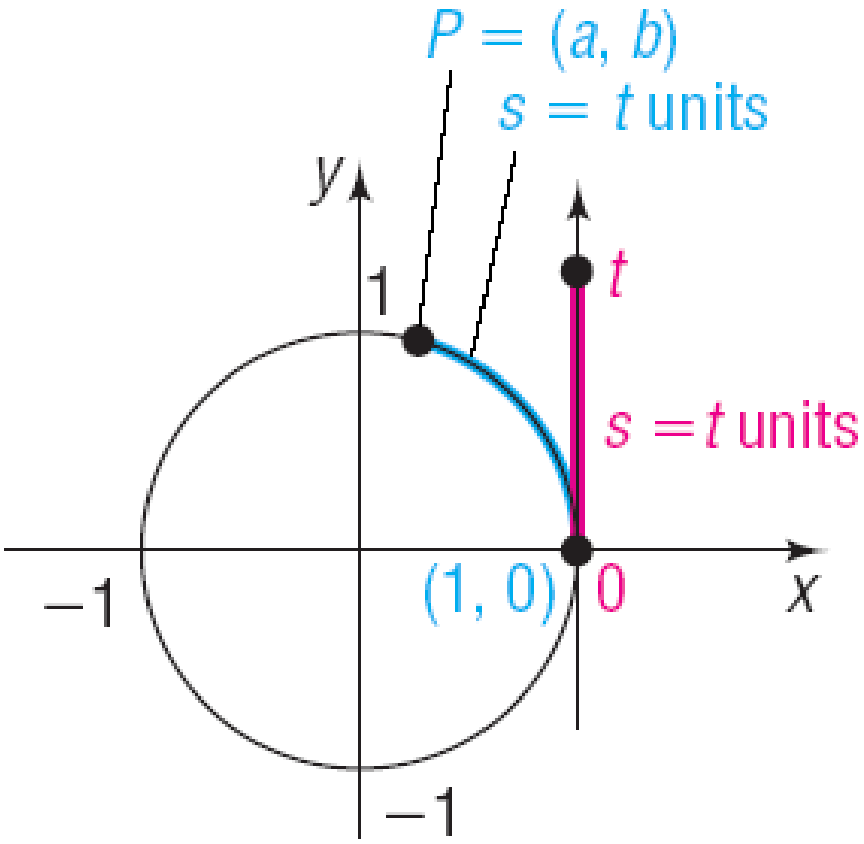
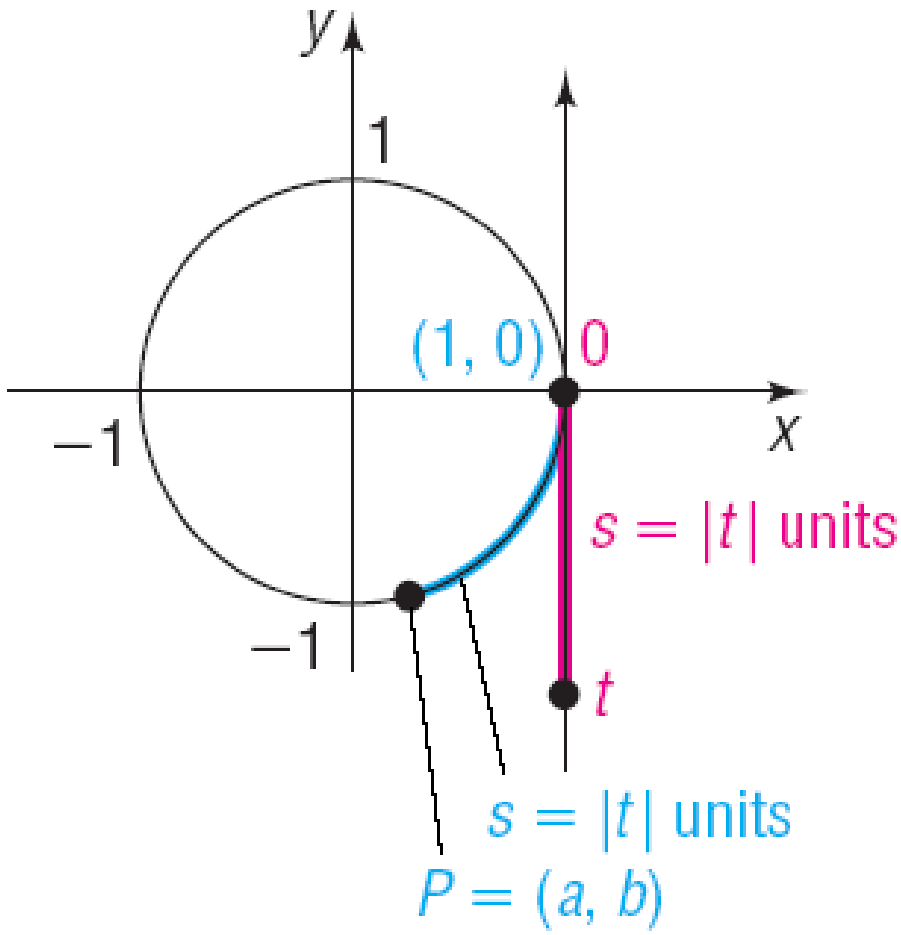
Section 5.2

Trigonometric Functions:

Unit Circle Approach

The Unit Circle





Let t be a real number and let $P = (x, y)$ be the point on the unit circle that corresponds to t.

The **sine function** associates with t the y-coordinate of P and is denoted by

$$\sin t = y$$

The **cosine function** associates with t the x-coordinate of P and is denoted by

$$\cos t = x$$

If $x \neq 0$, the **tangent function** is defined as

$$\tan t = \frac{y}{x}$$

Let t be a real number and let $P = (x, y)$ be the point on the unit circle that corresponds to t.

If $y \neq 0$, the **cosecant function** is defined as

$$\csc t = \frac{1}{y}$$

If $x \neq 0$, the **secant function** is defined as

$$\sec t = \frac{1}{x}$$

If $y \neq 0$, the **cotangent function** is defined as

$$\cot t = \frac{x}{y}$$

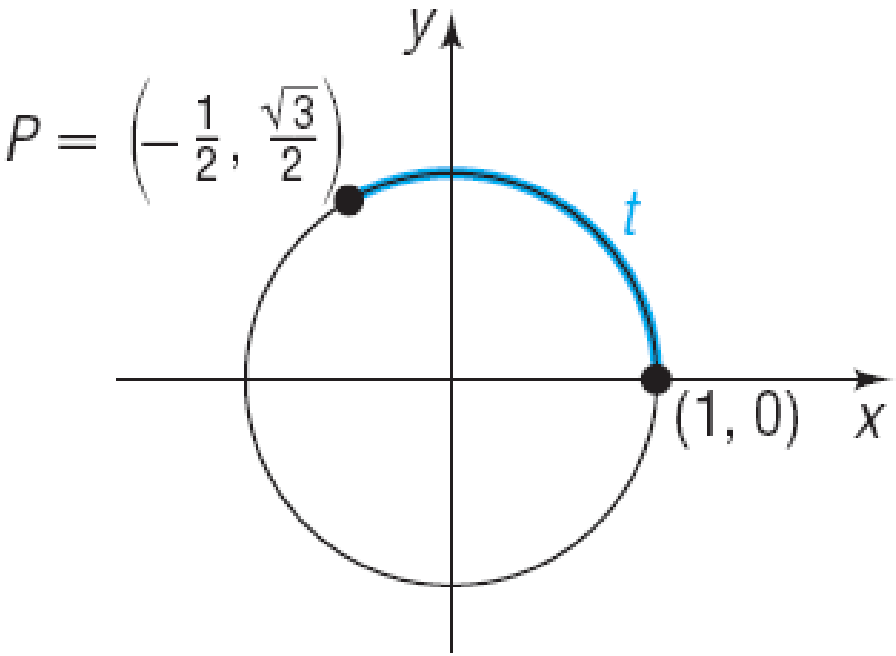
OBJECTIVE 1

1. Find the Exact Values of the Trigonometric Functions Using a Point on the Unit Circle

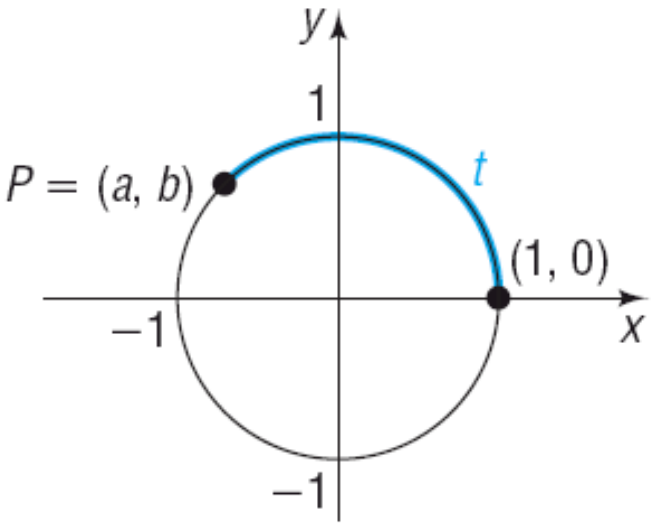
EXAMPLE

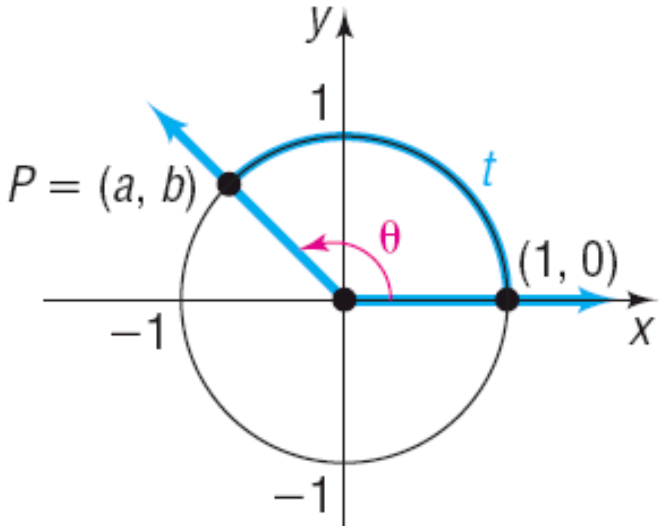
Finding the Values of the Six Trigonometric Functions Using a Point on the Unit Circle

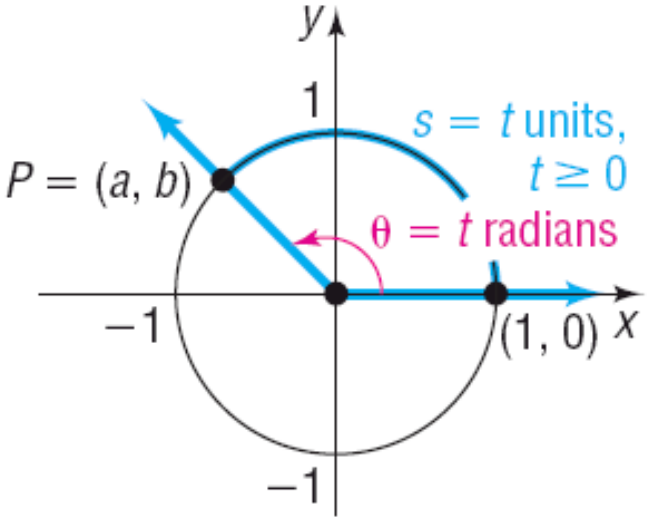
Find the values of sin t, cos t, tan t, csc t, sec t, and cot t if $P = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ is the point on the unit circle that corresponds to the real number t.

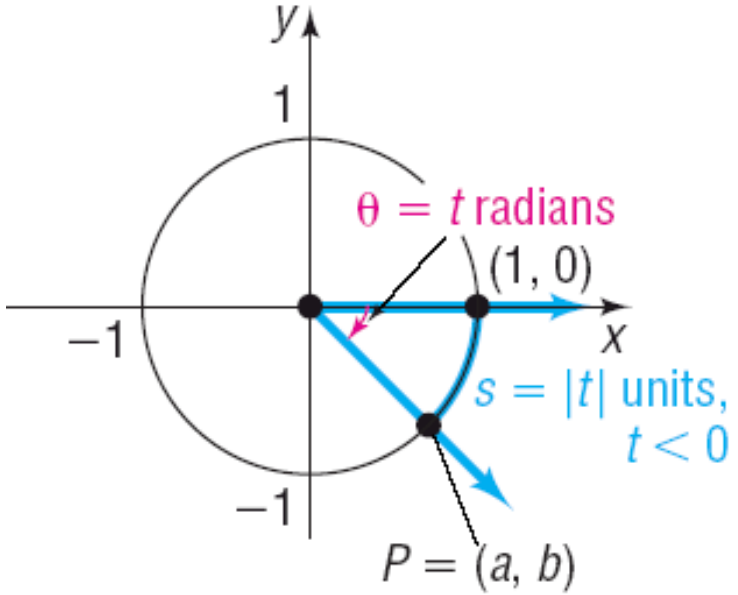


Trigonometric Functions of Angles









If $\theta = t$ radians, the six **trigonometric functions of the angle θ** are defined as

$$\sin\theta = \sin t \qquad \cos\theta = \cos t \qquad \tan\theta = \tan t$$
$$\csc\theta = \csc t \qquad \sec\theta = \sec t \qquad \cot\theta = \cot t$$

OBJECTIVE 2

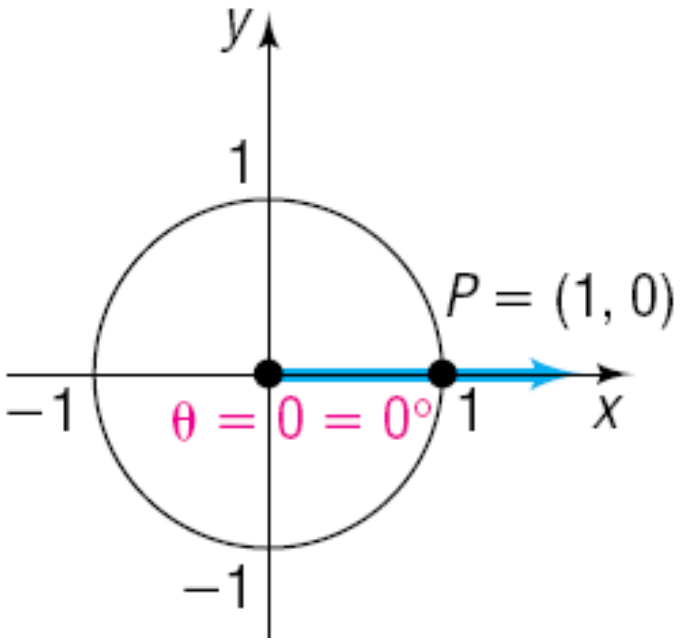
2 Find the Exact Values of the Trigonometric Functions of Quadrantal Angles

EXAMPLE

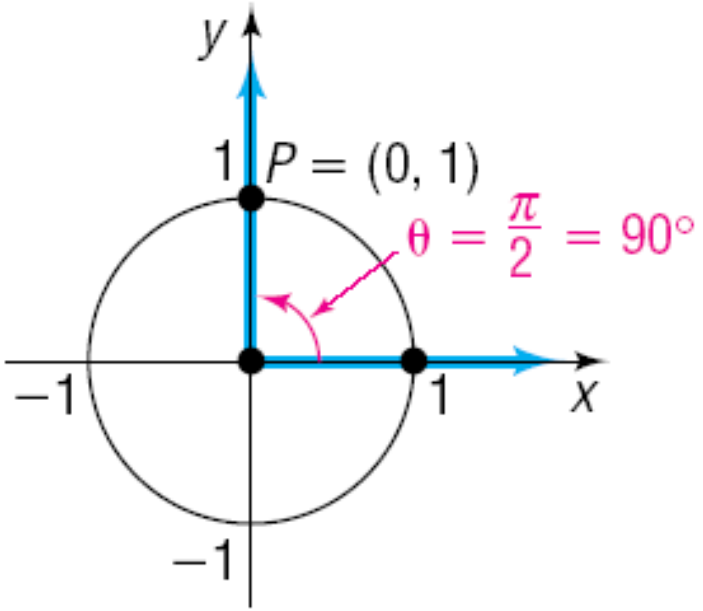
Finding the Exact Values of the Six Trigonometric Functions of Quadrantal Angles

Find the exact values of the six trigonometric functions of:

(a) $\theta = 0 = 0°$



(b) $\theta = \frac{\pi}{2} = 90°$

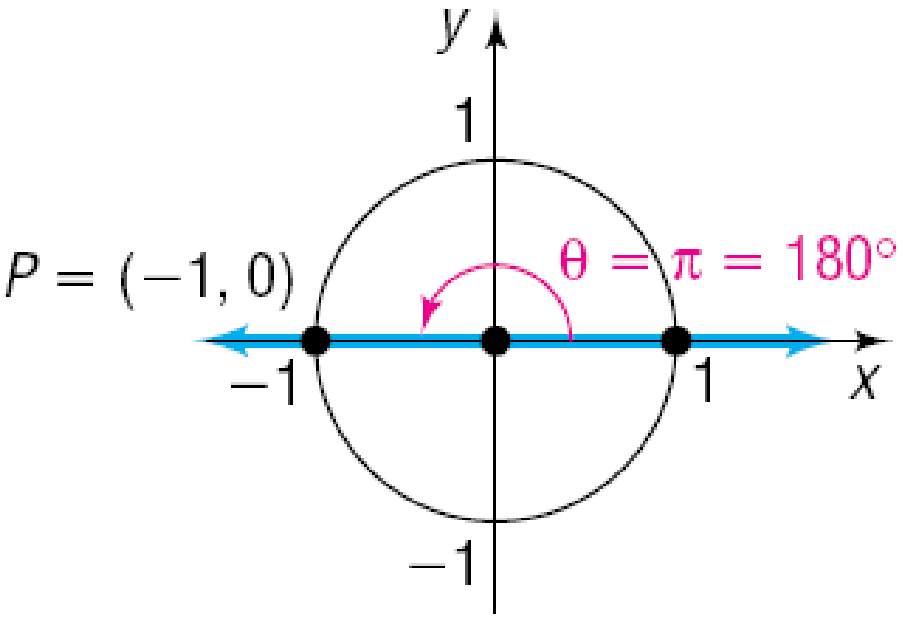


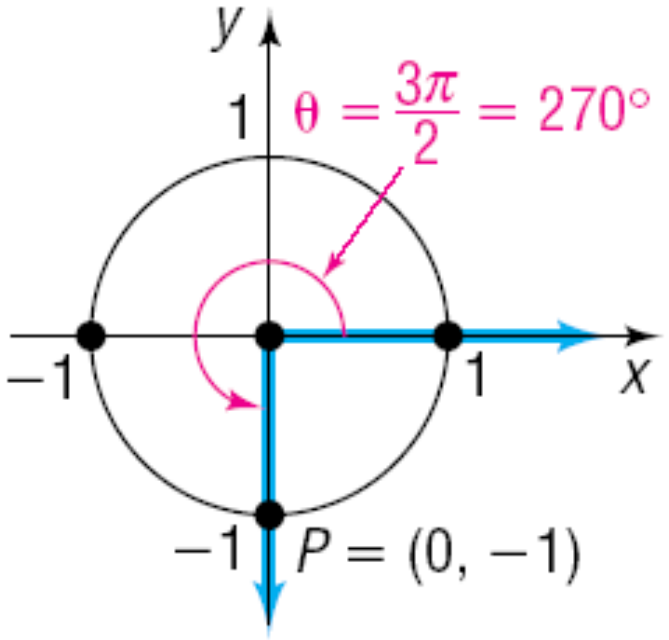
Finding the Exact Values of the Six Trigonometric Functions of Quadrantal Angles

Find the exact values of the six trigonometric functions of:

(c) $\theta = \pi = 180°$

(d) $\theta = \dfrac{3\pi}{2} = 270°$





Quadrantal Angles

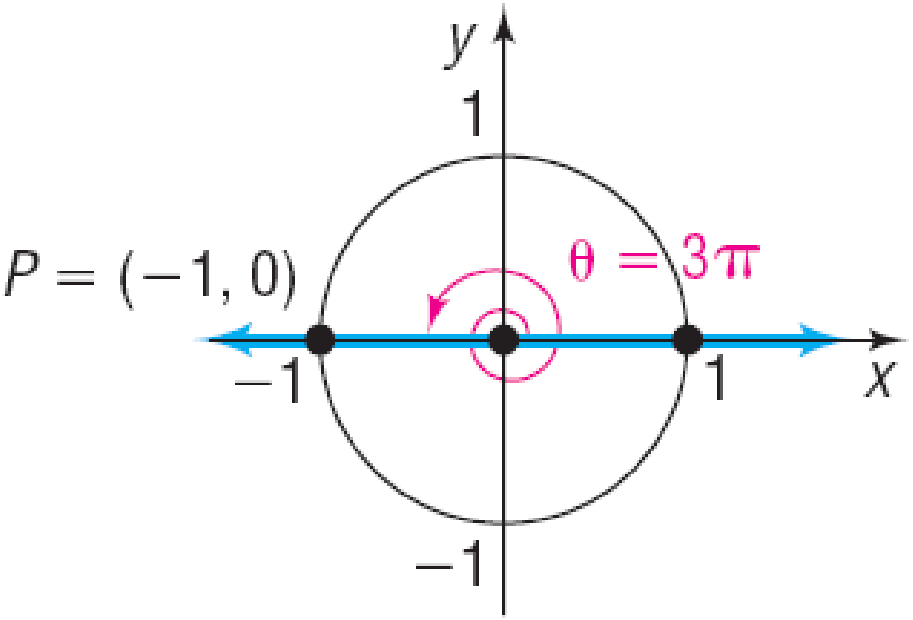
θ (Radians)	θ (Degrees)	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
0	0°	0	1	0	Not defined	1	Not defined
$\frac{\pi}{2}$	90°	1	0	Not defined	1	Not defined	0
π	180°	0	-1	0	Not defined	-1	Not defined
$\frac{3\pi}{2}$	270°	-1	0	Not defined	-1	Not defined	0

EXAMPLE

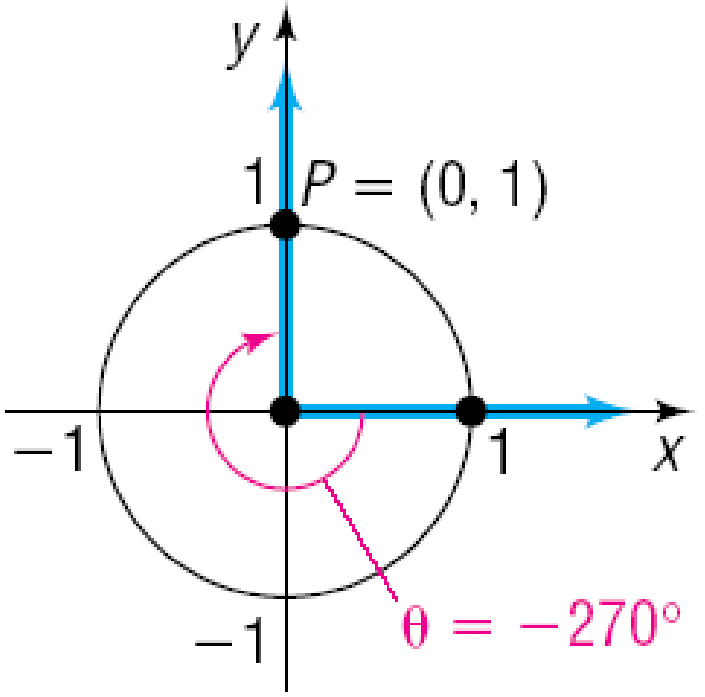
Finding Exact Values of the Trigonometric Functions of Angles That Are Integer Multiples of Quadrantal Angles

Find the exact value of:

(a) $\sin(3\pi)$



(b) $\cos(-270°)$



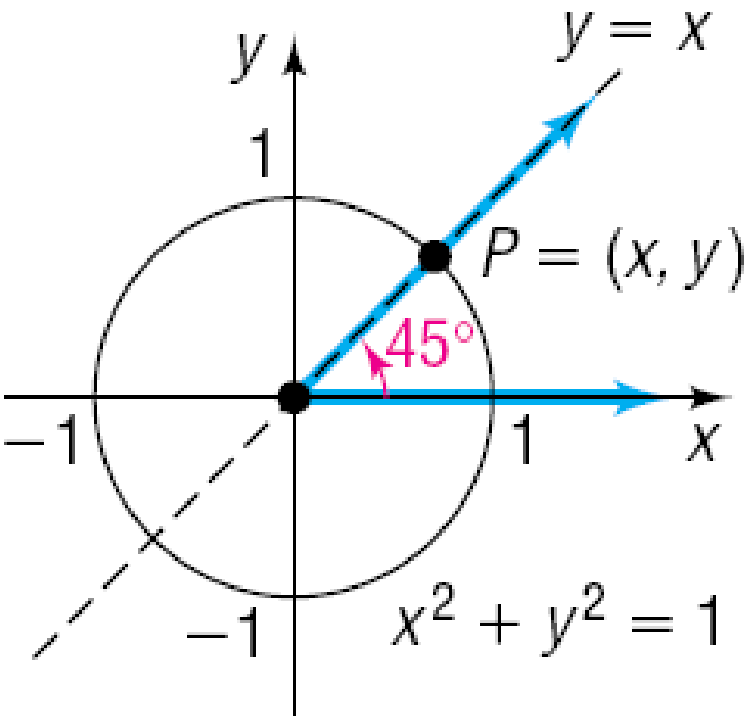
OBJECTIVE 3

3 Find the Exact Values of theTrigonometric Functions of $\frac{\pi}{4} = 45°$

EXAMPLE

Finding the Exact Values of the Trigonometric Functions of $\frac{\pi}{4} = 45°$

Find the exact values of the six trigonometric functions of $\frac{\pi}{4} = 45°$.



EXAMPLE

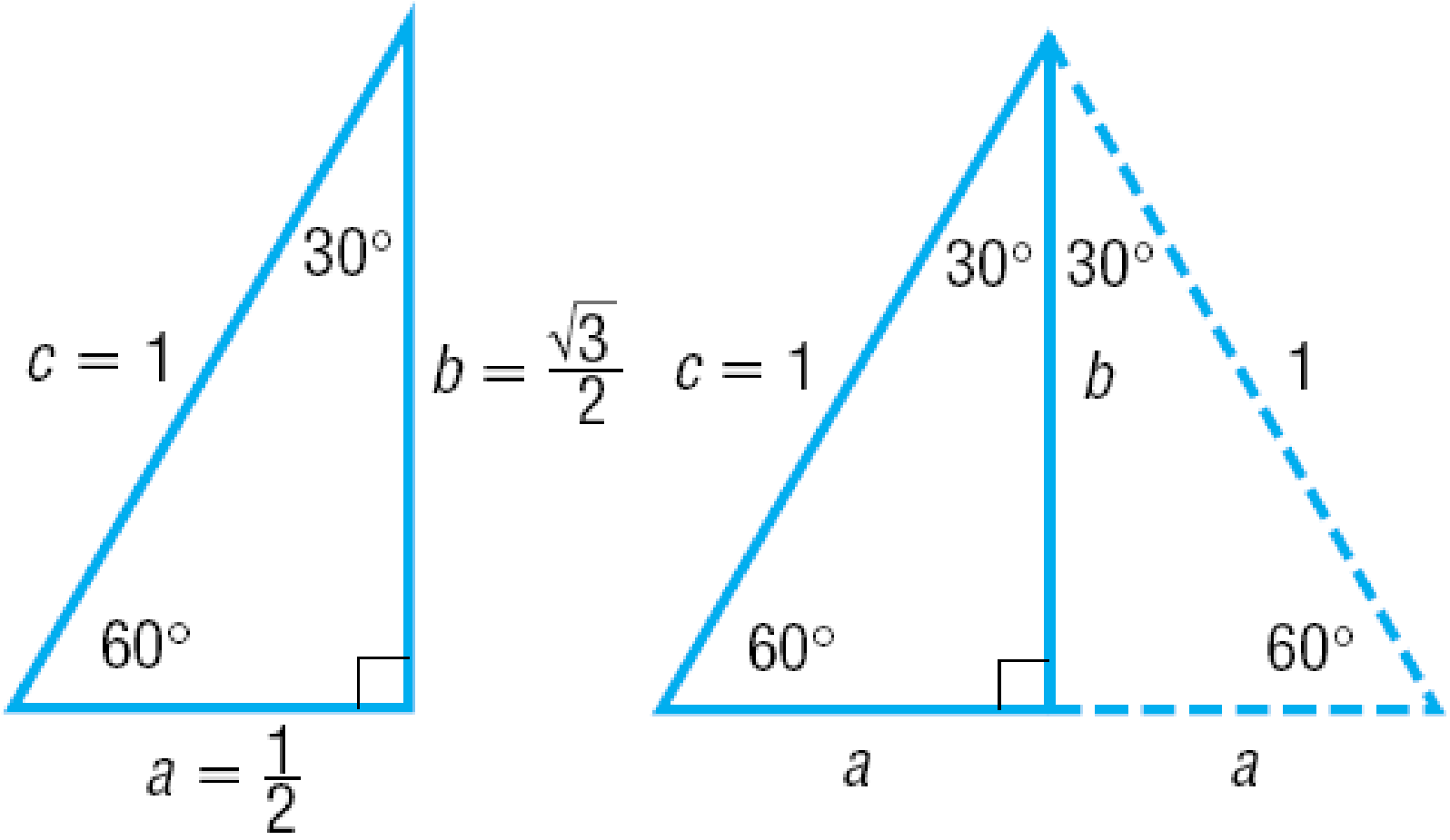
Finding the Exact Value of a Trigonometric Expression

Find the exact value of each expression.

(a) $\sin 45^\circ \cos 180^\circ$ (b) $\tan \frac{\pi}{4} - \sin \frac{3\pi}{2}$ (c) $\left(\sec \frac{\pi}{4}\right)^2 + \csc \frac{\pi}{2}$

OBJECTIVE 4

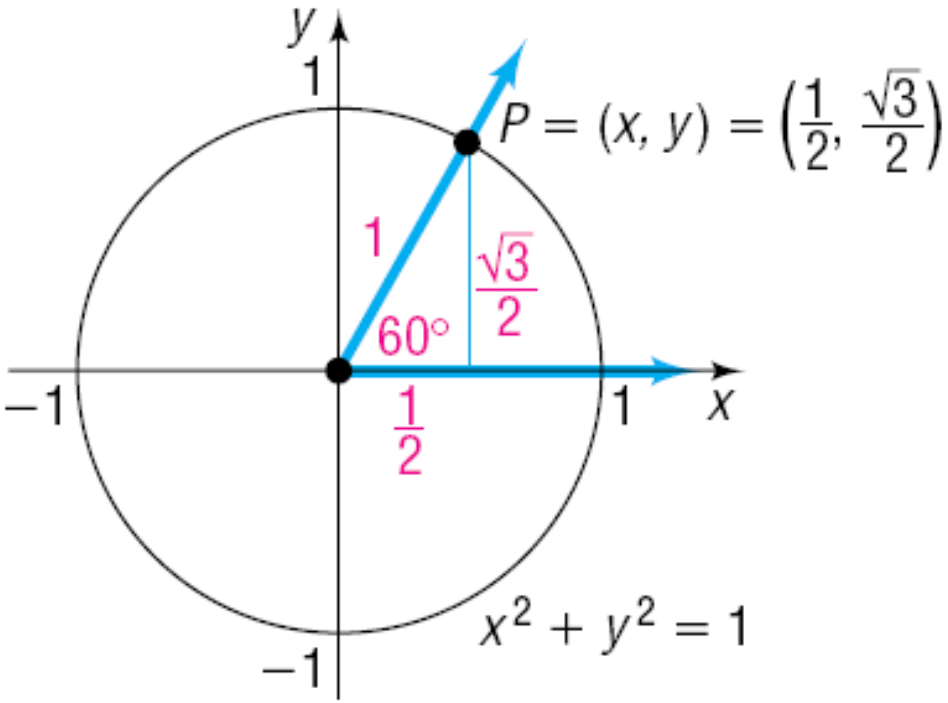
4 **Find the Exact Values of theTrigonometric Functions of $\frac{\pi}{6} = 30^\circ$ and $\frac{\pi}{3} = 60^\circ$**


30°
$c = 1$
$b = \frac{\sqrt{3}}{2}$
60°
$a = \frac{1}{2}$
30°
30°
$c = 1$
b
1
60°
60°
a
a

EXAMPLE

Finding the Exact Values of the Trigonometric Functions of $\frac{\pi}{3} = 60°$

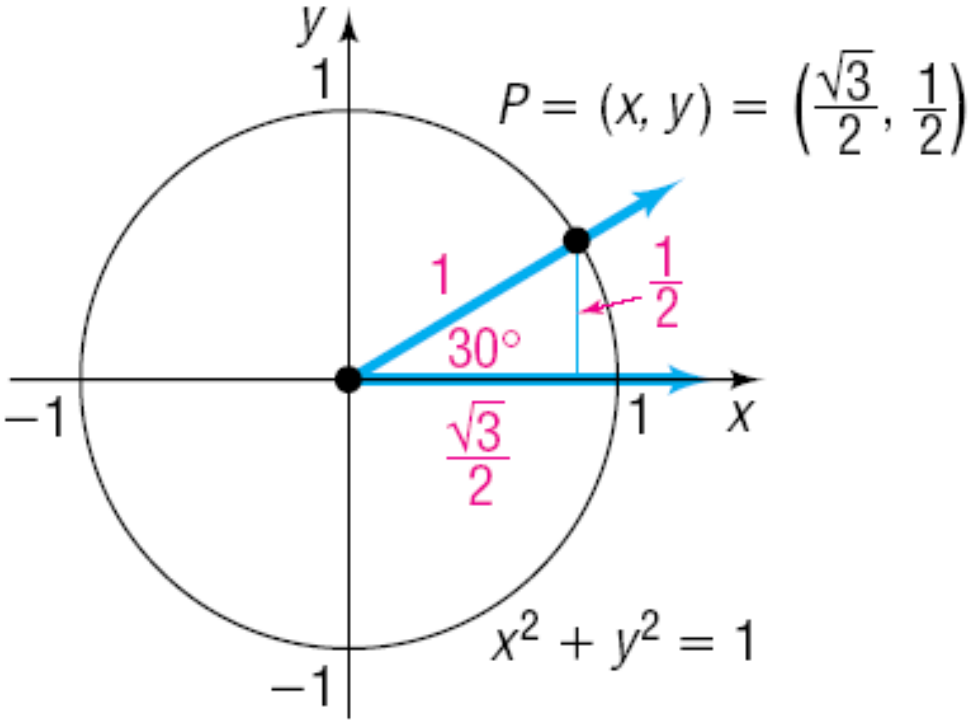
Find the exact values of the six trigonometric functions of $\frac{\pi}{3} = 60°$.



EXAMPLE

Finding the Exact Values of the Trigonometric Functions of $\frac{\pi}{6} = 30°$

Find the exact values of the trigonometric functions of $\frac{\pi}{6} = 30°$.



θ (Radians)	θ (Degrees)	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
$\frac{\pi}{6}$	30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
$\frac{\pi}{4}$	45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
$\frac{\pi}{3}$	60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$

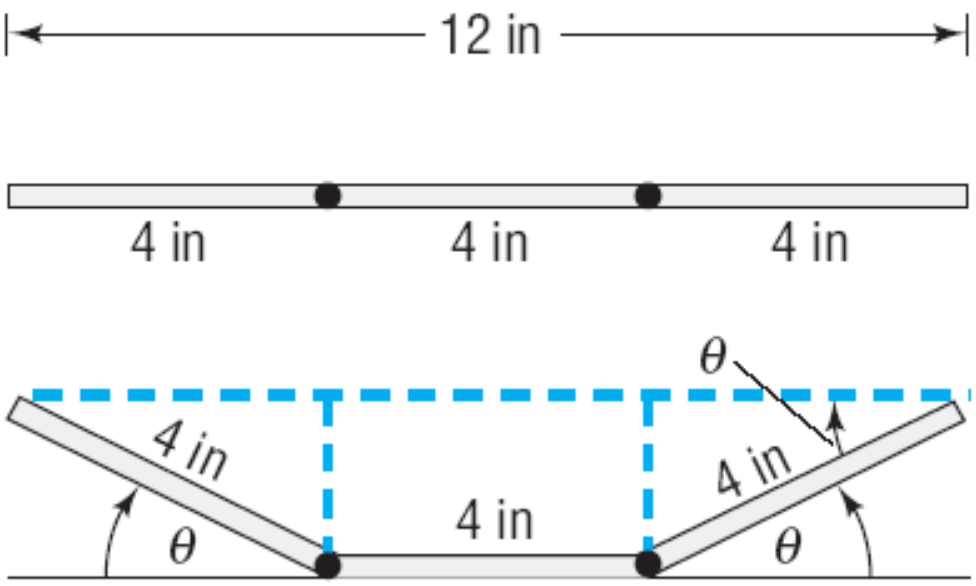
EXAMPLE

Constructing a Rain Gutter

A rain gutter is to be constructed of aluminum sheets 12 inches wide. After marking off a length of 4 inches from each edge, this length is bent up at an angle θ. See Figure 30. The area A of the opening may be expressed as a function of θ as

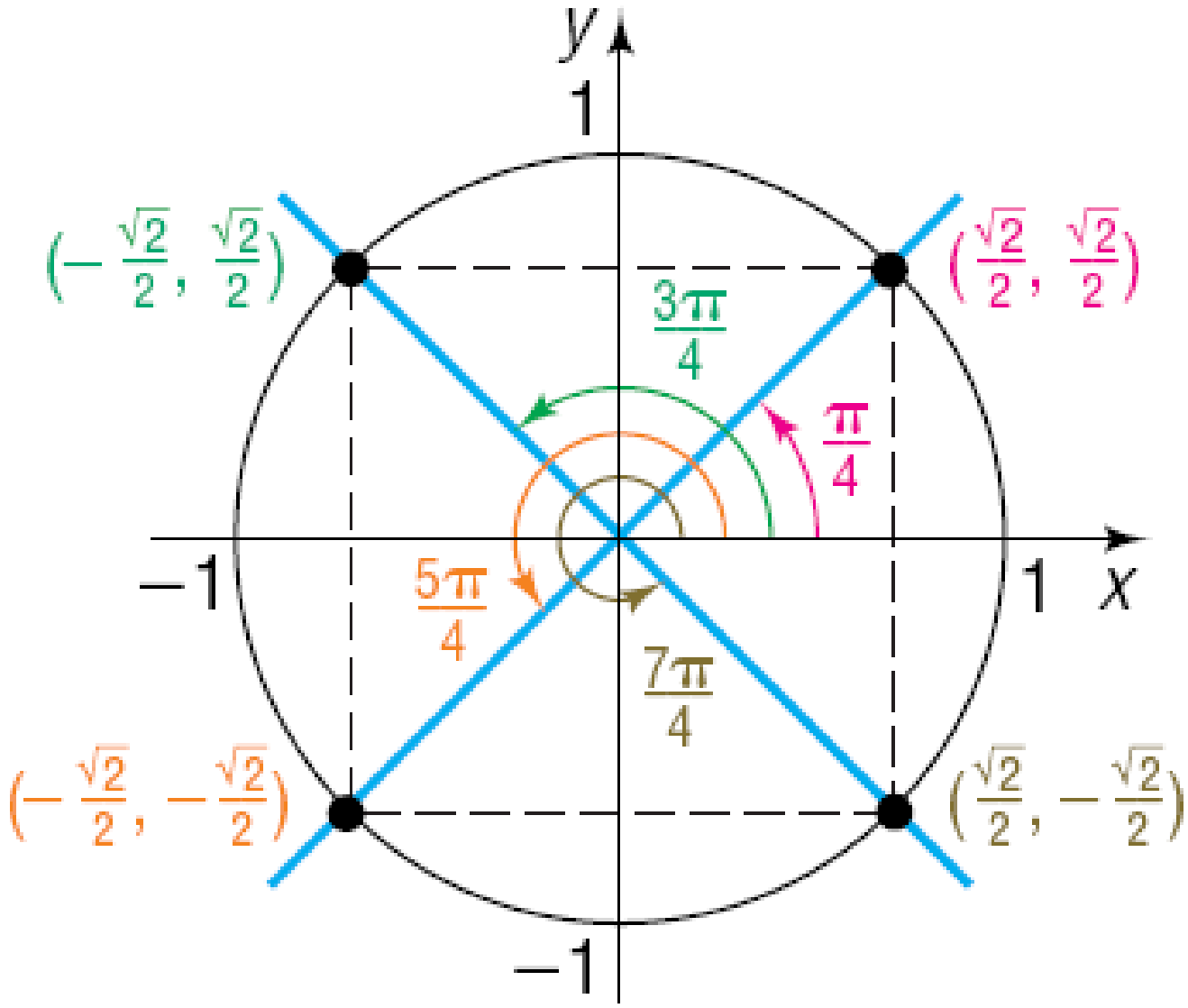
$$A(\theta) = 16 \sin \theta(\cos \theta + 1)$$

Find the area A of the opening for $\theta = 30°$, $\theta = 45°$, and $\theta = 60°$.



OBJECTIVE 5

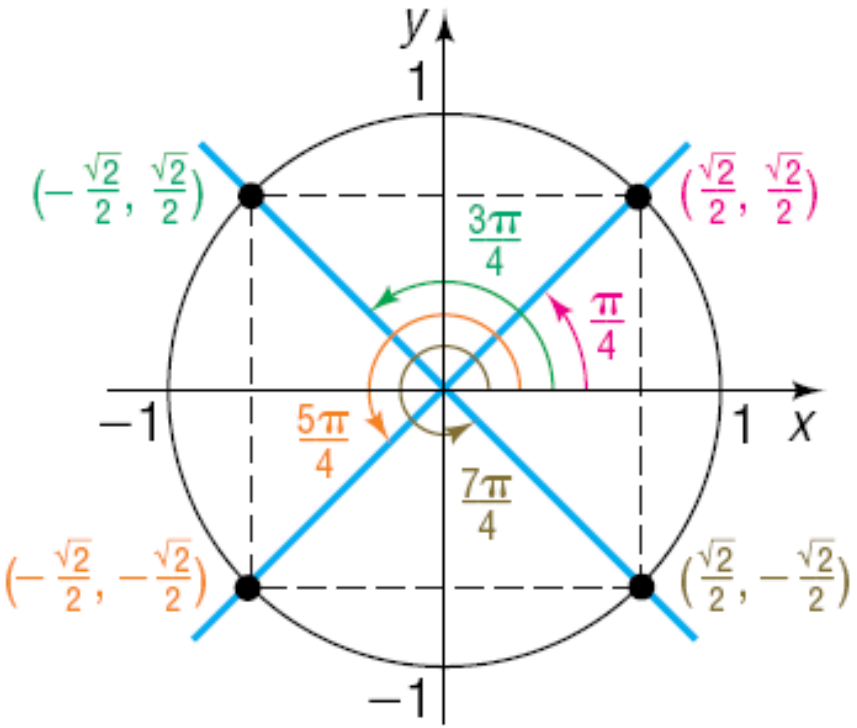
5 **Find the Exact Values of the Trigonometric Functions for Integer Multiples of $\frac{\pi}{6} = 30°$, $\frac{\pi}{4} = 45°$, and $\frac{\pi}{3} = 60°$**

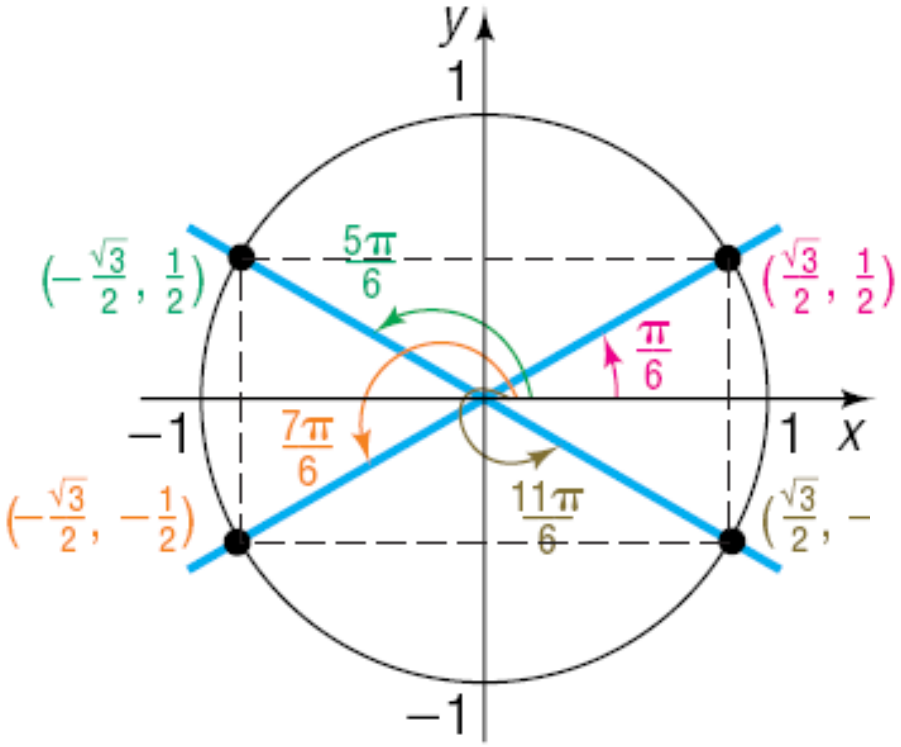

y
1
−1
1
x
−1
$\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
$\frac{3\pi}{4}$
$\frac{\pi}{4}$
$\frac{5\pi}{4}$
$\frac{7\pi}{4}$
$\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$
$\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

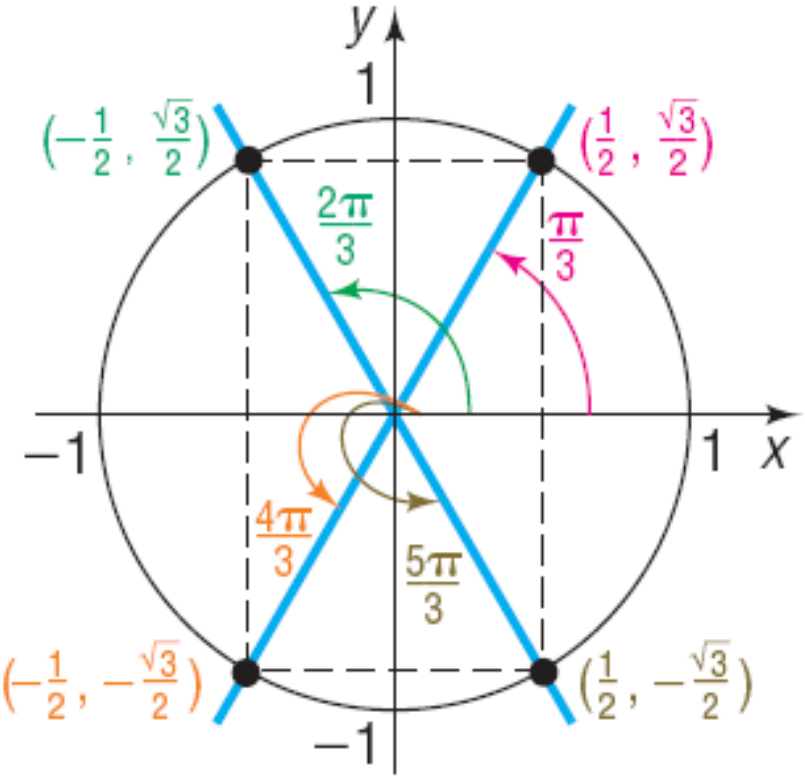
EXAMPLE

Finding Exact Values for Multiples of $\frac{\pi}{4} = 45°$

Find: (a) $\sin 135°$ (b) $\cos \frac{5\pi}{4}$ (c) $\tan 315°$




y
1
−1
1
x
−1
$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
$\frac{5\pi}{6}$
$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
$\frac{\pi}{6}$
$\frac{7\pi}{6}$
$\frac{11\pi}{6}$
$\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$
$\left(\frac{\sqrt{3}}{2}, -\right.$


y
1
−1
1
x
−1
$\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
$\frac{2\pi}{3}$
$\frac{\pi}{3}$
$\frac{4\pi}{3}$
$\frac{5\pi}{3}$
$\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$
$\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

EXAMPLE Using Figures 32 and 33

Find: (a) $\cos 210°$ (b) $\sin(-60°)$ (c) $\tan \dfrac{5\pi}{3}$

Figure 32

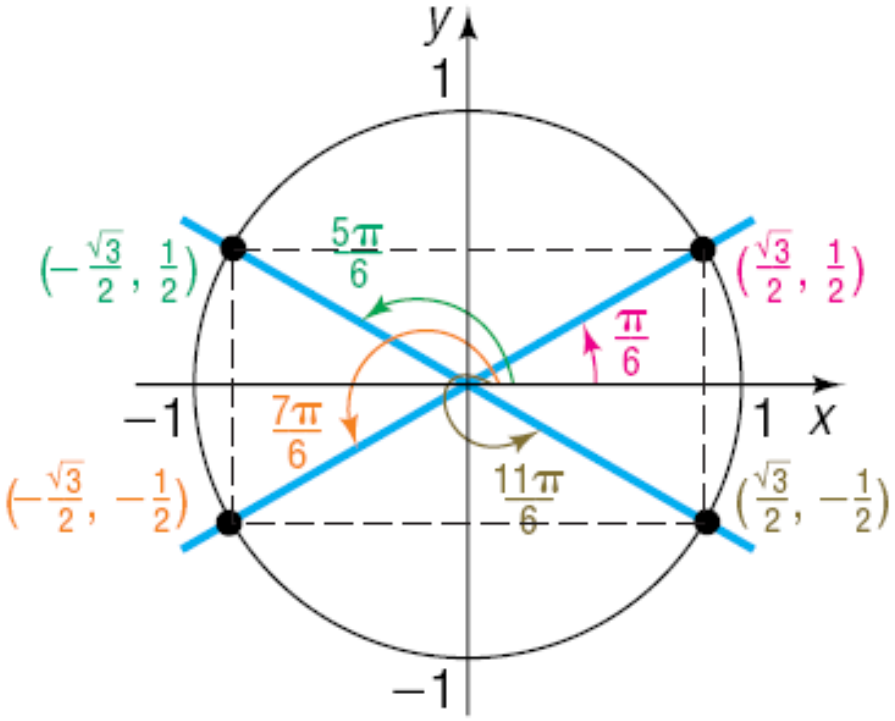
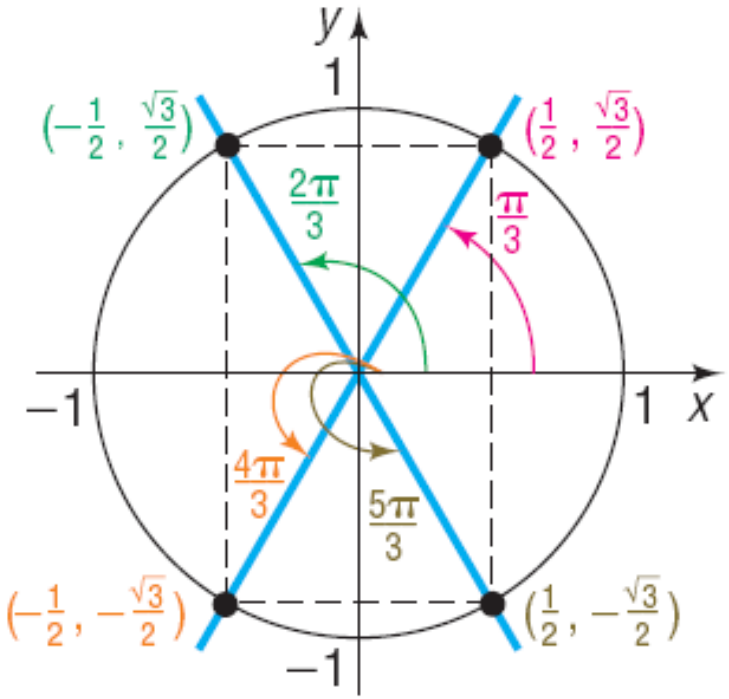


Figure 33



OBJECTIVE 6

Use a Calculator to Approximate the Value of a Trigonometric Function

$$\sec\theta = \frac{1}{x} = \frac{1}{\cos\theta}$$

$$\csc\theta = \frac{1}{y} = \frac{1}{\sin\theta}$$

$$\cot\theta = \frac{x}{y} = \frac{1}{\frac{y}{x}} = \frac{1}{\tan\theta}$$

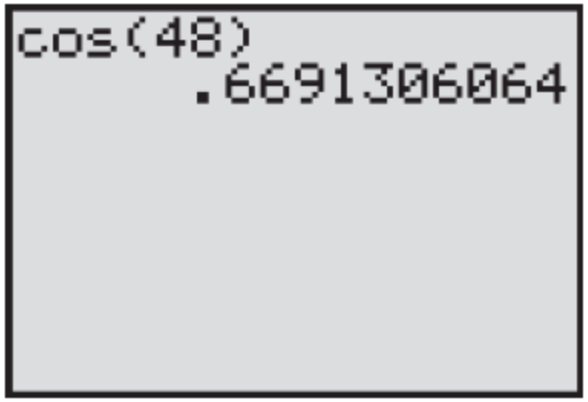
EXAMPLE

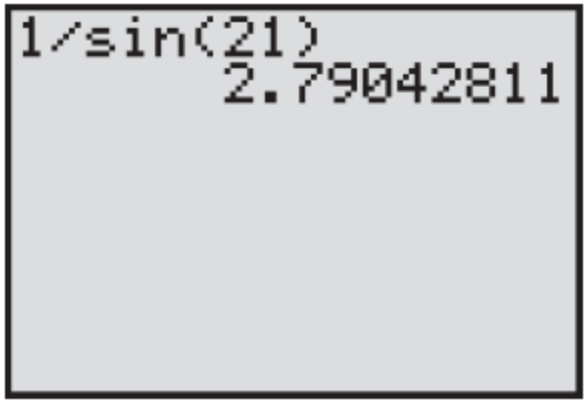
Using a Calculator to Approximate the Value of a Trigonometric Function

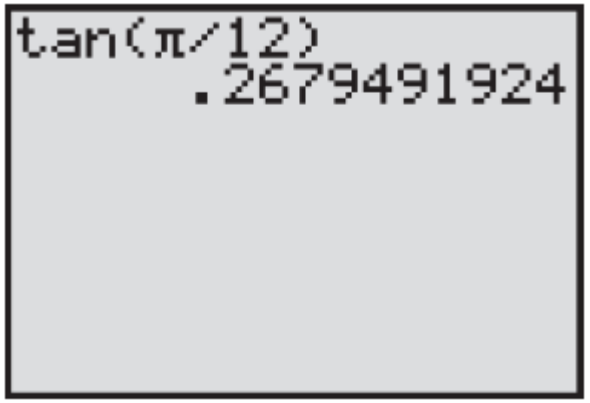
Use a calculator to find the approximate value of:

(a) $\cos 48°$ (b) $\csc 21°$ (c) $\tan \frac{\pi}{12}$

Express your answers rounded to two decimal places.



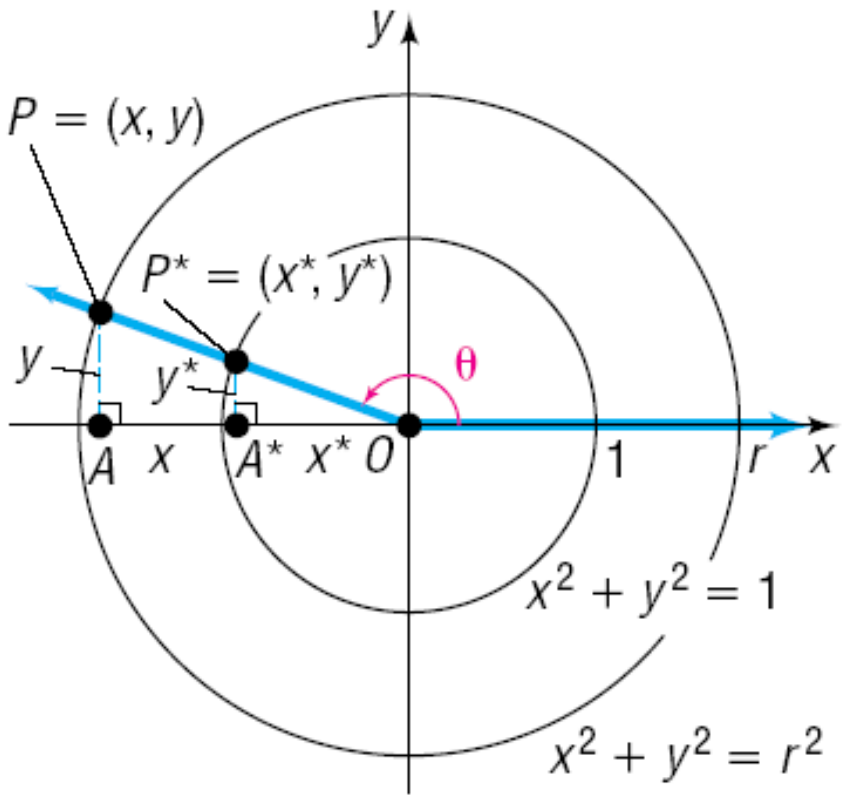




CHANGE MODE TO RADIANS

OBJECTIVE 7

Use a Circle of Radius *r* to Evaluate the Trigonometric Functions



Theorem

For an angle θ in standard position, let $P = (x, y)$ be the point on the terminal side of θ that is also on the circle $x^2 + y^2 = r^2$. Then

$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x}, \quad x \neq 0$$

$$\csc\theta = \frac{r}{y}, \quad y \neq 0 \qquad \sec\theta = \frac{r}{x}, \quad x \neq 0 \qquad \cot\theta = \frac{x}{y}, \quad y \neq 0$$

EXAMPLE

Finding the Exact Values of the Six Trigonometric Functions

Find the exact values of each of the six trigonometric functions of an angle θ if $(4, -3)$ is a point on its terminal side.