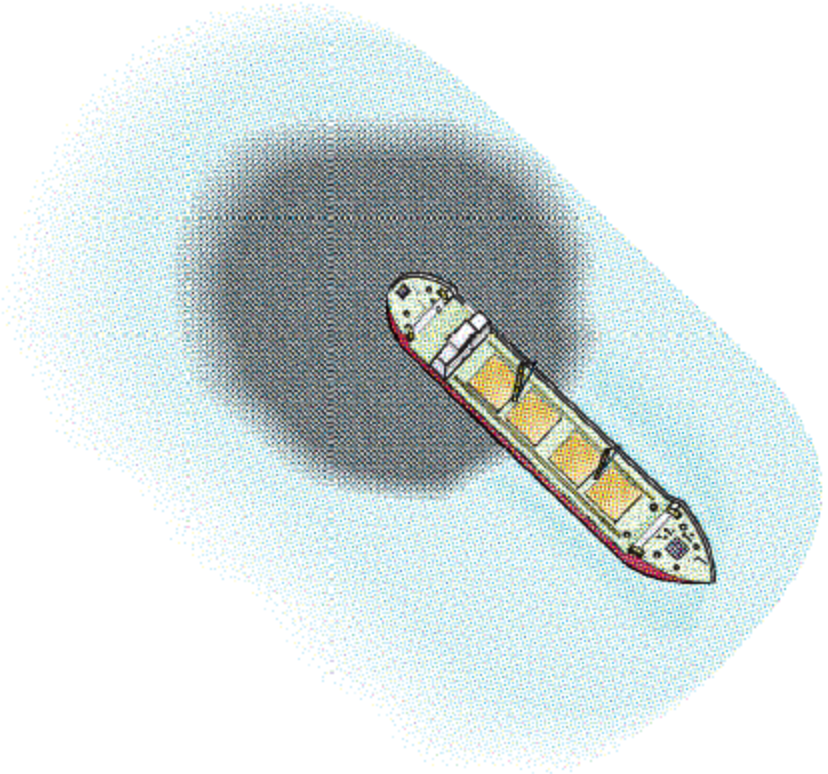
Section 4.1

Composite Functions

OBJECTIVE 1

Form a Composite Function

Suppose that an oil tanker is leaking oil and we want to be able to determine the area of the circular oil patch around the ship. It is determined that the oil is leaking from the tanker in such a way that the radius of the circular oil patch around the ship is increasing at a rate of 3 feet per minute.



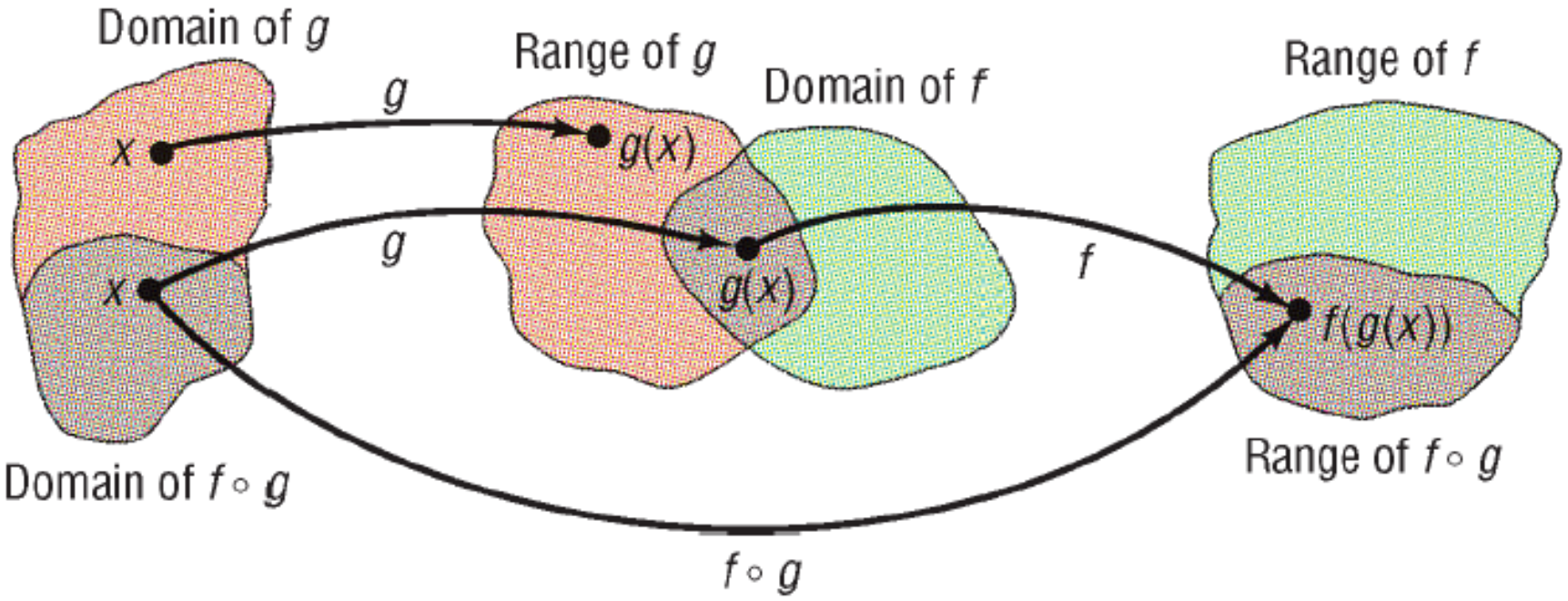
$$r(t) = 3t$$

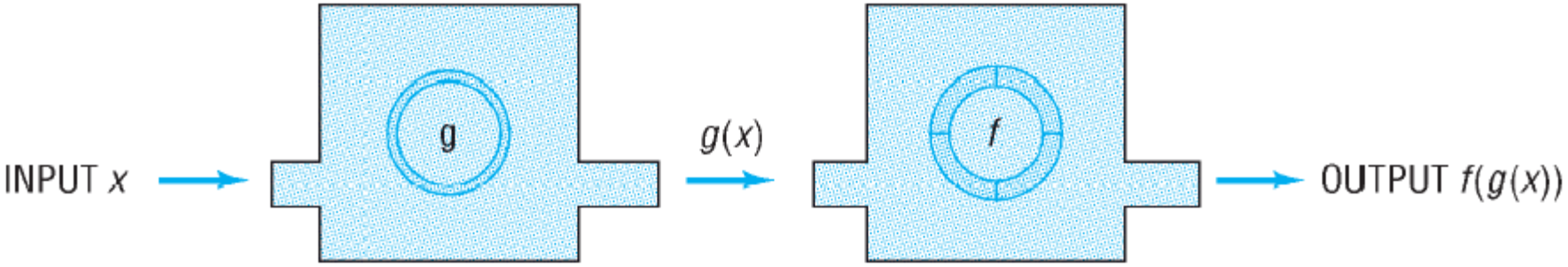
$$A(r) = \pi r^2$$

Given two functions f and g, the **composite function**, denoted by $f \circ g$ (read as "f composed with g"), is defined by

$$(f \circ g)(x) = f(g(x))$$

The domain of $f \circ g$ is the set of all numbers x in the domain of g such that $g(x)$ is in the domain of f.


Domain of g
Range of g
Domain of f
Range of f
g
x
$g(x)$
g
x
$g(x)$
f
$f(g(x))$
Domain of $f \circ g$
Range of $f \circ g$
$f \circ g$

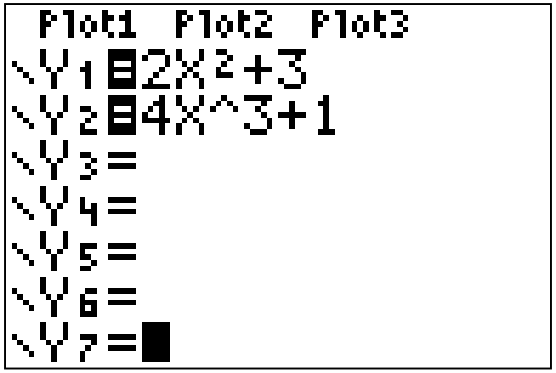

INPUT x
g
$g(x)$
f
OUTPUT $f(g(x))$

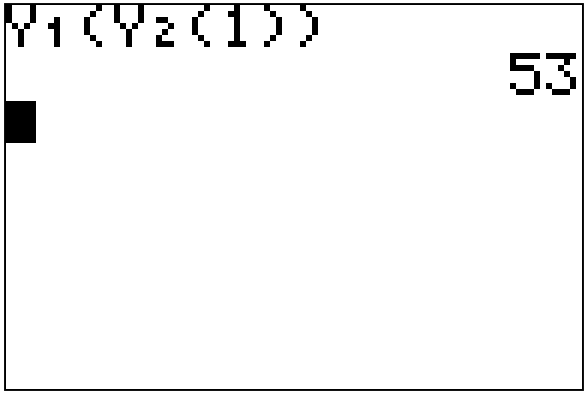
EXAMPLE

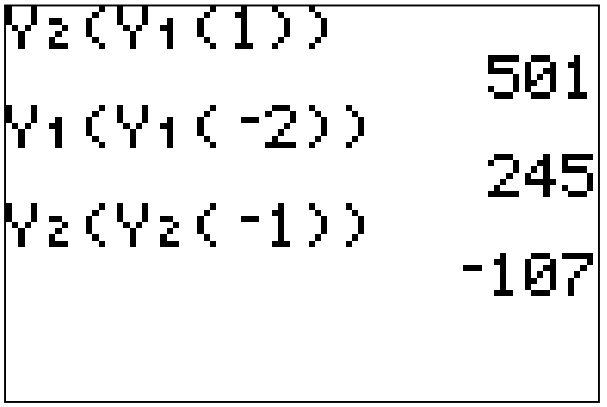
Evaluating a Composite Function

Suppose that $f(x) = 2x^2 + 3$ $g(x) = 4x^3 + 1$. Find:

(a) $(f \circ g)(1)$ (b) $(g \circ f)(1)$ (c) $(f \circ f)(-2)$ (d) $(g \circ g)(-1)$







OBJECTIVE 2

Find the Domain of a Composite Function

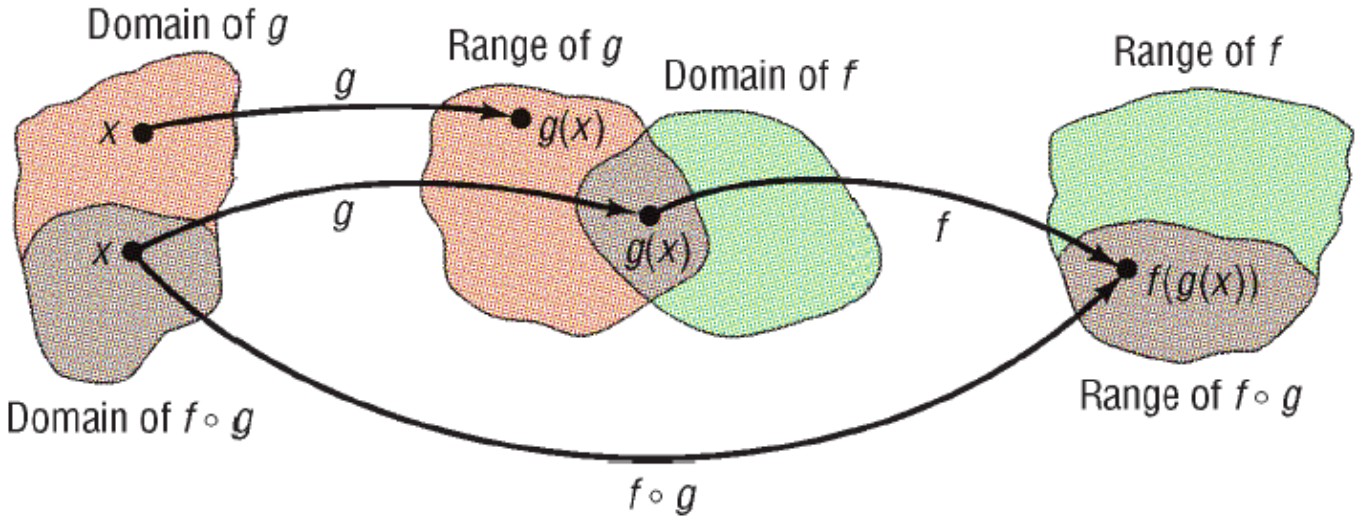
EXAMPLE

Finding a Composite Function and Its Domain

Suppose that $f(x) = 2x^2 + 3$ $g(x) = 4x^3 + 1$. Find:

Find: (a) $f \circ g$ (b) $g \circ f$

Then find the domain of each composite function.



1. $g(x)$ must be defined so any x not in the domain of g must be excluded.
2. $f(g(x))$ must be defined so any x for which $g(x)$ is not in the domain of f must be excluded.

EXAMPLE

Finding a Composite Function and Its Domain

Suppose that $f(x) = \dfrac{1}{x}$ and $g(x) = \sqrt{x-1}$

Find: (a) $f \circ g$ (b) $f \circ f$

Then find the domain of each composite function.

EXAMPLE

Showing That Two Composite Functions Are Equal

If $f(x) = 3x - 4$ and $g(x) = \frac{1}{3}(x + 4)$, show that

$$(f \circ g)(x) = (g \circ f)(x) = x$$

for every x in the domain of $f \circ g$ and $g \circ f$.

Seeing the Concept

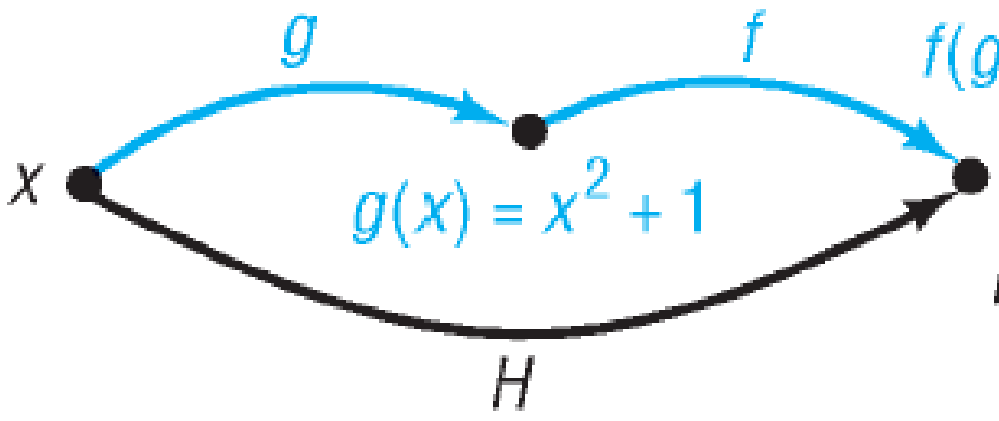
Using a graphing calculator, let $Y_1 = f(x) = 3x - 4$, $Y_2 = g(x) = \frac{1}{3}(x + 4)$, $Y_3 = f \circ g$, and $Y_4 = g \circ f$. Using the viewing window $-3 \le x \le 3, -2 \le y \le 2$, graph only Y_3 and Y_4. What do you see? TRACE to verify that $Y_3 = Y_4$.

Calculus Application

EXAMPLE

Finding the Components of a Composite Function

Find functions f and g such that $f \circ g = H$ if $H(x) = (x^2 + 1)^{50}$.



$$f(g(x)) = f(x^2 + 1)$$
$$= (x^2 + 1)^{50}$$
$$H(x) = (x^2 + 1)^{50}$$