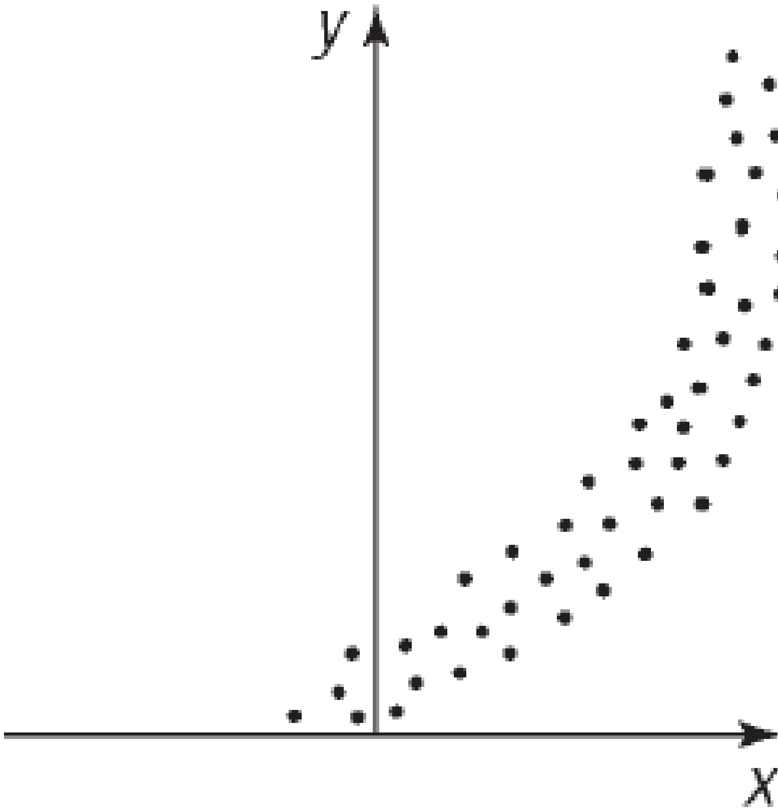
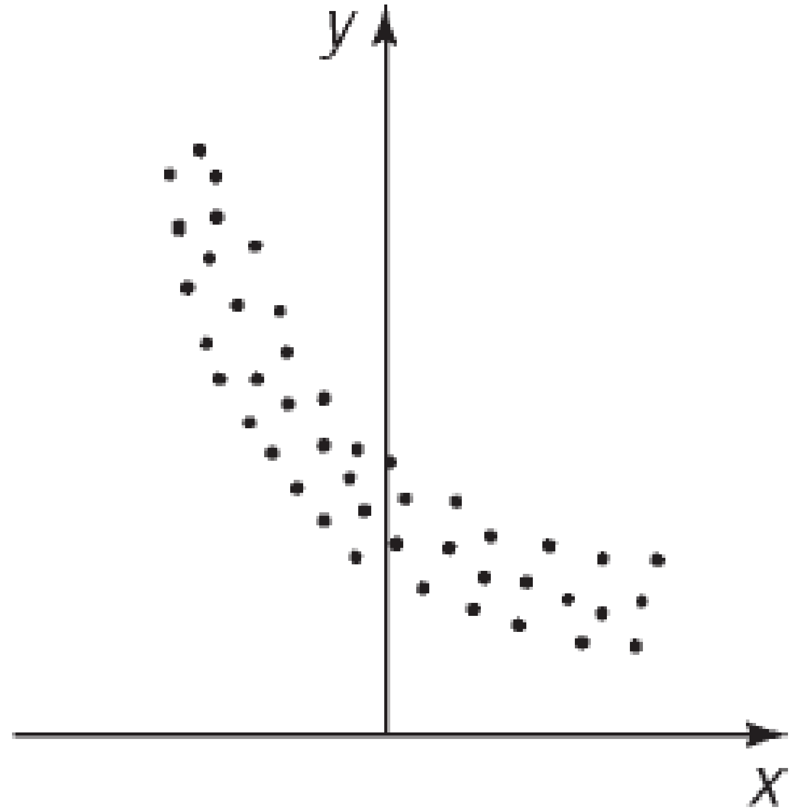
Section 4.9

Building Exponential, Logarithmic, and Logistic Models from Data



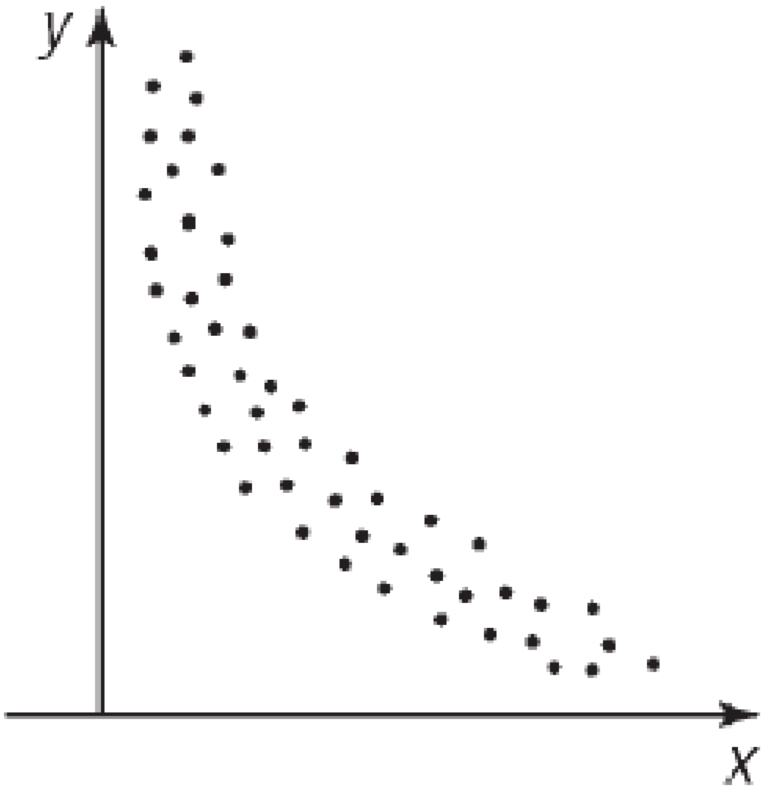
$y = ab^x, a > 0, b > 1$

Exponential



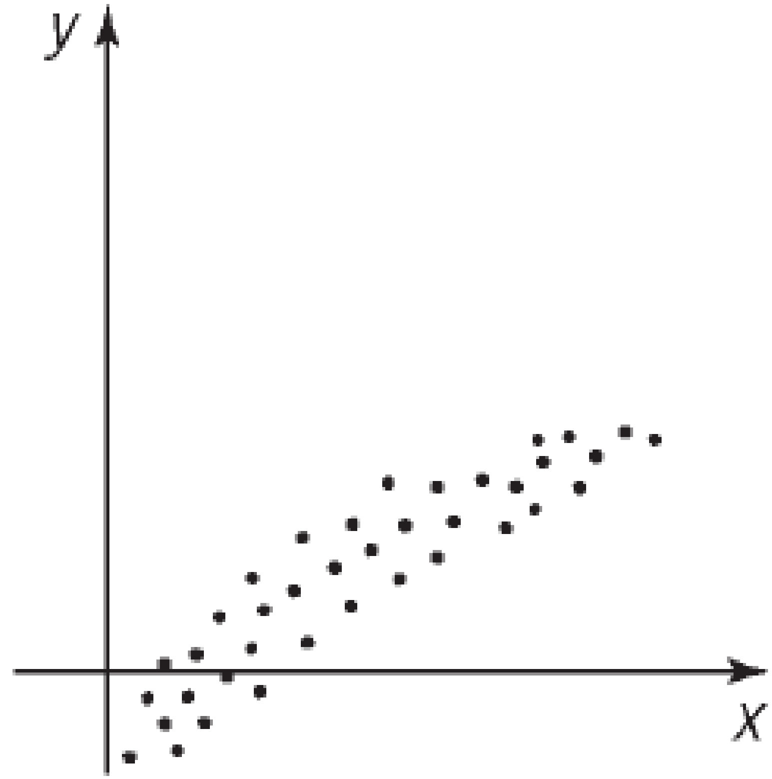
$y = ab^x, 0 < b < 1, a > 0$

Exponential



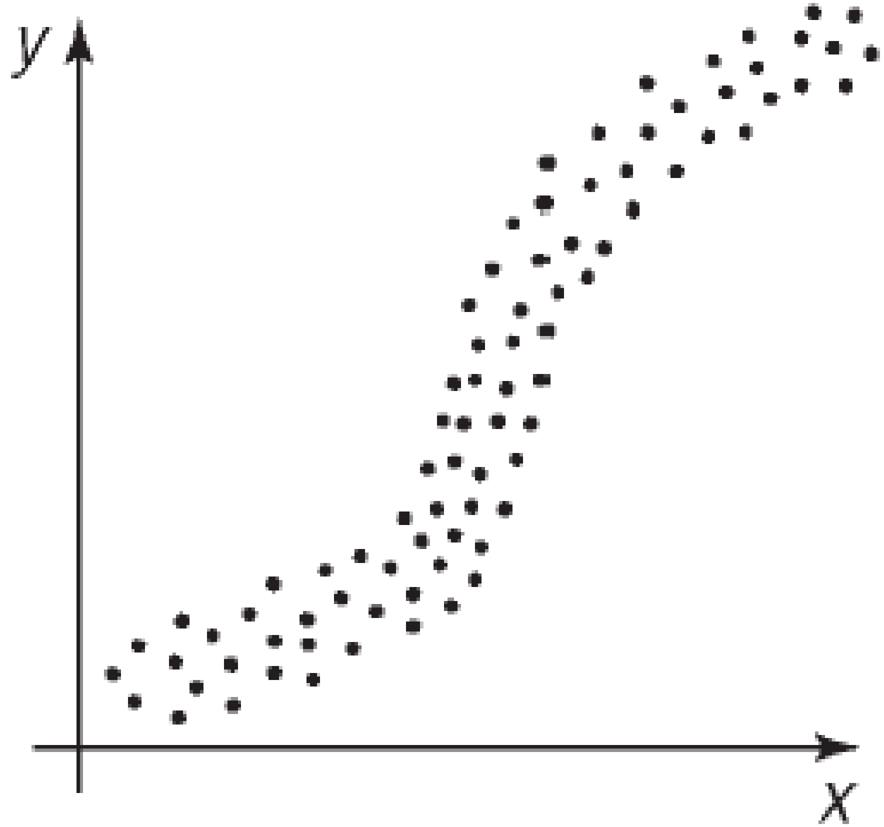
$y = a + b \ln x,\ a > 0,\ b < 0$

Logarithmic



$y = a + b \ln x,\ a > 0,\ b > 0$

Logarithmic



$$y = \frac{c}{1 + ae^{-bx}},\ a > 0,\ b > 0,\ c > 0$$

Logistic

OBJECTIVE 1

1 Use a Graphing Utility to Fit an Exponential Function to Data

EXAMPLE

Fitting an Exponential Function to Data

Beth is interested in finding a function that explains the closing price of Harley Davidson stock at the end of each year. She obtains the data shown in Table 10.

(a) Using a graphing utility, draw a scatter diagram with year as the independent variable.

(b) Using a graphing utility, fit an exponential function to the data.

(c) Express the function found in part (b) in the form $A = A_0e^{kt}$.

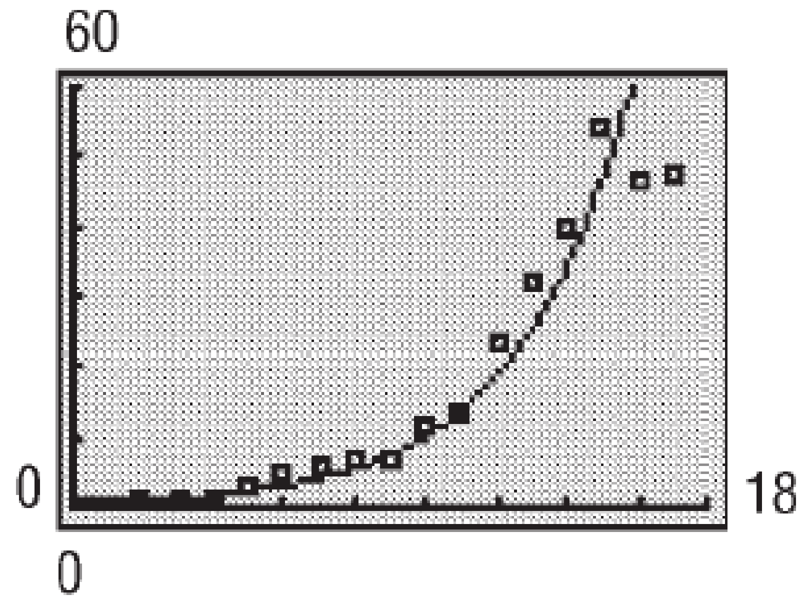
(d) Graph the exponential function found in part (b) or (c) on the scatter diagram.

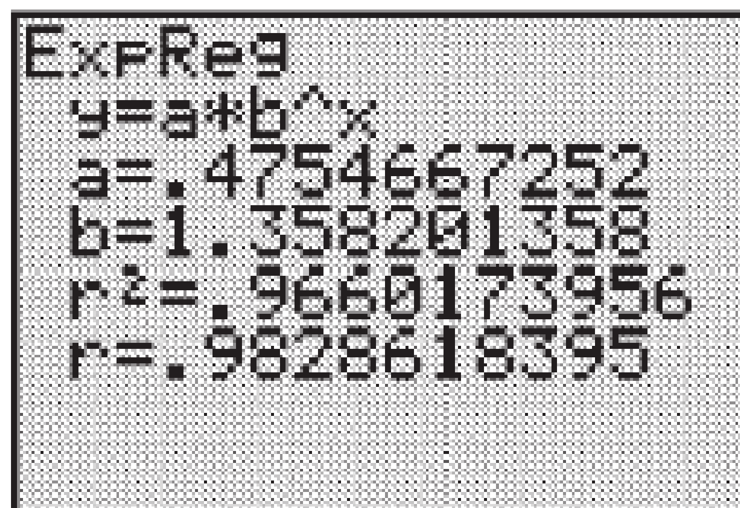
(e) Using the solution to part (b) or (c), predict the closing price of Harley Davidson stock at the end of 2004.

(f) Interpret the value of k found in part (c).

Table on next slide

Year, *x*	Closing Price, *y*
1987 ($x = 1$)	0.392
1988 ($x = 2$)	0.7652
1989 ($x = 3$)	1.1835
1990 ($x = 4$)	1.1609
1991 ($x = 5$)	2.6988
1992 ($x = 6$)	4.5381
1993 ($x = 7$)	5.3379
1994 ($x = 8$)	6.8032
1995 ($x = 9$)	7.0328
1996 ($x = 10$)	11.5585
1997 ($x = 11$)	13.4799
1998 ($x = 12$)	23.5424
1999 ($x = 13$)	31.9342
2000 ($x = 14$)	39.7277
2001 ($x = 15$)	54.31
2002 ($x = 16$)	46.20
2003 ($x = 17$)	47.53





OBJECTIVE 2

2 Use a Graphing Utility to Fit a Logarithmic Function to Data

EXAMPLE

Fitting a Logarithmic Function to Data

Jodi, a meteorologist, is interested in finding a function that explains the relation between the height of a weather balloon (in kilometers) and the atmospheric pressure (measured in millimeters of mercury) on the balloon. She collects the data shown in Table 11.

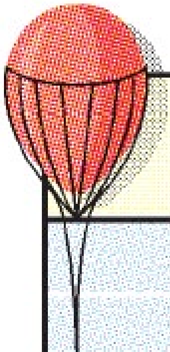
(a) Using a graphing utility, draw a scatter diagram of the data with atmospheric pressure as the independent variable.

(b) It is known that the relation between atmospheric pressure and height follows a logarithmic model. Using a graphing utility, fit a logarithmic function to the data.

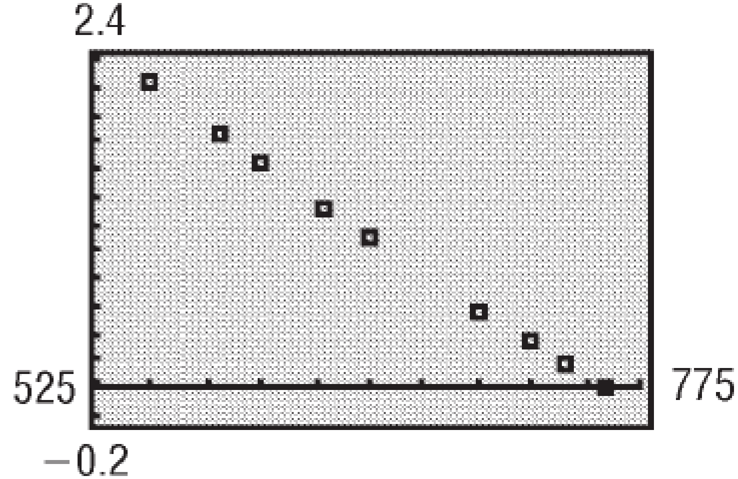
(c) Draw the logarithmic function found in part (b) on the scatter diagram.

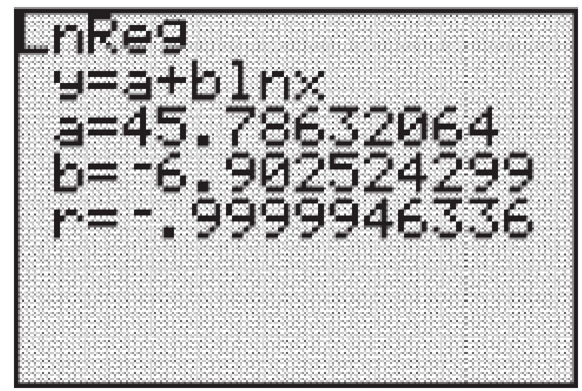
(d) Use the function found in part (b) to predict the height of the weather balloon if the atmospheric pressure is 560 millimeters of mercury.

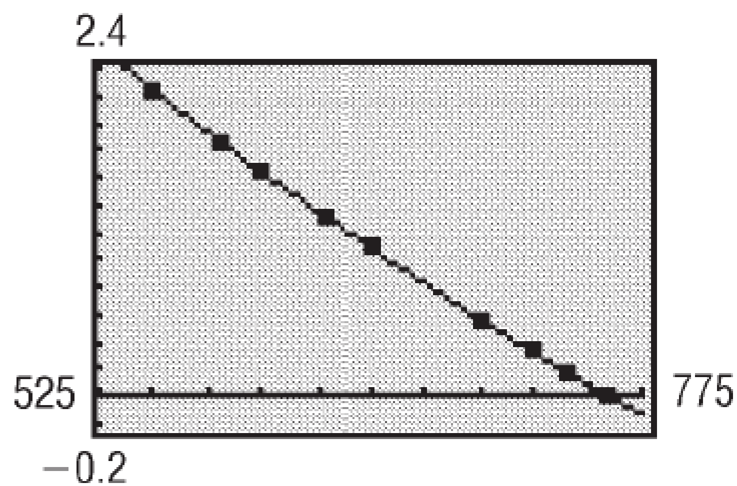
Table on next slide



Atmospheric Pressure, *p*	Height, *h*
760	0
740	0.184
725	0.328
700	0.565
650	1.079
630	1.291
600	1.634
580	1.862
550	2.235







OBJECTIVE 3

3 **Use a Graphing Utility to Fit a Logistic Function to Data**

EXAMPLE Fitting a Logistic Function to Data

The data in Table 12 represent the amount of yeast biomass in a culture after t hours.

(a) Using a graphing utility, draw a scatter diagram of the data with time as the independent variable.

(b) Using a graphing utility, fit a logistic function to the data.

(c) Using a graphing utility, graph the function found in part (b) on the scatter diagram.

(d) What is the predicted carrying capacity of the culture?

(e) Use the function found in part (b) to predict the population of the culture at $t = 19$ hours.

Table on next slide

Time (in hours)	Yeast Biomass	Time (in hours)	Yeast Biomass
0	9.6	10	513.3
1	18.3	11	559.7
2	29.0	12	594.8
3	47.2	13	629.4
4	71.1	14	640.8
5	119.1	15	651.1
6	174.6	16	655.9
7	257.3	17	659.6
8	350.7	18	661.8
9	441.0