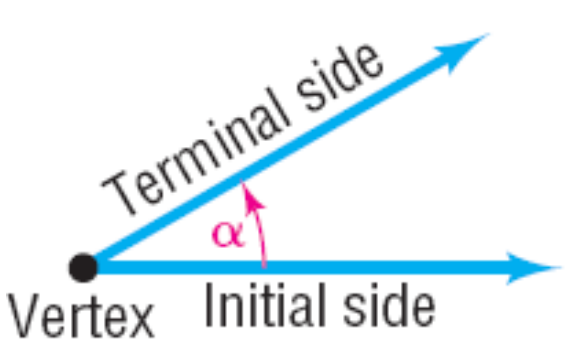
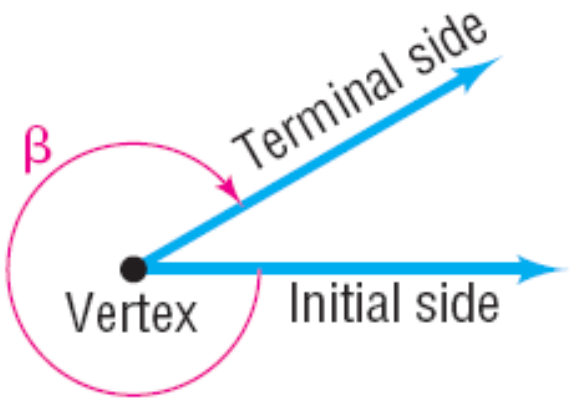
Section 5.1

Angles and Their Measure



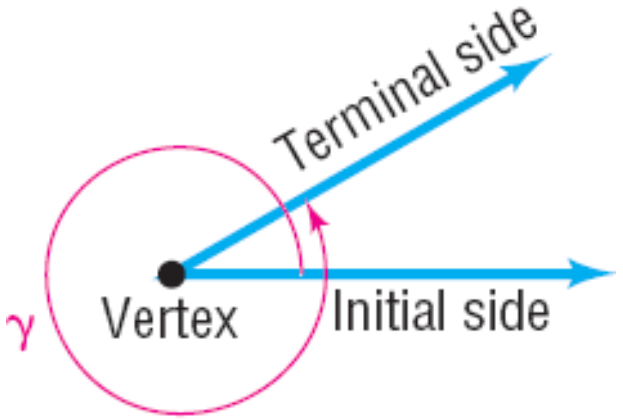
Counterclockwise rotation

Positive angle



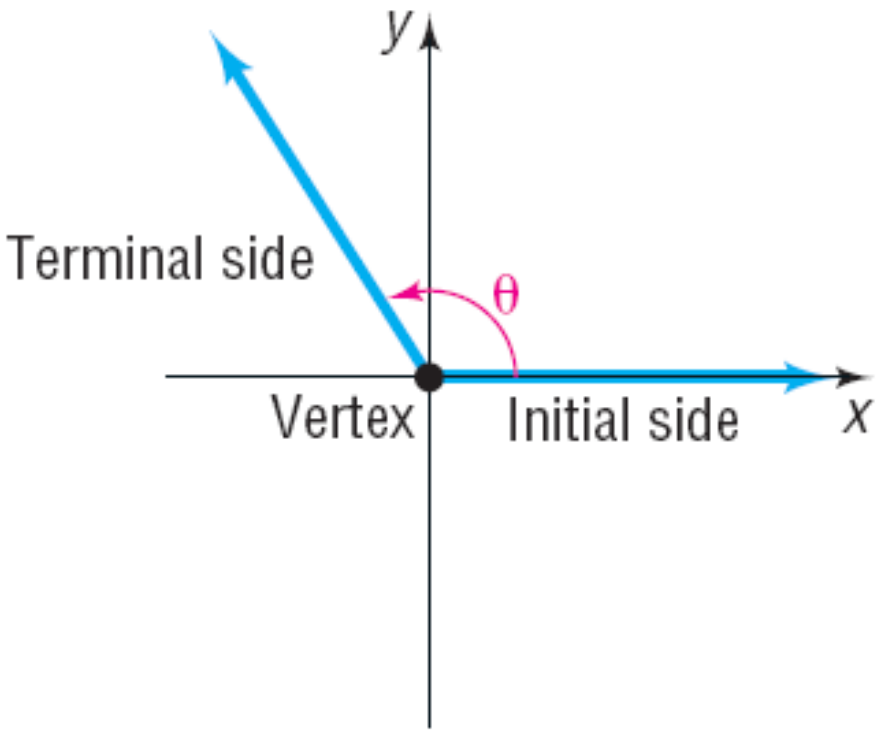
Clockwise rotation

Negative angle

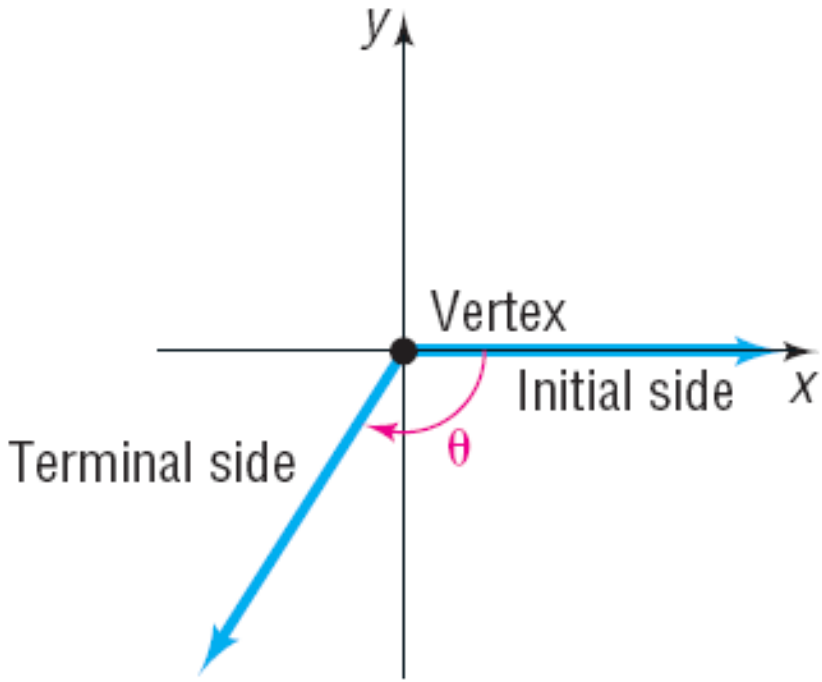


Counterclockwise rotation

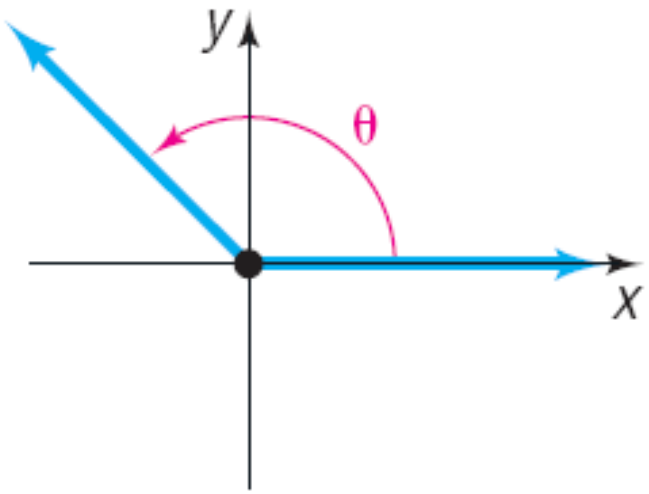
Positive angle



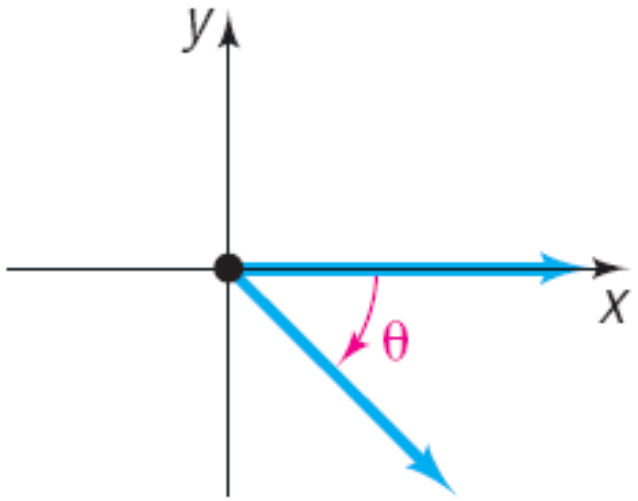
(a) θ is in standard position; θ is positive



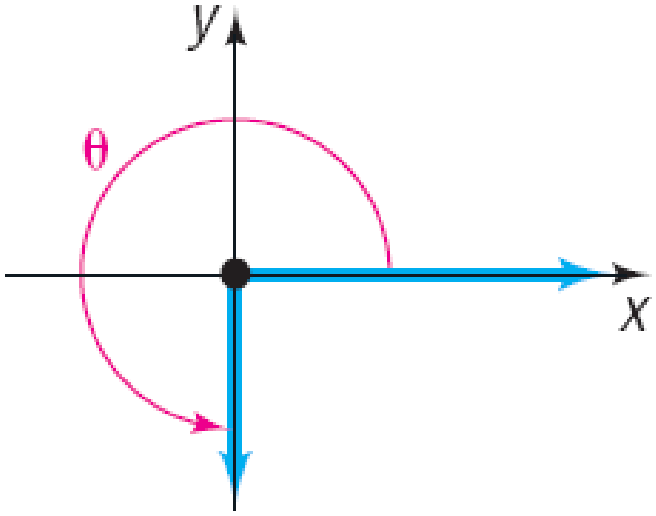
(b) θ is in standard position; θ is negative



(a) θ lies in quadrant II

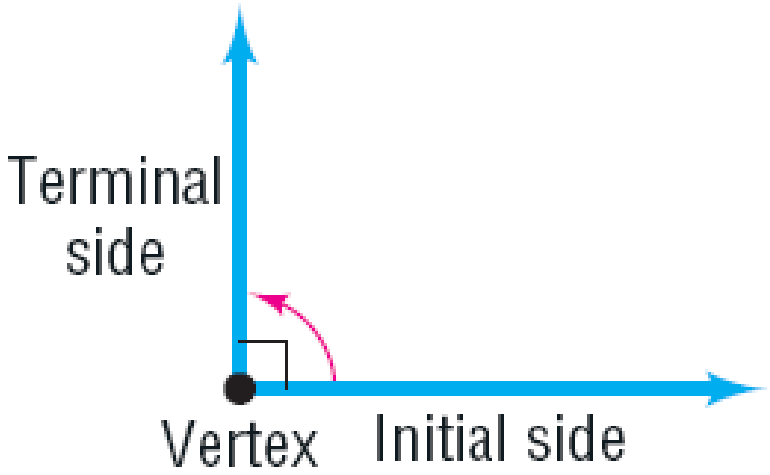


(b) θ lies in quadrant IV



(c) θ is a quadrantal angle

(a) 1 revolution counterclockwise, 360°



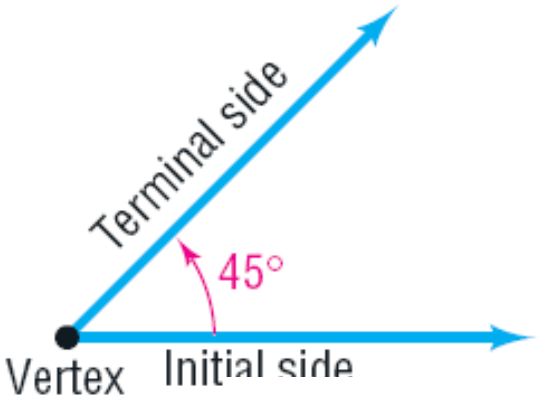
(b) right angle, $\frac{1}{4}$ revolution counter-clockwise, 90°

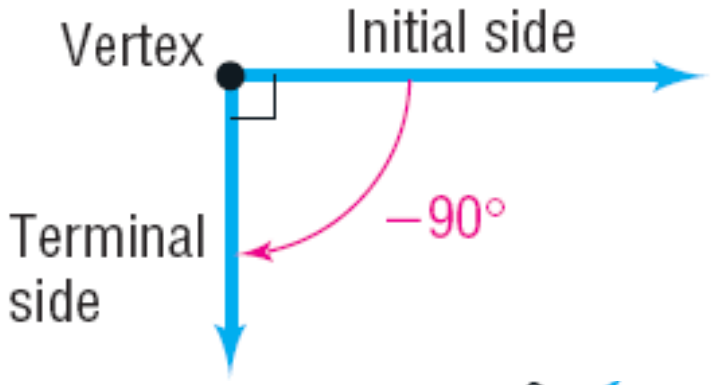
(c) straight angle, $\frac{1}{2}$ revolution counter-clockwise, 180°

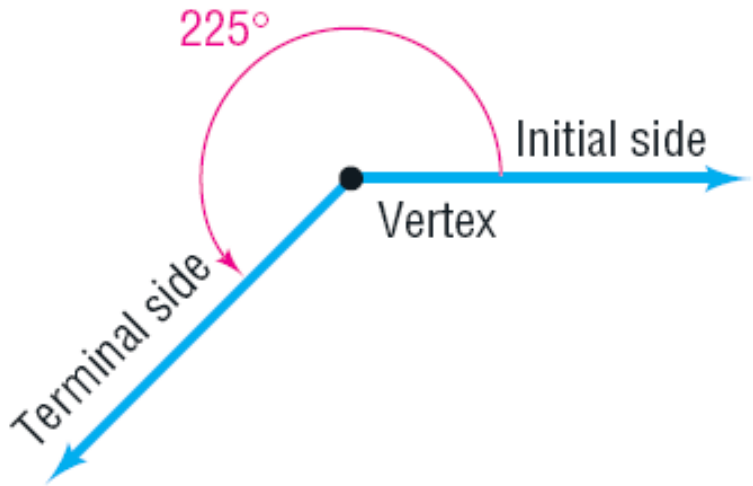
EXAMPLE Drawing an Angle

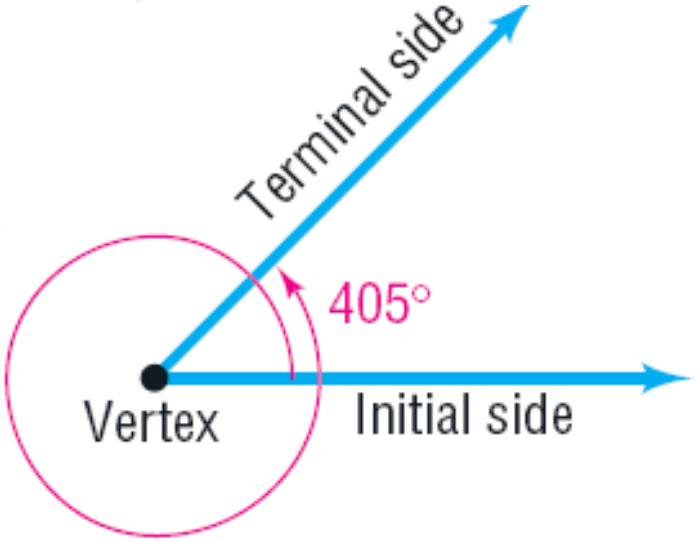
Draw each angle.

(a) $45°$ (b) $-90°$ (c) $225°$ (d) $405°$









OBJECTIVE 1

1. Convert between Degrees, Minutes, Seconds, Forms for Angles

$$1 \text{ counterclockwise revolution} = 360^\circ$$

$$1^\circ = 60' \qquad 1' = 60''$$

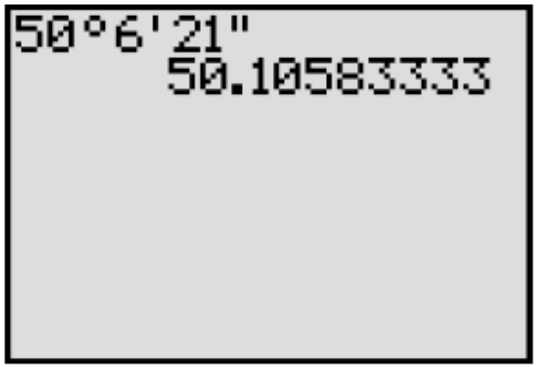
EXAMPLE

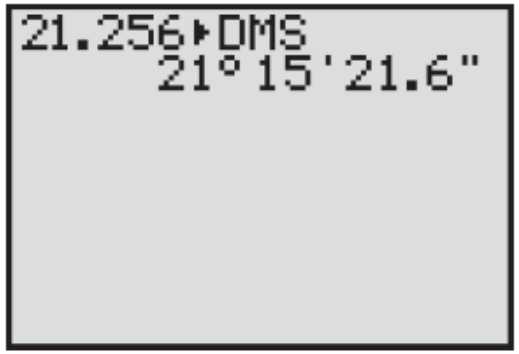
Converting between Degrees, Minutes, Seconds, and Decimal Forms

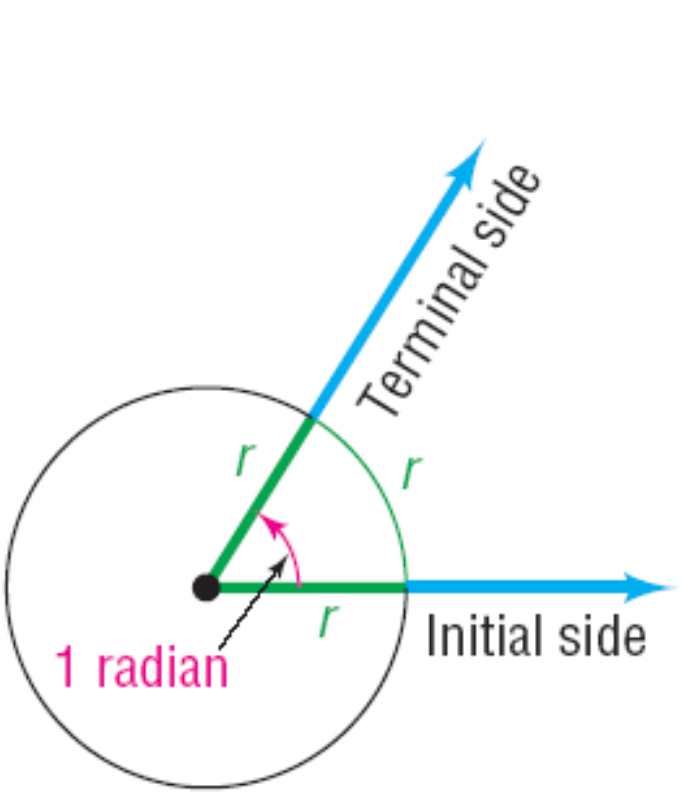
(a) Convert $50°6'21''$ to a decimal in degrees.

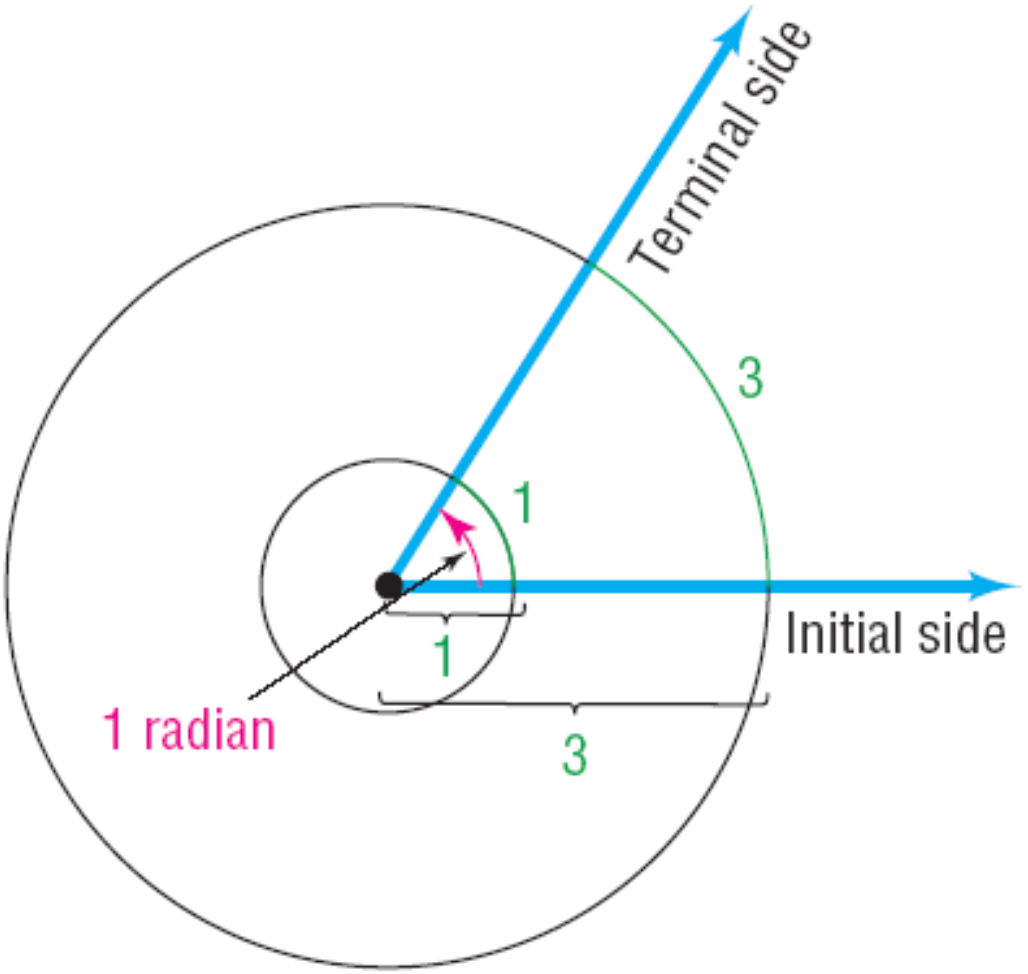
(b) Convert $21.256°$ to the D°M′S″ form.

Graphing Solution



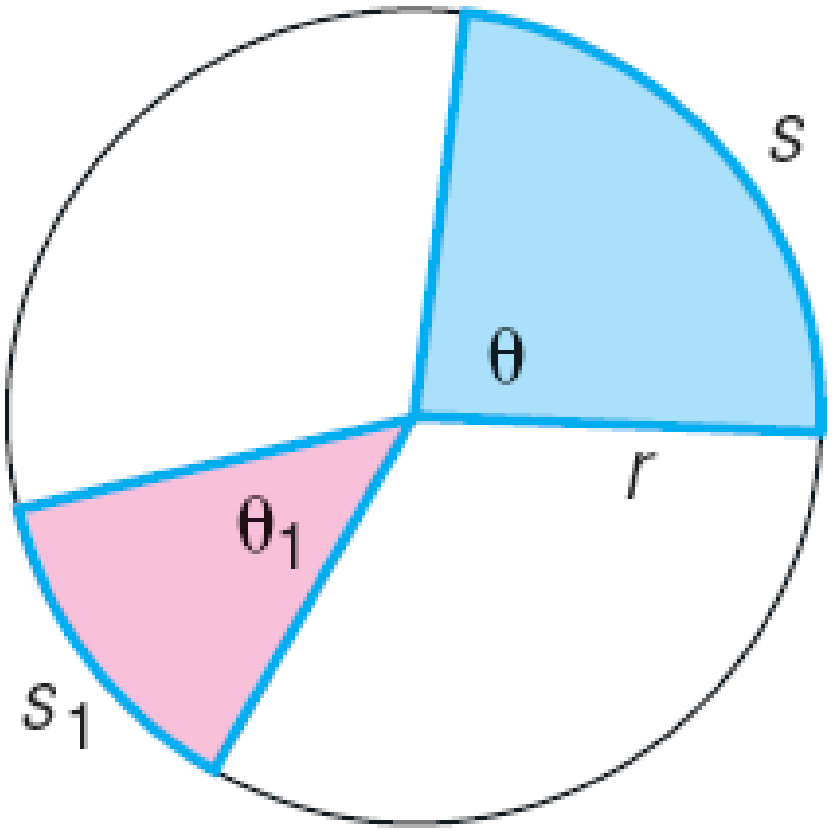



Terminal side
r
r
r
1 radian
Initial side


Terminal side
3
1
1
3
1 radian
Initial side

OBJECTIVE 2

2 Find the Arc Length of a Circle



$$\frac{\theta}{\theta_1} = \frac{s}{s_1}$$

Theorem

Arc Length

For a circle of radius r, a central angle of θ radians subtends an arc whose length s is

$$s = r\theta$$

EXAMPLE

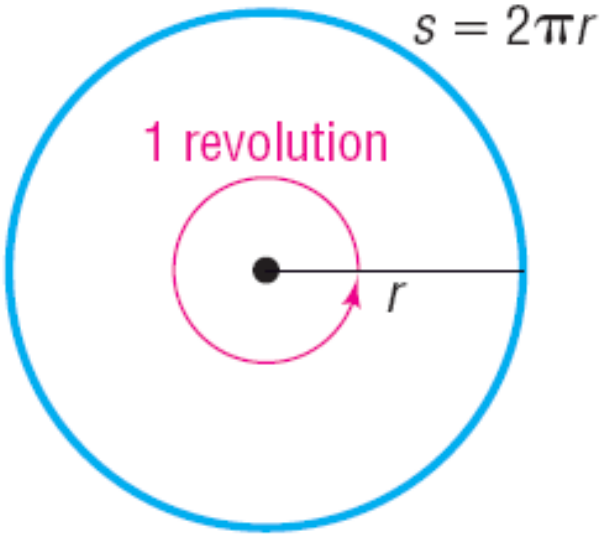
Finding the Length of an Arc of a Circle

Find the length of the arc of a circle of radius 4 meters subtended by a central angle of 0.5 radian.

$$s = r\theta$$

OBJECTIVE 3

3 Convert from Degrees to Radians and from Radians to Degrees



$$1 \text{ revolution} = 2\pi \text{ radians}$$

$$180^\circ = \pi \text{ radians}$$

$$1 \text{ degree} = \frac{\pi}{180} \text{ radian} \qquad 1 \text{ radian} = \frac{180}{\pi} \text{ degrees}$$

EXAMPLE Converting from Degrees to Radians

Convert each angle in degrees to radians.

(a) $60°$ (b) $150°$ (c) $-45°$ (d) $90°$ (e) $107°$

$$1 \text{ degree} = \frac{\pi}{180} \text{ radian} \qquad 1 \text{ radian} = \frac{180}{\pi} \text{ degrees}$$

EXAMPLE Converting Radians to Degrees

Convert each angle in radians to degrees.

(a) $\frac{\pi}{6}$ radian (b) $\frac{3\pi}{2}$ radians (c) $-\frac{3\pi}{4}$ radians

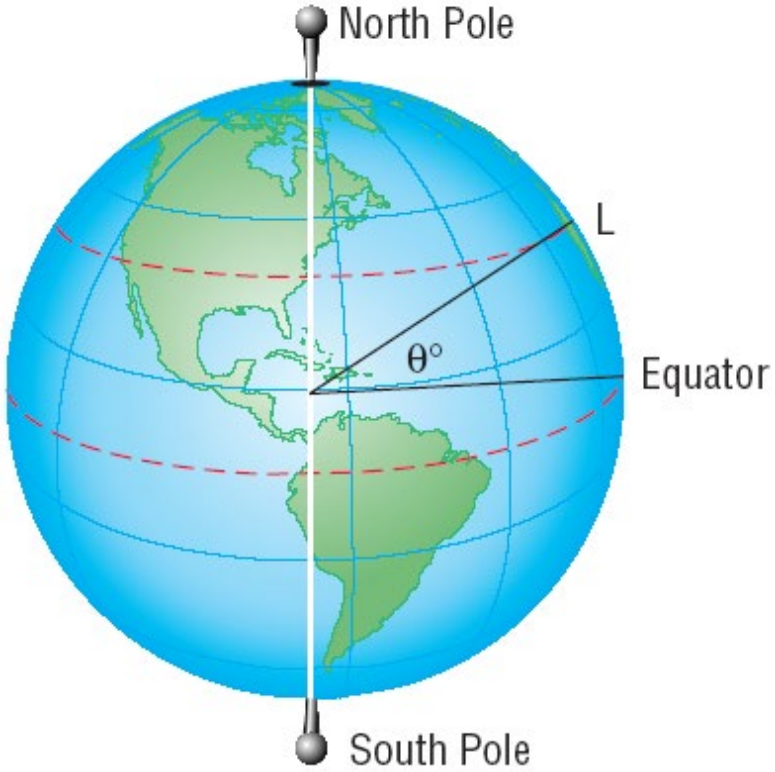
(d) $\frac{7\pi}{3}$ radians (e) 3 radians

Degrees	$0°$	$30°$	$45°$	$60°$	$90°$	$120°$	$135°$	$150°$	$180°$
Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
Degrees		$210°$	$225°$	$240°$	$270°$	$300°$	$315°$	$330°$	$360°$
Radians		$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π

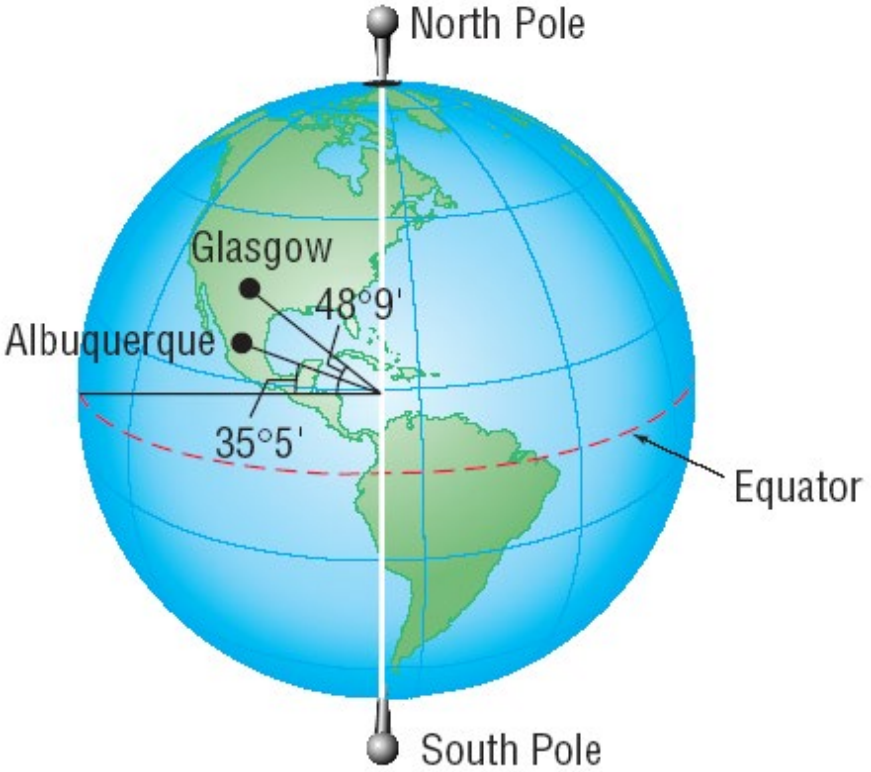
EXAMPLE

Finding the Distance between Two Cities

See Figure 15(a). The latitude of a location L is the angle formed by a ray drawn from the center of Earth to the Equator and a ray drawn from the center of Earth to L. See Figure 15(b). Glasgow, Montana, is due north of Albuquerque, New Mexico. Find the distance between Glasgow (48°9′ north latitude) and Albuquerque (35°5′ north latitude). Assume that the radius of Earth is 3960 miles.



(a)

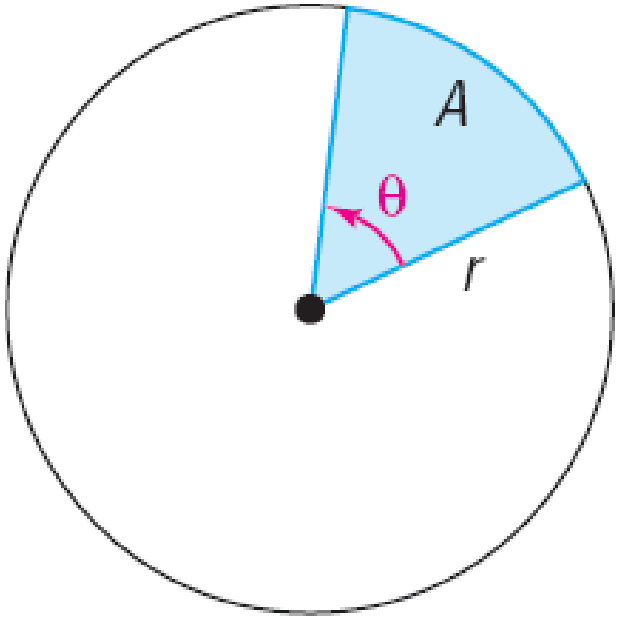


(b)

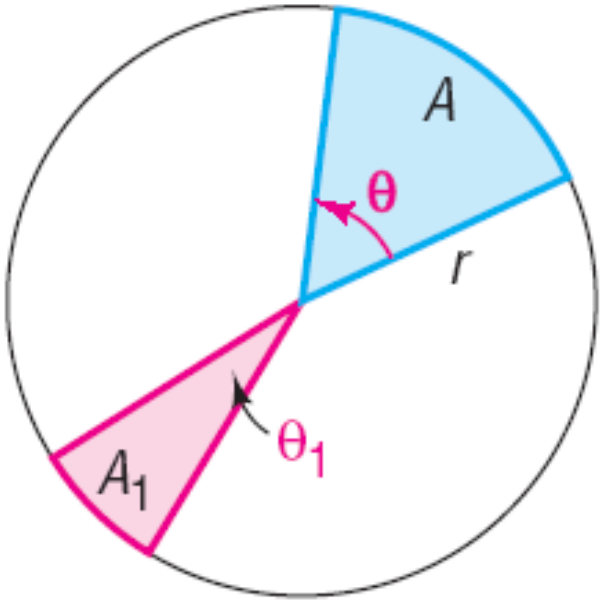
OBJECTIVE 4

4 Find the Area of a Sector of a Circle

Area of a Sector



$$\frac{\theta}{\theta_1} = \frac{A}{A_1}$$



The area A of the sector of a circle of radius r formed by a central angle of θ radians is

$$A = \frac{1}{2}r^2\theta$$

EXAMPLE

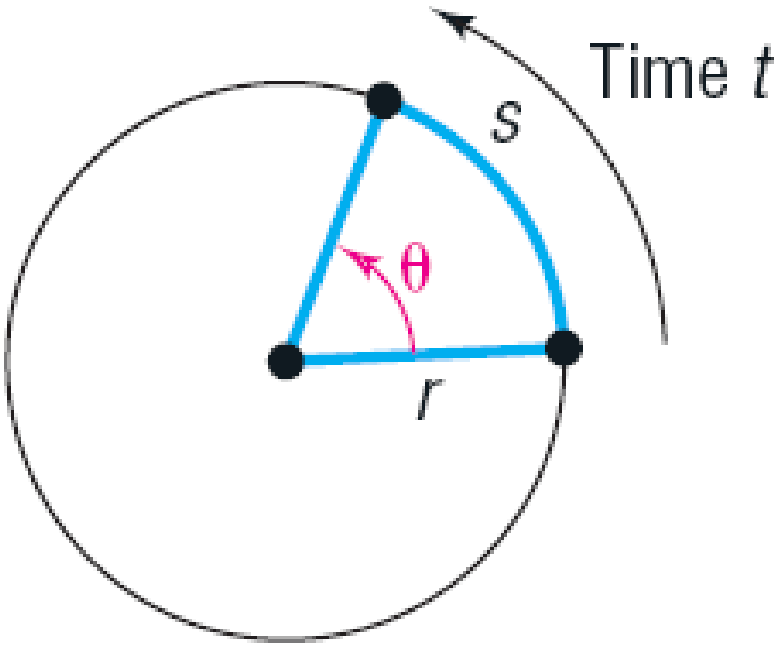
Finding the Area of a Sector of a Circle

Find the area of the sector of a circle of radius 5 feet formed by an angle of 40°. Round the answer to two decimal places.

$$A = \frac{1}{2}r^2\theta$$

OBJECTIVE 5

- Find the Linear Speed of an Object Traveling in Circular Motion



$$v = \frac{s}{t}$$

$$\omega = \frac{\theta}{t}$$

$$v = r\omega$$

EXAMPLE

Finding Linear Speed

A child is spinning a rock at the end of a 2-foot rope at the rate of 180 revolutions per minute (rpm). Find the linear speed of the rock when it is released.