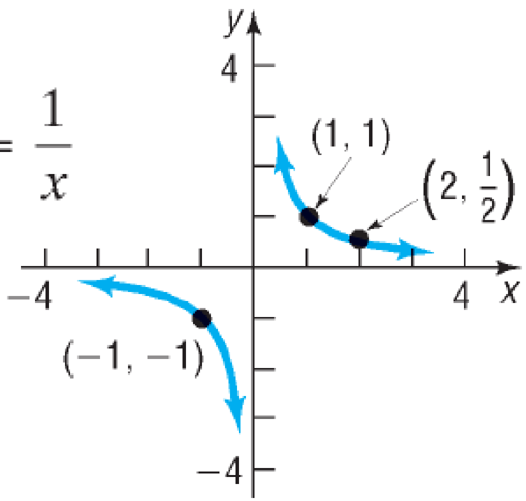
2. Shift up 3 units: Add 3. $y = |x + 2| + 3$

3. Reflect about the *y*-axis: Replace x by $-x$. $y = |-x + 2| + 3$

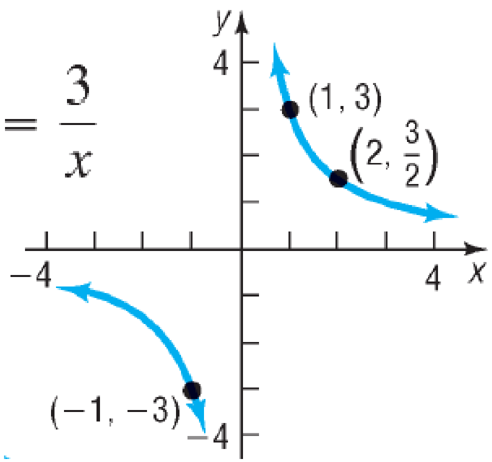
EXAMPLE Combining Graphing Procedures

Graph the function: $f(x) = \dfrac{3}{x-2} + 1$

Step 1: $y = \dfrac{1}{x}$

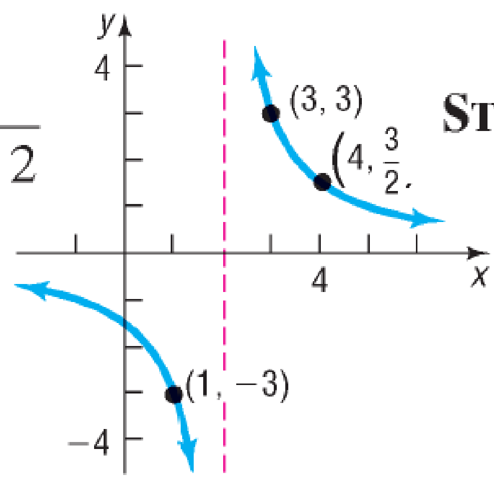


Step 2: $y = \dfrac{3}{x}$



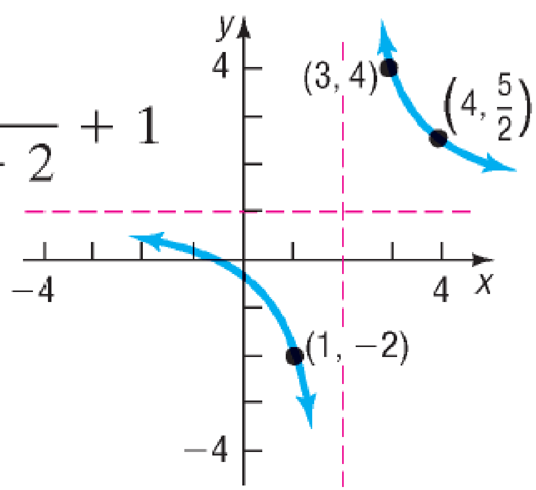
Multiply by 3;
Vertical stretch

Step 3: $y = \dfrac{3}{x-2}$



Replace x by $x - 2$;
Horizontal shift
right 2 units

Step 4: $y = \dfrac{3}{x-2} + 1$

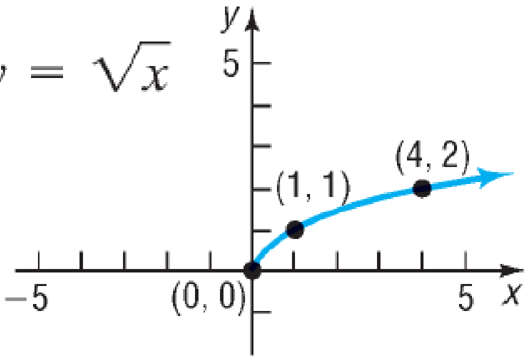


Add 1;
Vertical shift
up 1 unit

EXAMPLE Combining Graphing Procedures

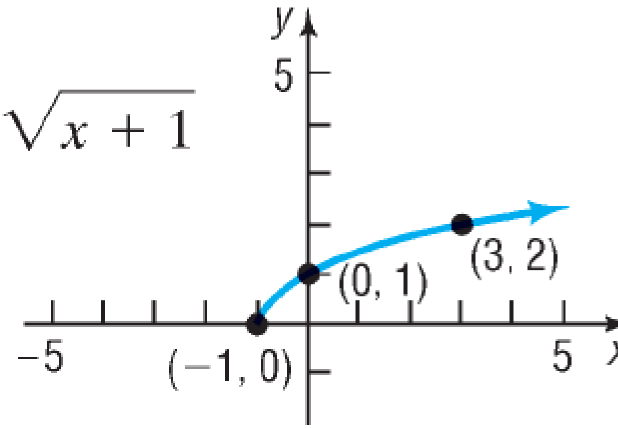
Graph the function: $f(x) = \sqrt{1 - x} + 2$

STEP 1: $y = \sqrt{x}$



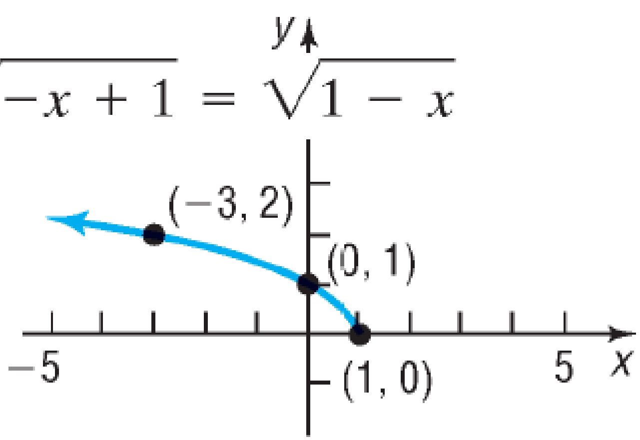
Replace x by $x + 1$;
Horizontal shift
left 1 unit

STEP 2: $y = \sqrt{x + 1}$



Replace x by $-x$;
Reflect
about y-axis

STEP 3: $y = \sqrt{-x + 1} = \sqrt{1 - x}$



Add 2;
Vertical shift
up 2 units

STEP 4: $y = \sqrt{1 - x} + 2$