

## **Section 3.5**

# **Polynomial and Rational Inequalities**



## Steps for Solving Polynomial and Rational Inequalities Algebraically

**STEP 1:** Write the inequality so that a polynomial or rational expression  $f$  is on the left side and zero is on the right side in one of the following forms:

$$f(x) > 0 \quad f(x) \geq 0 \quad f(x) < 0 \quad f(x) \leq 0$$

For rational expressions, be sure that the left side is written as a single quotient.

**STEP 2:** Determine the numbers at which the expression  $f$  on the left side equals zero and, if the expression is rational, the numbers at which the expression  $f$  on the left side is undefined.

**STEP 3:** Use the numbers found in Step 2 to separate the real number line into intervals.

**STEP 4:** Select a number in each interval and evaluate  $f$  at the number.

(a) If the value of  $f$  is positive, then  $f(x) > 0$  for all numbers  $x$  in the interval.

(b) If the value of  $f$  is negative, then  $f(x) < 0$  for all numbers  $x$  in the interval.

If the inequality is not strict ( $\geq$  or  $\leq$ ), include the solutions of  $f(x) = 0$  in the solution set, but be careful not to include values of  $x$  where the expression is undefined.



# OBJECTIVE 1

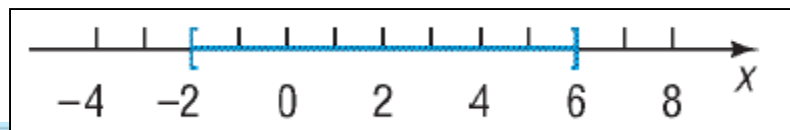
 **Solve Polynomial Inequalities Algebraically and Graphically**

## EXAMPLE

## Solving a Polynomial Inequality

Solve the inequality  $x^2 \leq 4x + 12$ , and graph the solution set.

### Algebraic Solution



**STEP 1:** Rearrange the inequality so that 0 is on the right side.

**STEP 2:** Determine the numbers at which the expression  $f$  on the left side equals zero

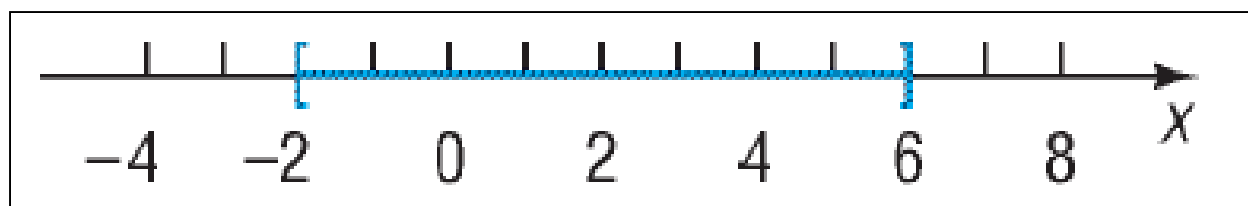
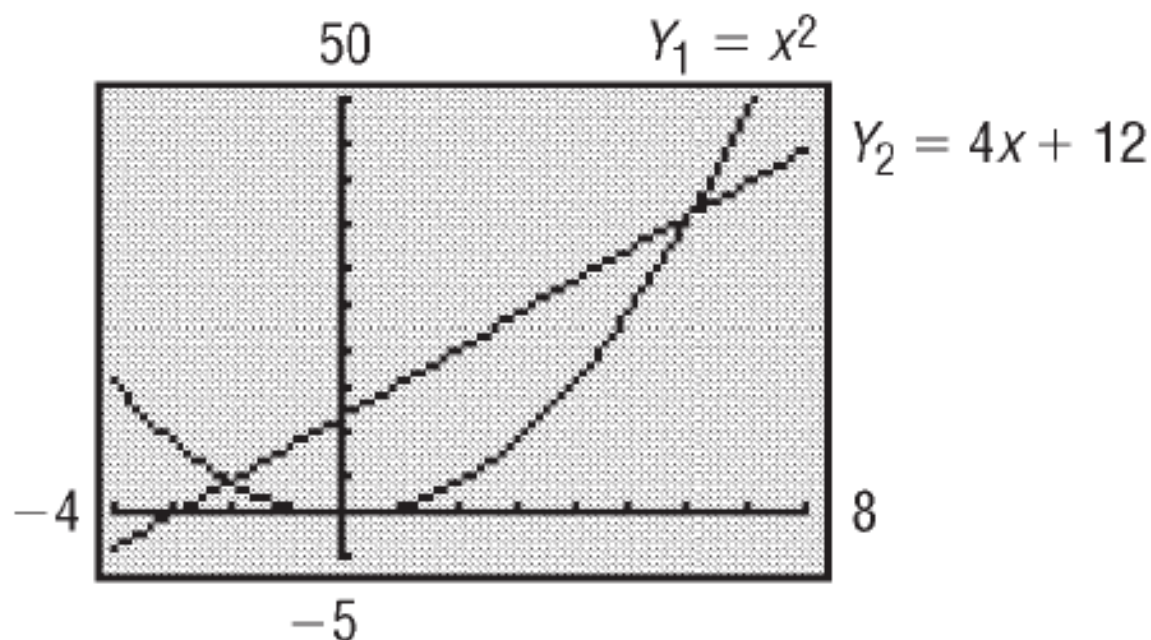
**STEP 3:** Use the numbers found in Step 2 to separate the real number line into intervals.

**STEP 4:** Select a number in each interval and evaluate  $f$  at the number.

	$-\infty$	$-2$	$6$	$\infty$
Interval	$(-\infty, -2)$		$(-2, 6)$	$(6, \infty)$
Number Chosen	$-3$		$0$	$7$
Value of $f$	$f(-3) = 9$		$f(0) = -12$	$f(7) = 9$
Conclusion	Positive		Negative	Positive

**EXAMPLE****Solving a Polynomial Inequality**

Solve the inequality  $x^2 \leq 4x + 12$ , and graph the solution set.

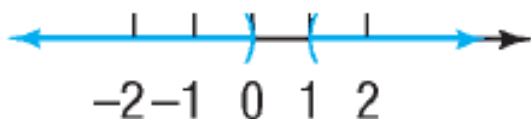
**Graphing Solution**

## EXAMPLE

## Solving a Polynomial Inequality

Solve the inequality  $x^4 > x$ , and graph the solution set.

### Algebraic Solution



**STEP 1:** Rearrange the inequality so that 0 is on the right side.

**STEP 2:** Determine the numbers at which the expression  $f$  on the left side equals zero

**STEP 3:** Use the numbers found in Step 2 to separate the real number line into intervals.

**STEP 4:** Select a number in each interval and evaluate  $f$  at the number.

	$-\infty$	0	1	$\infty$
Interval	$(-\infty, 0)$	$(0, 1)$	$(1, \infty)$	
Number Chosen	-1	$\frac{1}{2}$	2	
Value of $f$	$f(-1) = 2$	$f\left(\frac{1}{2}\right) = -\frac{7}{16}$	$f(2) = 14$	
Conclusion	Positive	Negative	Positive	

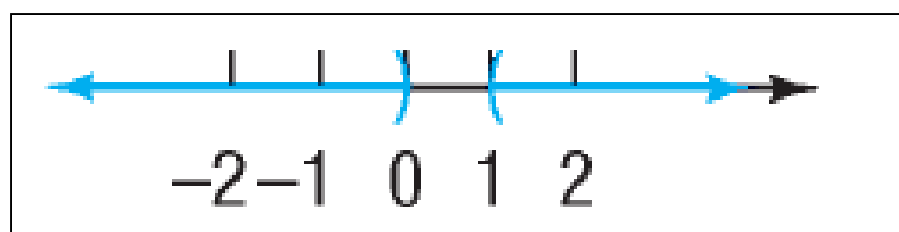
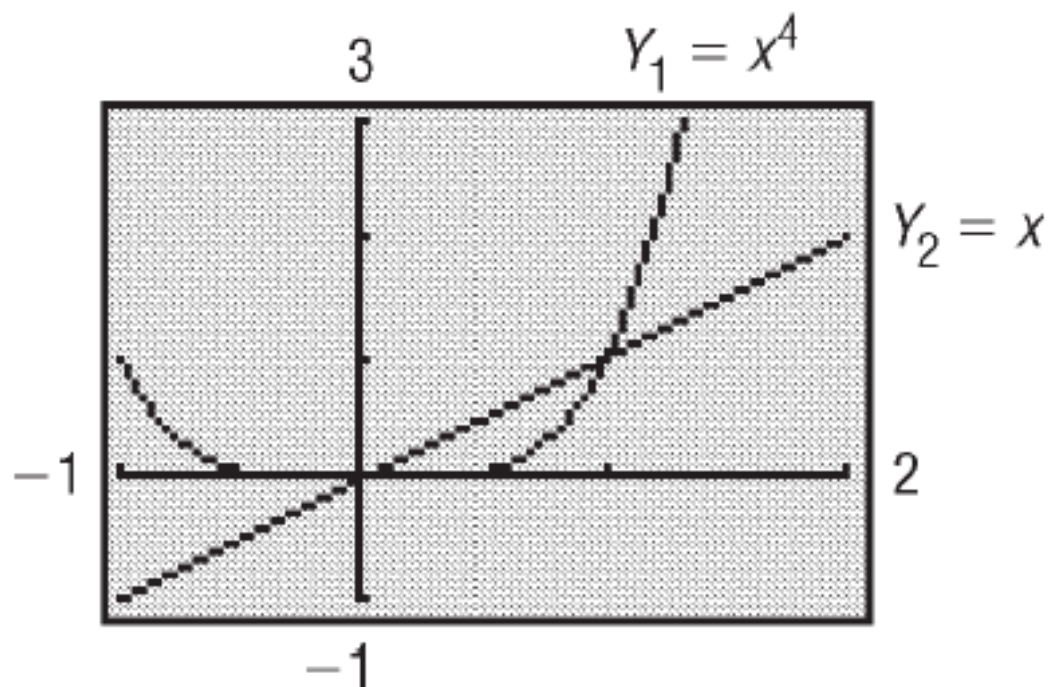


# EXAMPLE

## Solving a Polynomial Inequality

Solve the inequality  $x^4 > x$ , and graph the solution set.

### Graphing Solution



# OBJECTIVE 2

**2** Solve Rational Inequalities Algebraically and Graphically

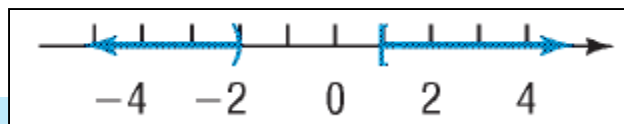


## EXAMPLE

## Solving a Rational Inequality

Solve the inequality  $\frac{4x + 5}{x + 2} \geq 3$ , and graph the solution set.

### Algebraic Solution



$$f(x) = \frac{x - 1}{x + 2}$$

**STEP 1:** Rearrange the inequality so that 0 is on the right side.

**STEP 2:** Determine the numbers at which the expression  $f$  on the left side equals zero and, if the expression is rational, the numbers at which the expression  $f$  on the left side is undefined.

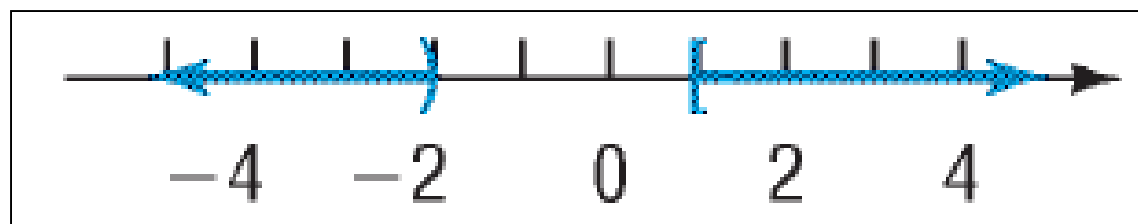
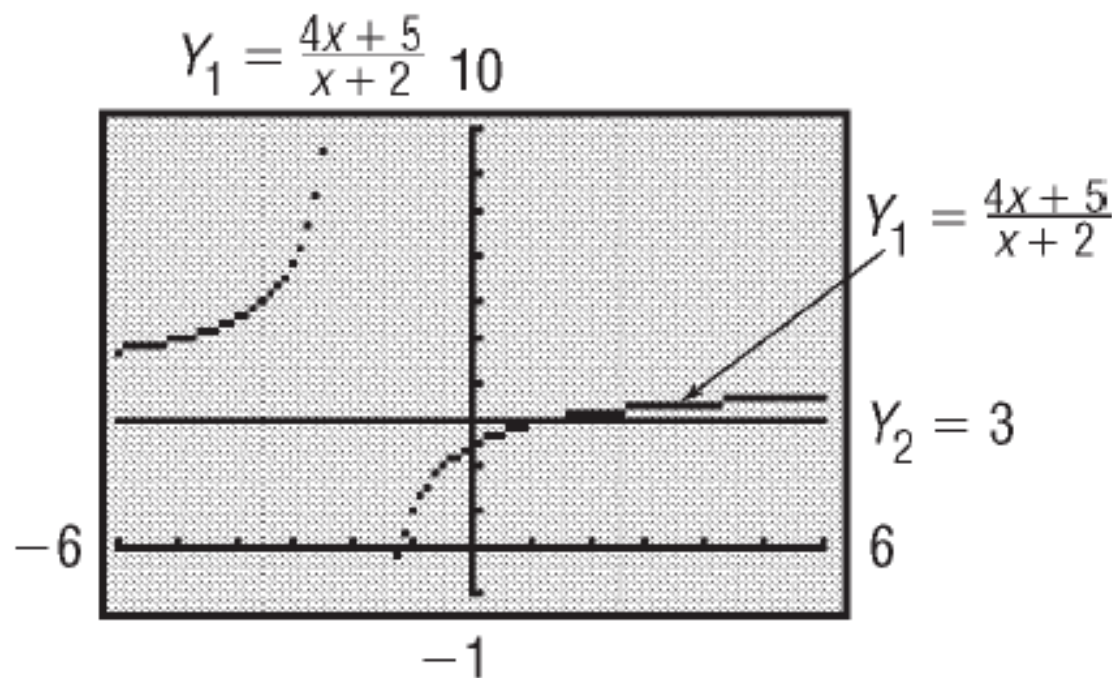
**STEP 3:** Use the numbers found in Step 2 to separate the real number line into intervals.

**STEP 4:** Select a number in each interval and evaluate  $f$  at the number.

	$-\infty$	$-2$	$1$	$\infty$
Interval	$(-\infty, -2)$		$(-2, 1)$	$(1, \infty)$
Number Chosen	$-3$		$0$	$2$
Value of $f$	$f(-3) = 4$		$f(0) = -\frac{1}{2}$	$f(2) = \frac{1}{4}$
Conclusion	Positive		Negative	Positive

**EXAMPLE****Solving a Rational Inequality**

Solve the inequality  $\frac{4x + 5}{x + 2} \geq 3$ , and graph the solution set.



## EXAMPLE

## Minimum Sales Requirements

Tami is considering leaving her \$30,000 a year job and buying a cookie company. According to the financial records of the firm, the relationship between pounds of cookies sold and profit is as exhibited by Table 20.

- Draw a scatter diagram of the data in Table 20 with the pounds of cookies sold as the independent variable.
- Use a graphing utility to find the quadratic function of best fit.
- Use the function found in part (b) to determine the number of pounds of cookies that Tami must sell for the profits to exceed \$30,000 a year and therefore make it worthwhile for her to quit her job.
- Using the function found in part (b), determine the number of pounds of cookies that Tami should sell to maximize profits.
- Using the function found in part (b), determine the maximum profit that Tami can expect to earn.

Table on next slide

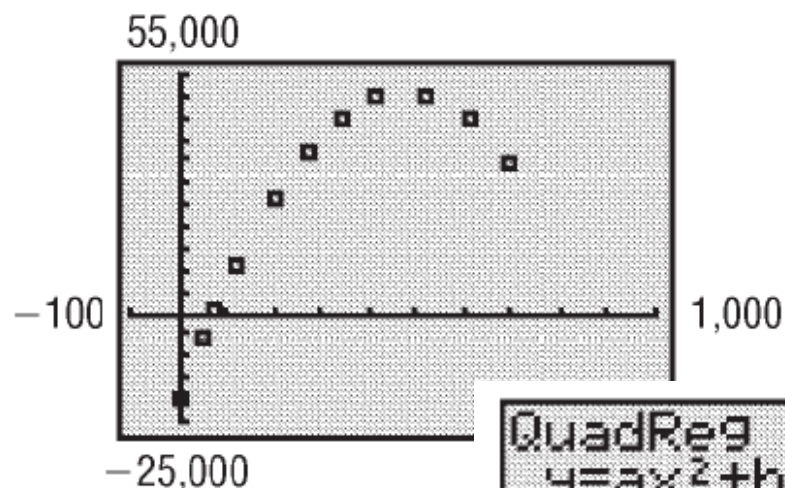


# EXAMPLE

## Minimum Sales Requirements



Pounds of Cookies (in Hundreds), $x$	Profit, $P$
0	-20,000
50	-5,990
75	412
120	10,932
200	26,583
270	36,948
340	44,381
420	49,638
525	49,225
610	44,381
700	34,220



```
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b=295.860223  
c=-20042.52454
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