# Section 5.4

# Graphs of the Sine and Cosine Functions

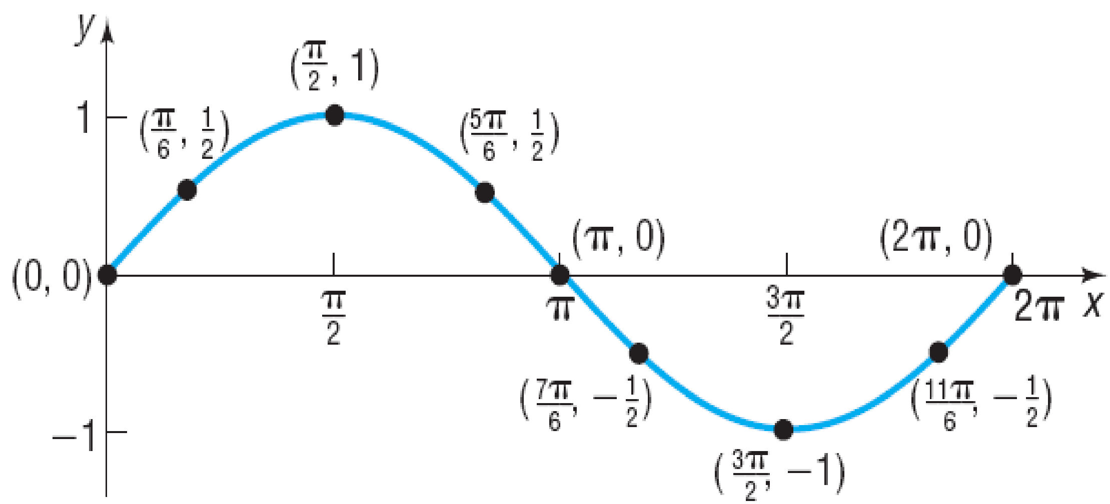
$$y = f(x) = \sin x \qquad y = f(x) = \cos x \qquad y = f(x) = \tan x$$

$$y = f(x) = \csc x \qquad y = f(x) = \sec x \qquad y = f(x) = \cot x$$

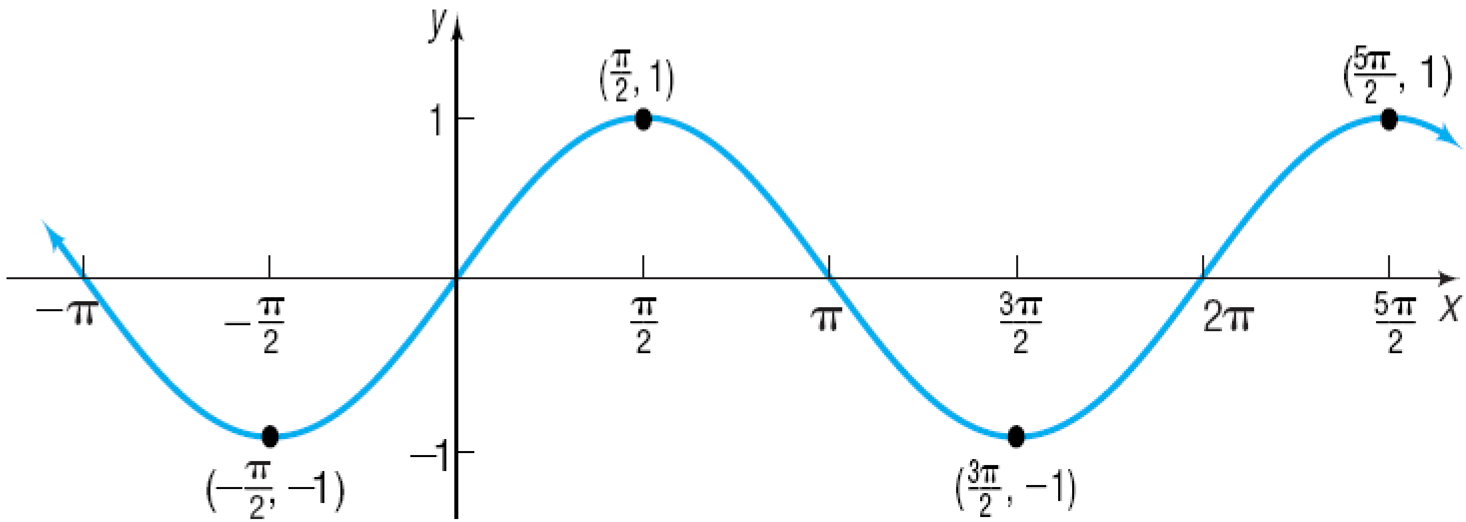
# OBJECTIVE 1

1 Graph Transformations of the Sine Function

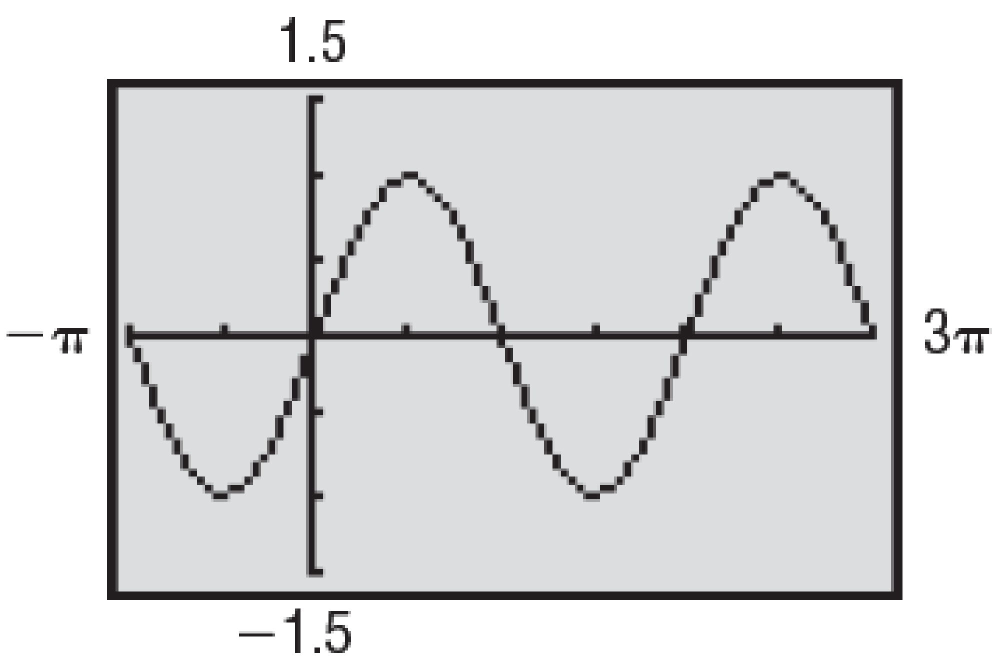
| $x$ | $y = \sin x$ | $(x, y)$ |
|---|---|---|
| $0$ | $0$ | $(0, 0)$ |
| $\frac{\pi}{6}$ | $\frac{1}{2}$ | $\left(\frac{\pi}{6}, \frac{1}{2}\right)$ |
| $\frac{\pi}{2}$ | $1$ | $\left(\frac{\pi}{2}, 1\right)$ |
| $\frac{5\pi}{6}$ | $\frac{1}{2}$ | $\left(\frac{5\pi}{6}, \frac{1}{2}\right)$ |
| $\pi$ | $0$ | $(\pi, 0)$ |
| $\frac{7\pi}{6}$ | $-\frac{1}{2}$ | $\left(\frac{7\pi}{6}, -\frac{1}{2}\right)$ |
| $\frac{3\pi}{2}$ | $-1$ | $\left(\frac{3\pi}{2}, -1\right)$ |
| $\frac{11\pi}{6}$ | $-\frac{1}{2}$ | $\left(\frac{11\pi}{6}, -\frac{1}{2}\right)$ |
| $2\pi$ | $0$ | $(2\pi, 0)$ |




$y = \sin x, 0 \leq x \leq 2\pi$



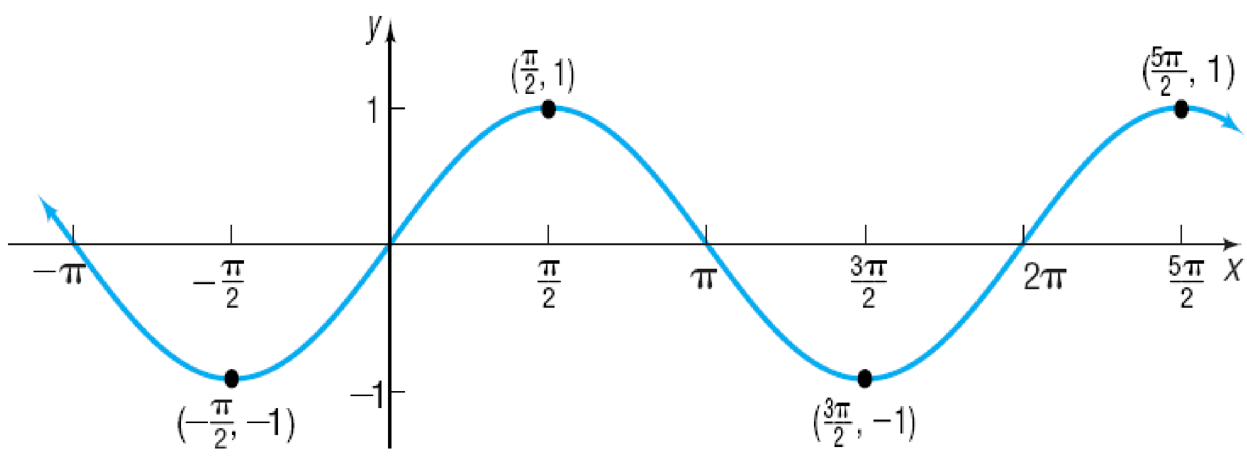

$$y = \sin x, \ -\infty < x < \infty$$



## Properties of the Sine Function

1. The domain is the set of all real numbers.
2. The range consists of all real numbers from $-1$ to $1$, inclusive.
3. The sine function is an odd function, as the symmetry of the graph with respect to the origin indicates.
4. The sine function is periodic, with period $2\pi$.
5. The $x$-intercepts are $\ldots, -2\pi, -\pi, 0, \pi, 2\pi, 3\pi, \ldots$; the $y$-intercept is $0$.
6. The maximum value is $1$ and occurs at $x = \ldots, -\frac{3\pi}{2}, \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \ldots$;

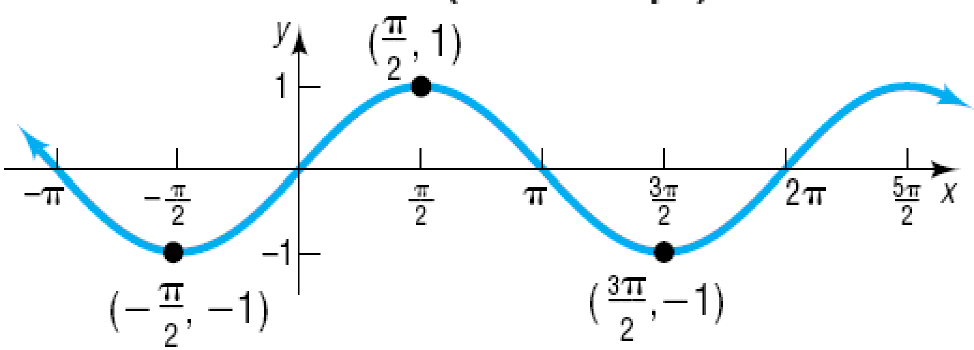
   the minimum value is $-1$ and occurs at $x = \ldots, -\frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2}, \ldots$.



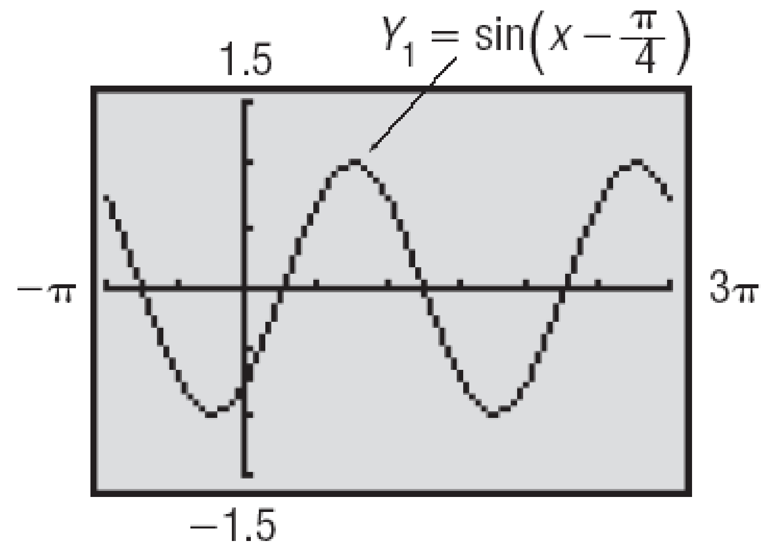
**EXAMPLE**

## Graphing Variations of $y = \sin x$ Using Transformations

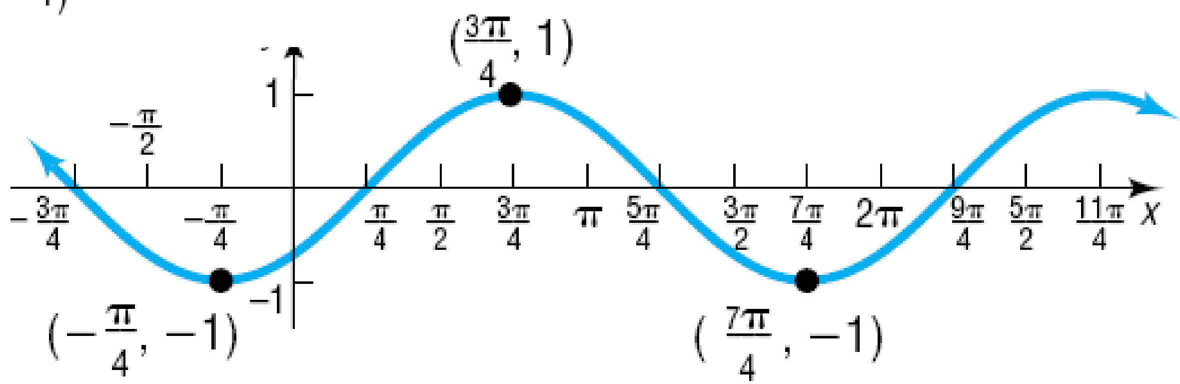
Use the graph of $y = \sin x$ to graph $y = \sin\left(x - \dfrac{\pi}{4}\right)$.




$y = \sin x$




Replace $x$ by $x - \frac{\pi}{4}$; horizontal shift to the right $\frac{\pi}{4}$ units.

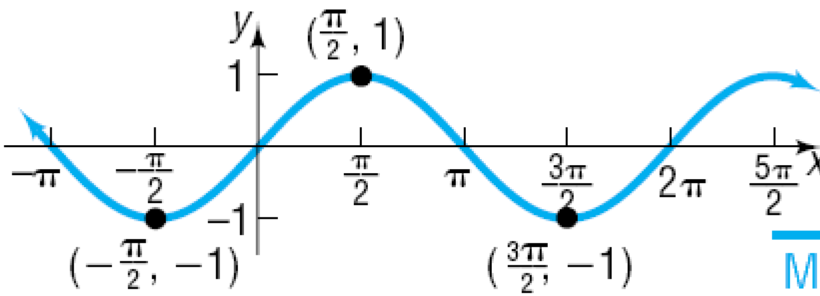



$y = \sin\left(x - \frac{\pi}{4}\right)$

# EXAMPLE

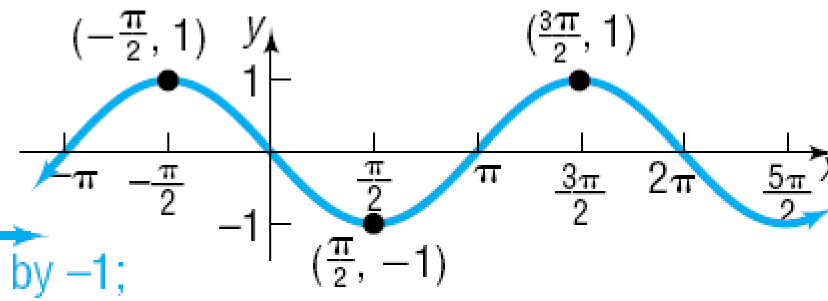
## Graphing Variations of $y = \sin x$ Using Transformations

Use the graph of $y = \sin x$ to graph $y = -\sin x + 2$.



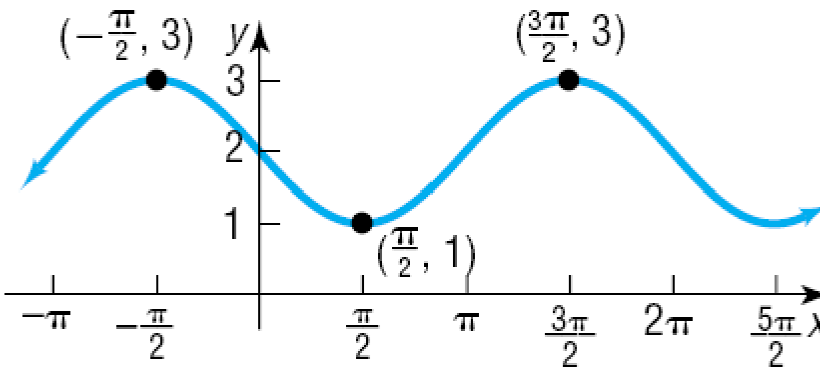

$y = \sin x$

Multiply by −1; reflect about *x*-axis.

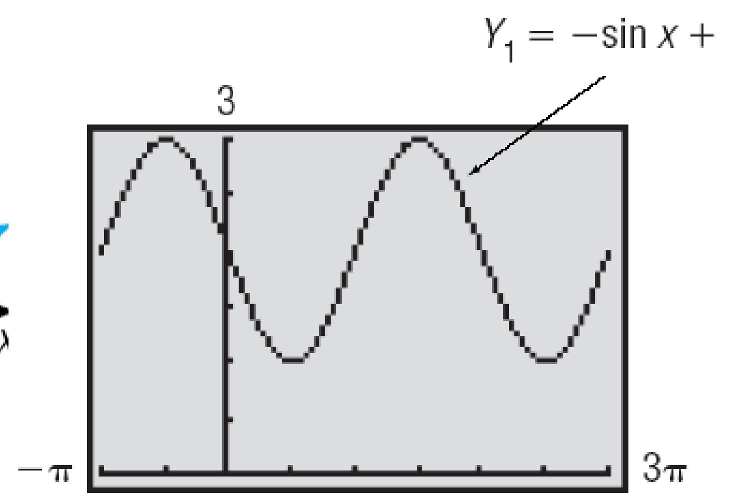



$y = -\sin x$

Add 2; vertical shift.



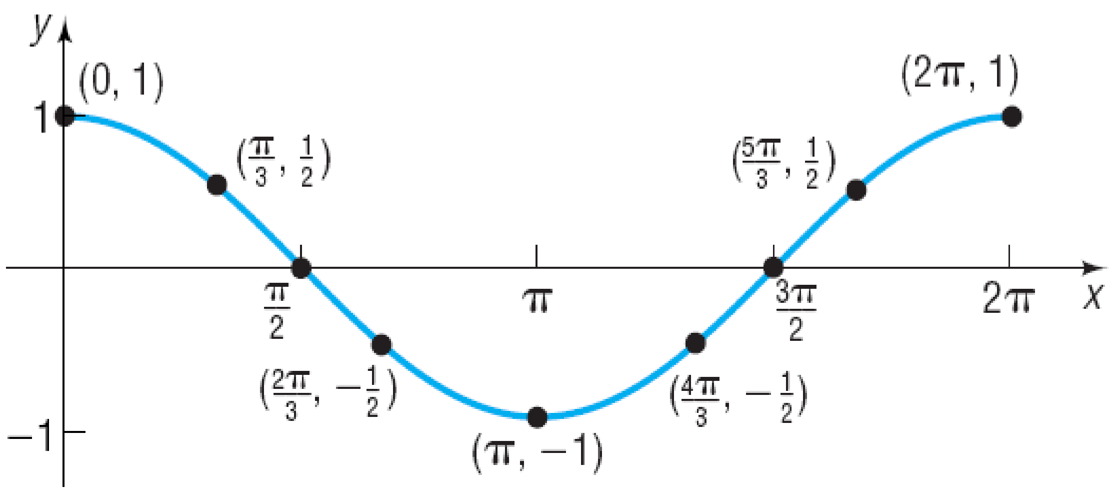

$y = -\sin x + 2$



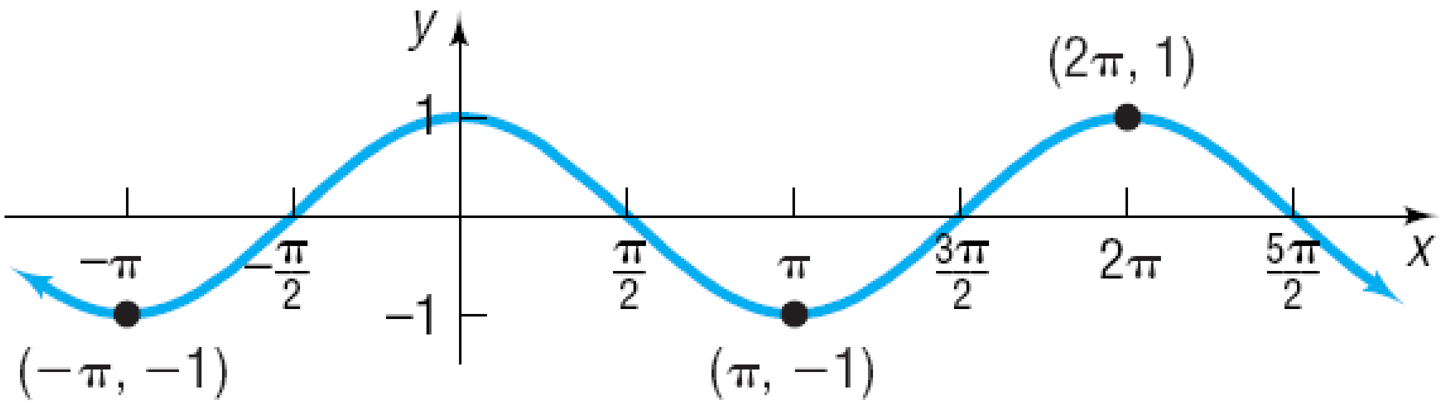
# OBJECTIVE 2

2 Graph Transformations of the Cosine Function

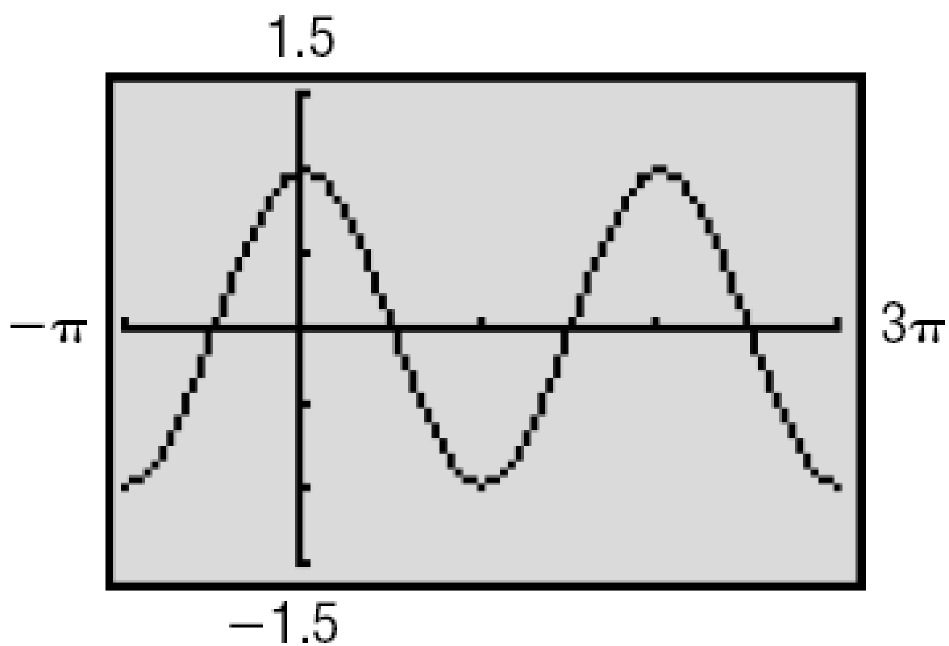
| x | y = cos x | (x, y) |
|---|---|---|
| $0$ | $1$ | $(0, 1)$ |
| $\frac{\pi}{3}$ | $\frac{1}{2}$ | $\left(\frac{\pi}{3}, \frac{1}{2}\right)$ |
| $\frac{\pi}{2}$ | $0$ | $\left(\frac{\pi}{2}, 0\right)$ |
| $\frac{2\pi}{3}$ | $-\frac{1}{2}$ | $\left(\frac{2\pi}{3}, -\frac{1}{2}\right)$ |
| $\pi$ | $-1$ | $(\pi, -1)$ |
| $\frac{4\pi}{3}$ | $-\frac{1}{2}$ | $\left(\frac{4\pi}{3}, -\frac{1}{2}\right)$ |
| $\frac{3\pi}{2}$ | $0$ | $\left(\frac{3\pi}{2}, 0\right)$ |
| $\frac{5\pi}{3}$ | $\frac{1}{2}$ | $\left(\frac{5\pi}{3}, \frac{1}{2}\right)$ |
| $2\pi$ | $1$ | $(2\pi, 1)$ |




$y = \cos x, 0 \leq x \leq 2\pi$

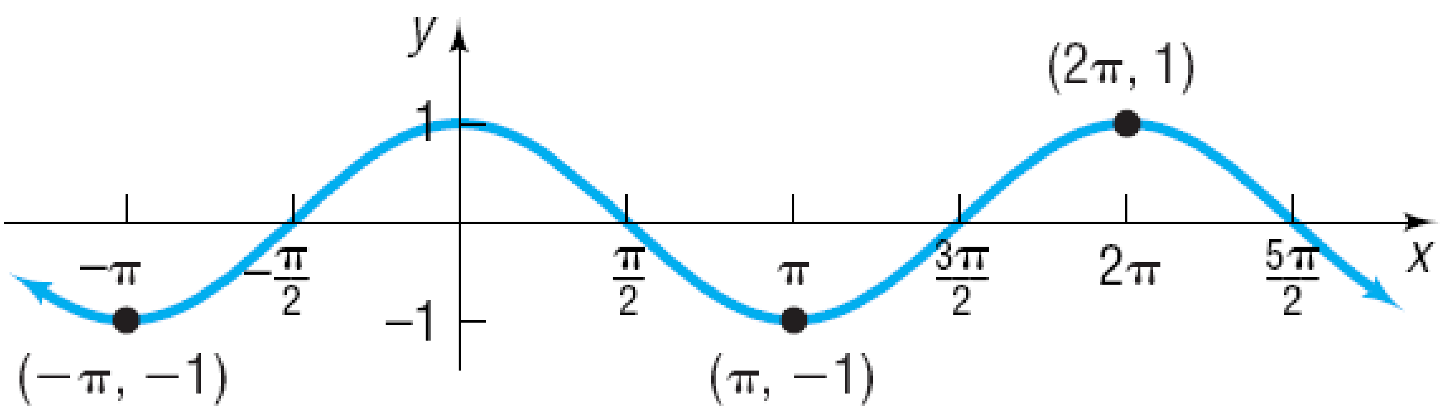



$$y = \cos x, -\infty < x < \infty$$



**Properties of the Cosine Function**

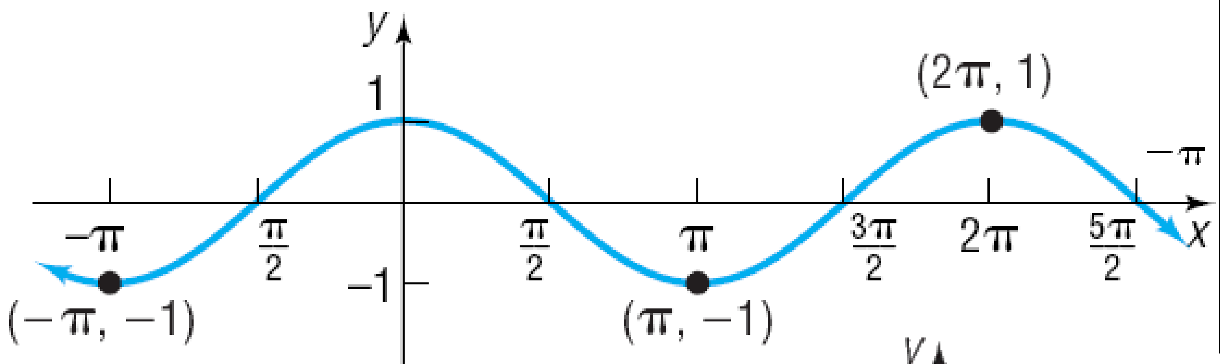
1. The domain is the set of all real numbers.
2. The range consists of all real numbers from $-1$ to 1, inclusive.
3. The cosine function is an even function, as the symmetry of the graph with respect to the $y$-axis indicates.
4. The cosine function is periodic, with period $2\pi$.
5. The $x$-intercepts are $\ldots, -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$; the $y$-intercept is 1.
6. The maximum value is 1 and occurs at $x = \ldots, -2\pi, 0, 2\pi, 4\pi, 6\pi, \ldots$; the minimum value is $-1$ and occurs at $x = \ldots, -\pi, \pi, 3\pi, 5\pi, \ldots$.

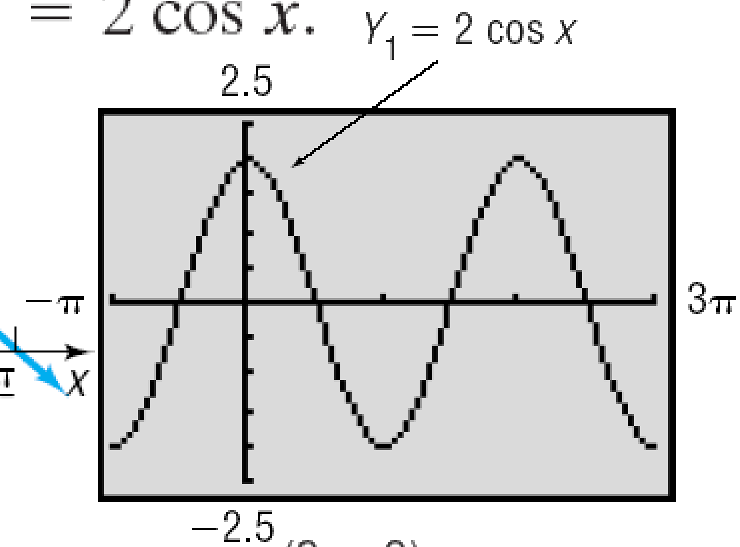


**EXAMPLE**

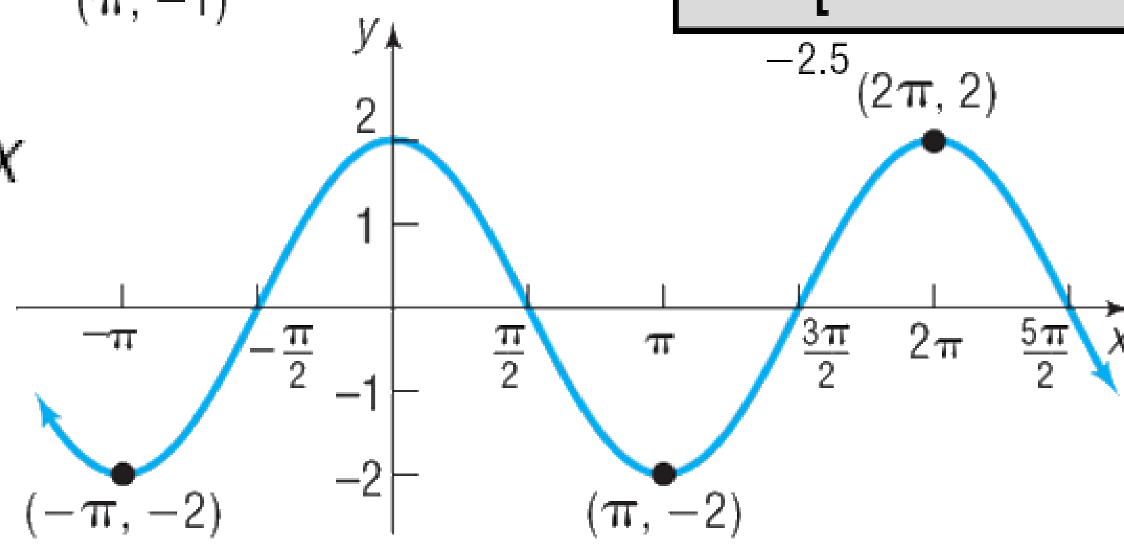
## Graphing Variations of y = cos x Using Transformations

Use the graph of $y = \cos x$ to graph $y = 2\cos x$.




$y = \cos x$



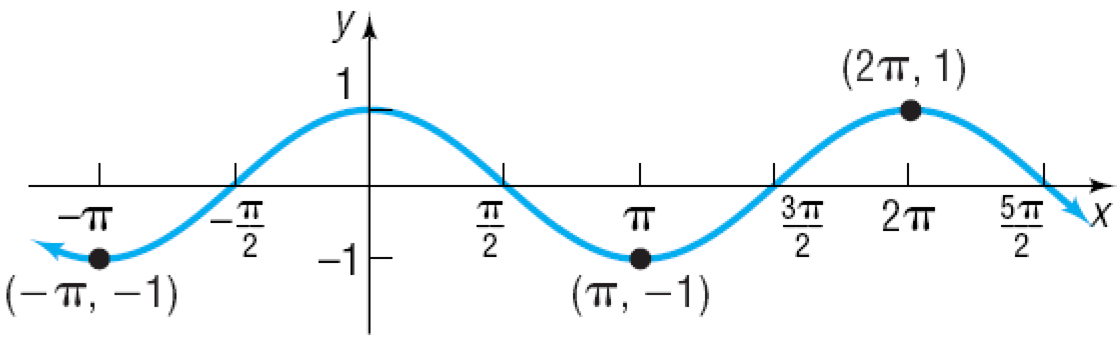

$y = 2\cos x$

Multiply by 2; vertical stretch by a factor of 2.

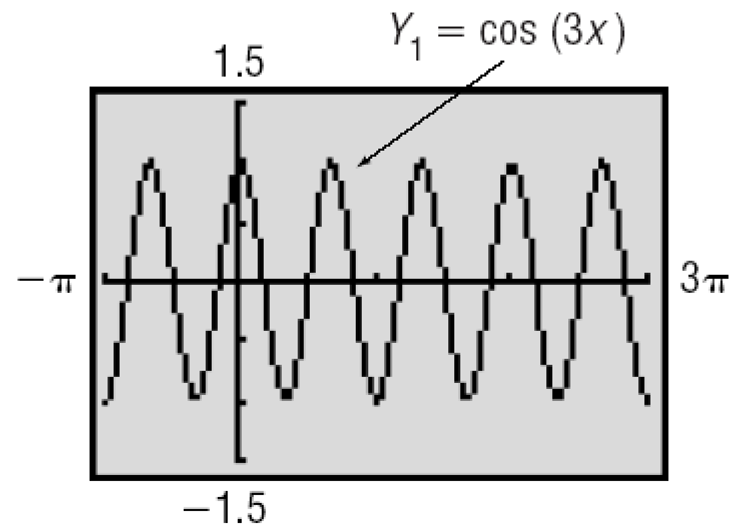
**EXAMPLE**

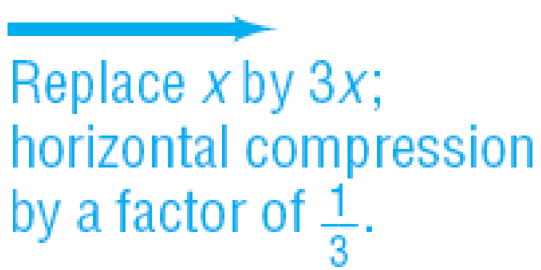
## Graphing Variations of $y = \cos x$ Using Transformations

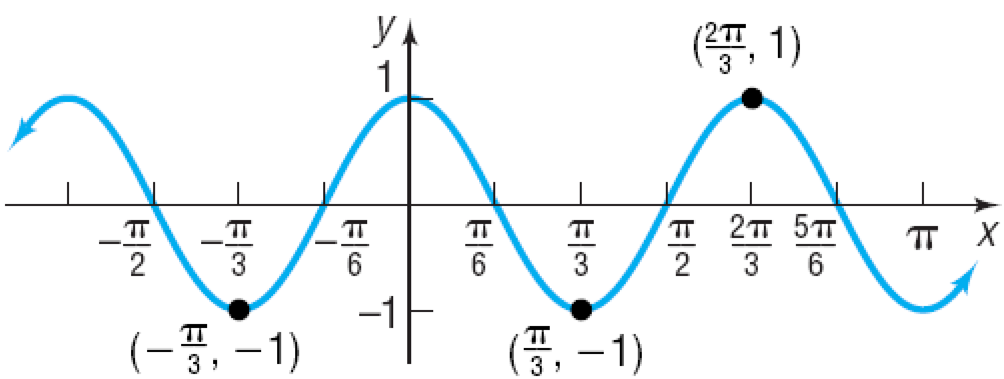
Use the graph of $y = \cos x$ to graph $y = \cos(3x)$.



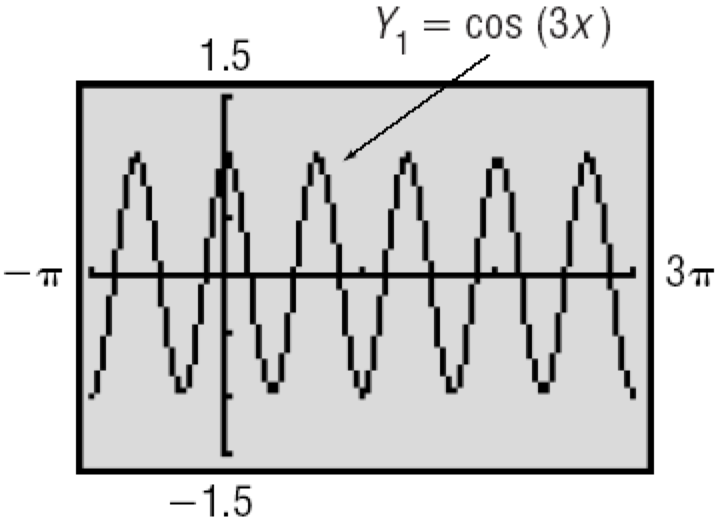

$y = \cos x$

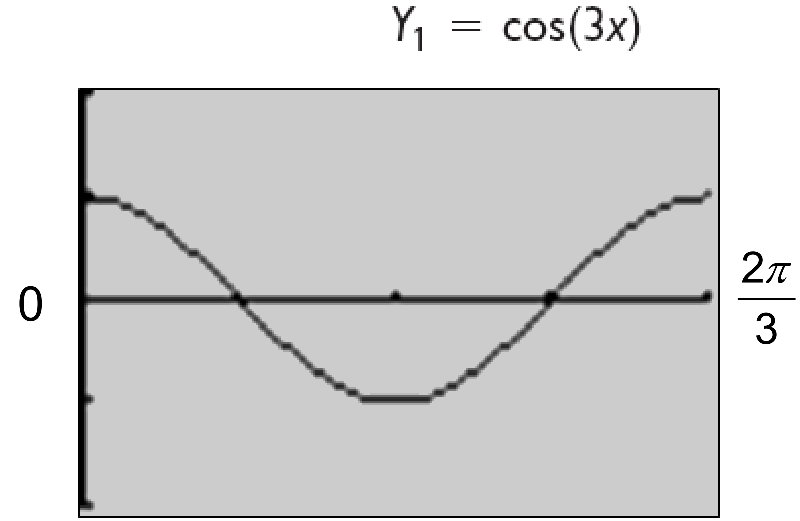



$y = \cos(3x)$

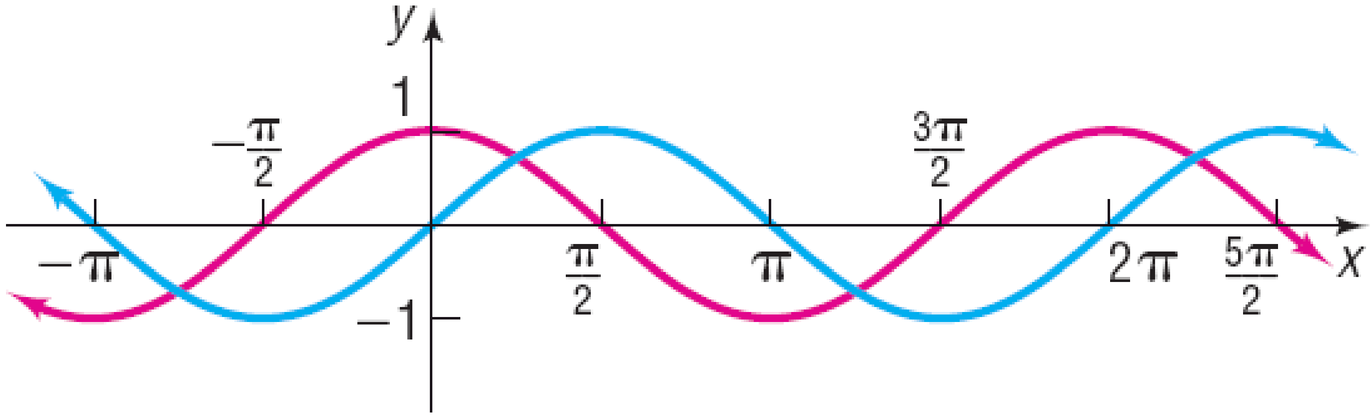


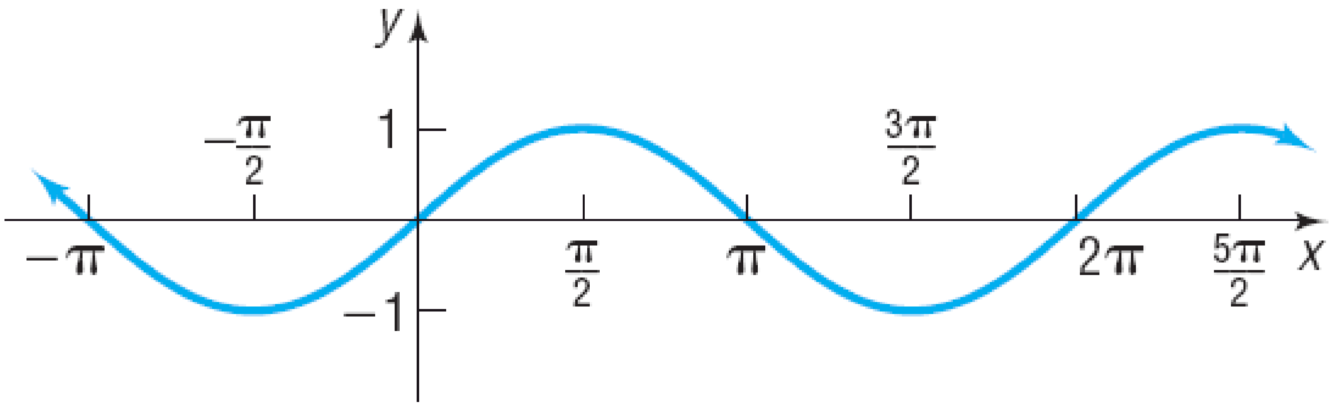

## Seeing the Concept

Graph $Y_1 = \cos(3x)$ with Xmin $= 0$, Xmax $= \frac{2\pi}{3}$, and Xscl $= \frac{\pi}{6}$ to verify that the period is $\frac{2\pi}{3}$.

# Sinusoidal Graphs




**(a)** $y = \cos x$ $y = \cos\left(x - \frac{\pi}{2}\right)$



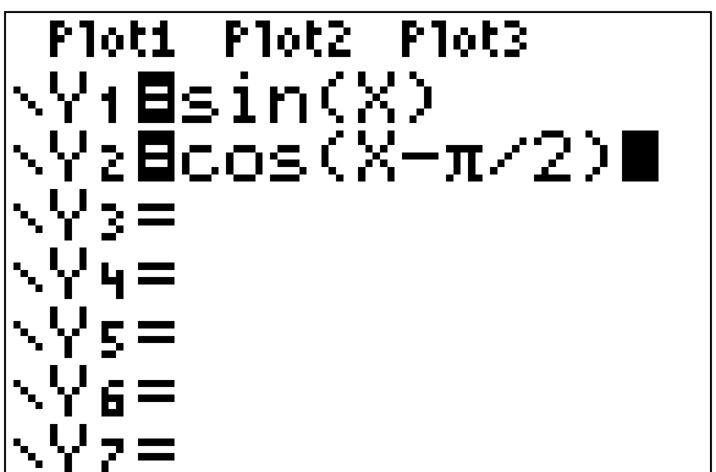

**(b)** $y = \sin x$

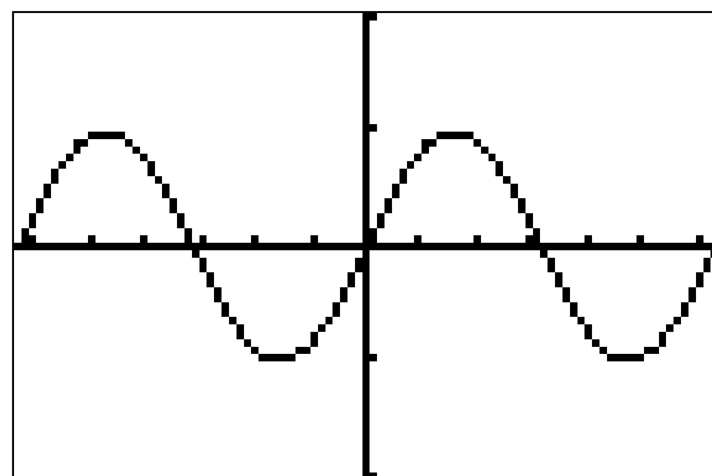
$$\sin x = \cos\left(x - \frac{\pi}{2}\right)$$

## Seeing the Concept

Graph $Y_1 = \sin x$ and $Y_2 = \cos\left(x - \frac{\pi}{2}\right)$.

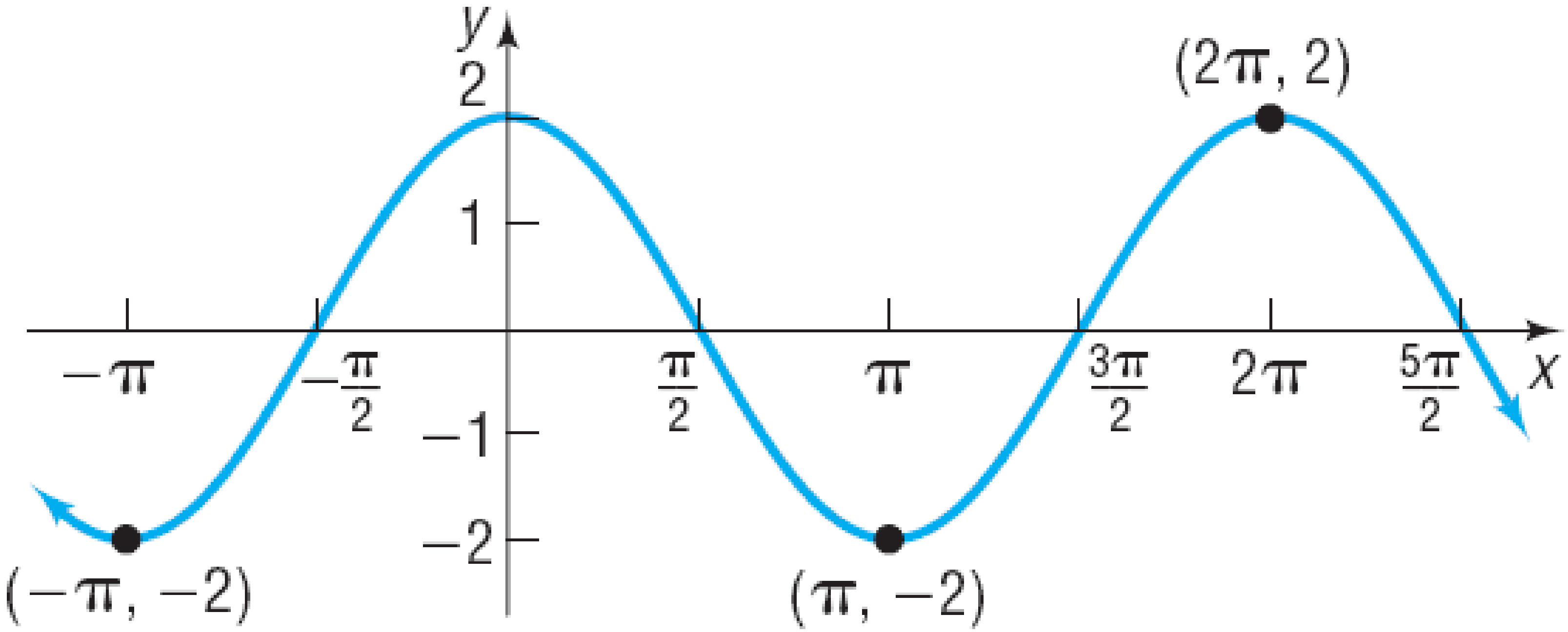
How many graphs do you see?



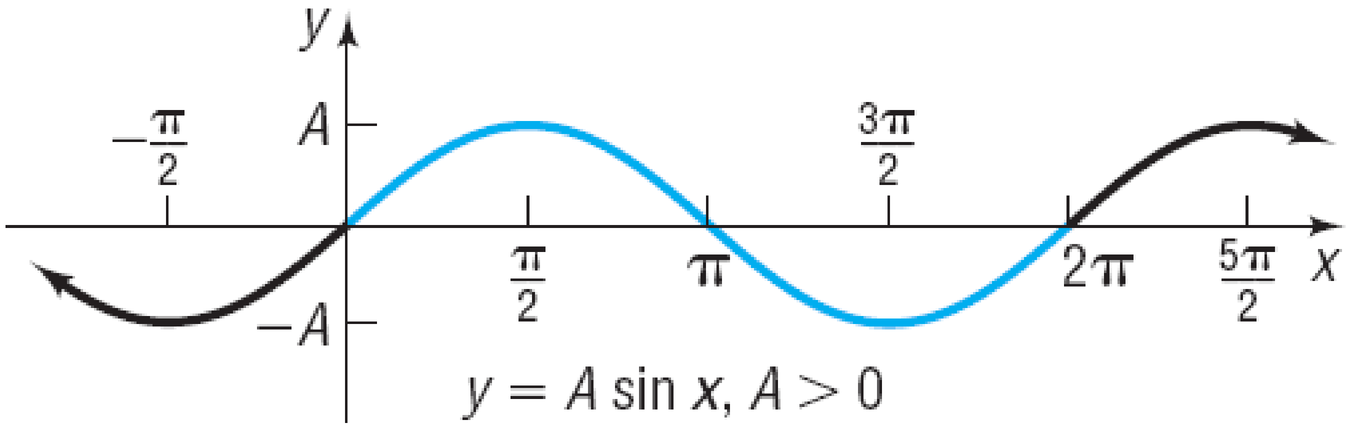



# OBJECTIVE 3

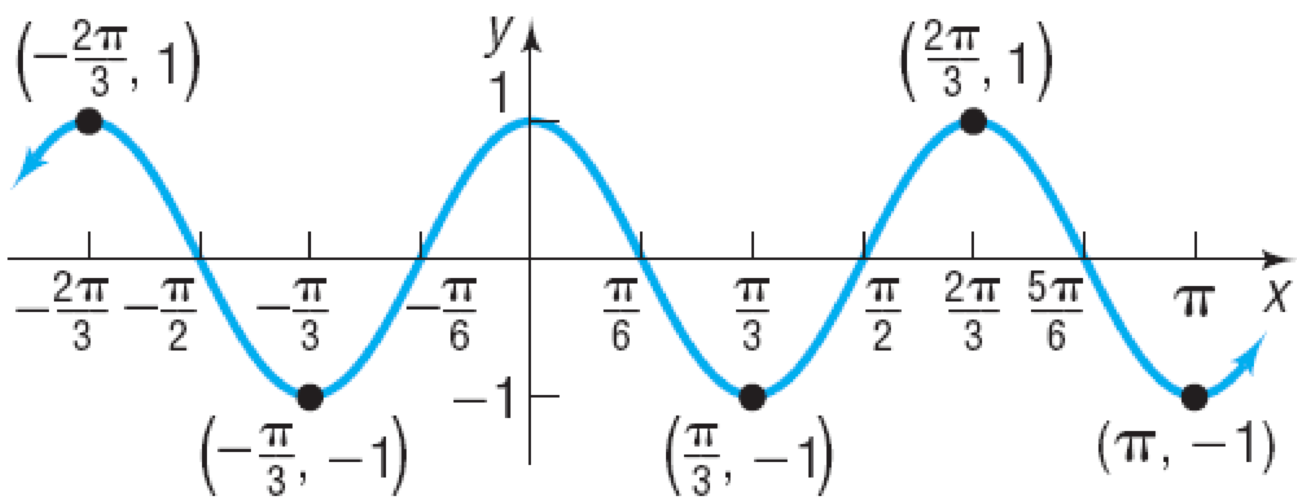
3 **Determine the Amplitude and Period of Sinusoidal Functions**



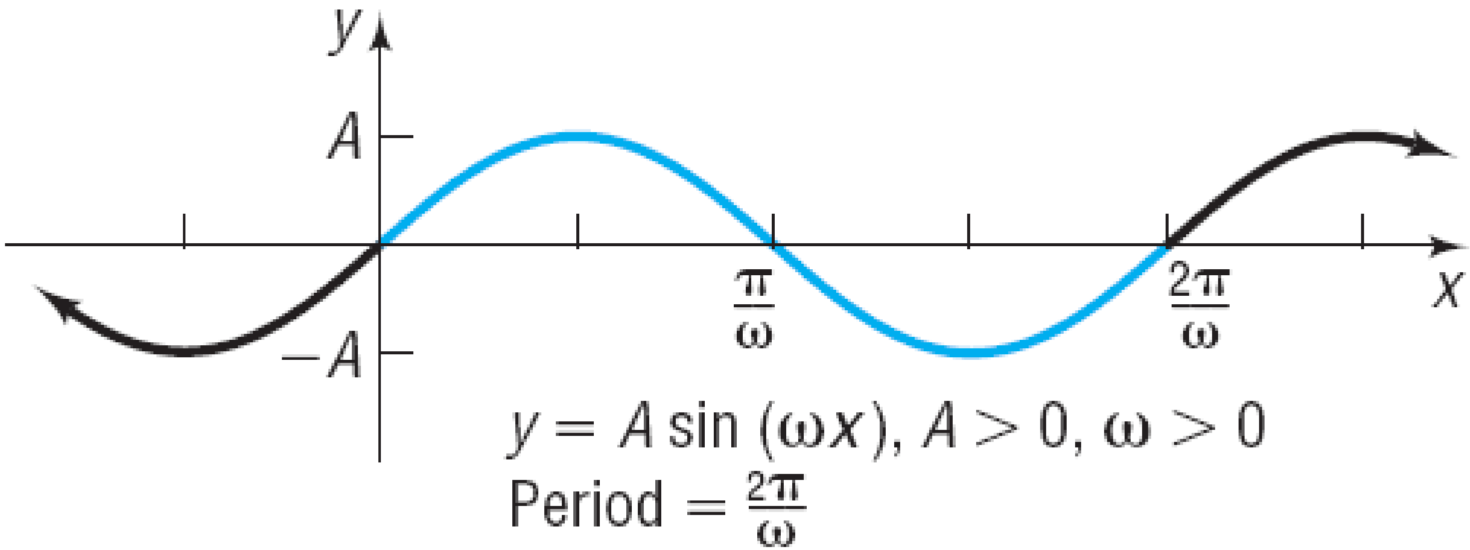

$y = 2 \cos x$




$y = A \sin x,\ A > 0$
Period $= 2\pi$




$y = \cos(3x)$




## Theorem

If $\omega > 0$, the amplitude and period of $y = A\sin(\omega x)$ and $y = A\cos(\omega x)$ are

$$\text{Amplitude} = |A| \qquad \text{Period} = T = \frac{2\pi}{\omega}$$

## EXAMPLE

### Finding the Amplitude and Period of a Sinusoidal Function

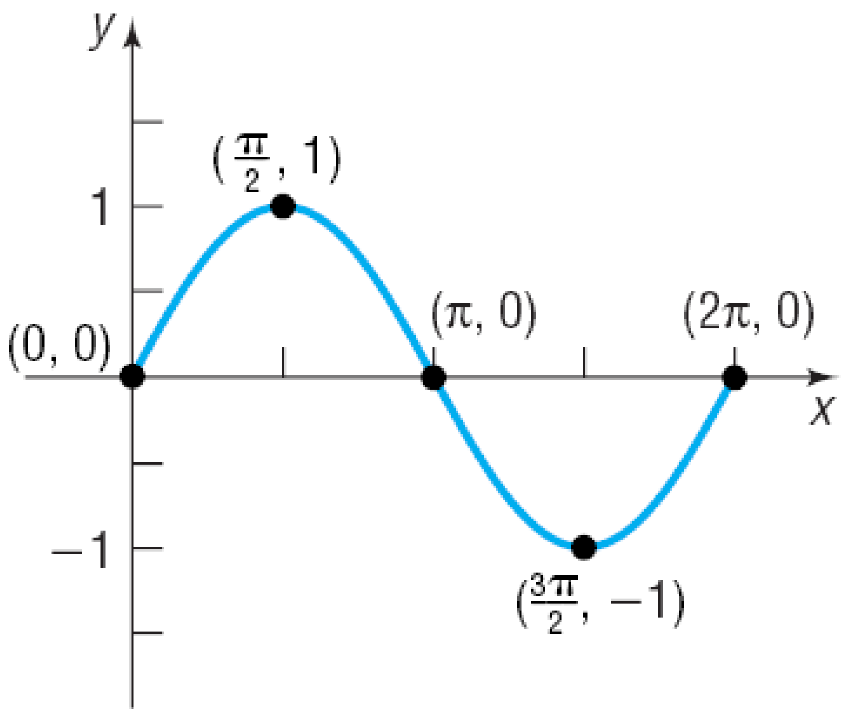
Determine the amplitude and period of $y = -4\cos(3x)$

If $\omega > 0$, the amplitude and period of $y = A\sin(\omega x)$ and $y = A\cos(\omega x)$ are

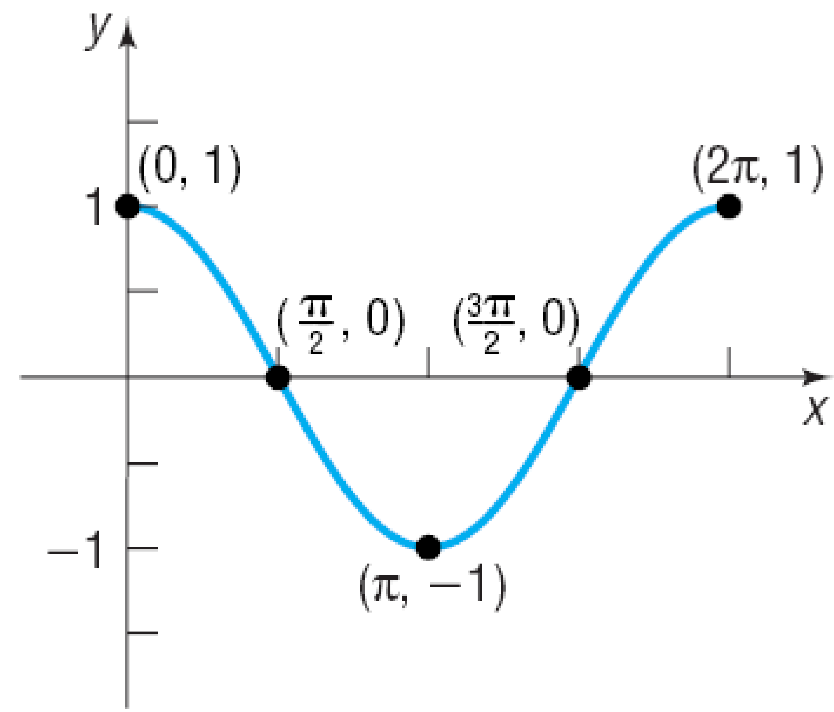
$$\text{Amplitude} = |A| \qquad \text{Period} = T = \frac{2\pi}{\omega}$$

# OBJECTIVE 4

4 Graph Sinusoidal Functions Using Key Points



y
(π/2, 1)
1
(0, 0)
(π, 0)
(2π, 0)
x
−1
(3π/2, −1)

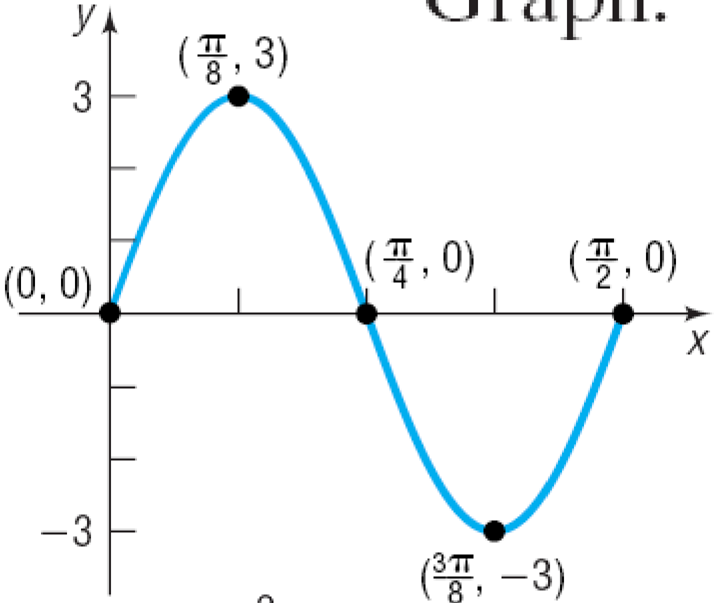



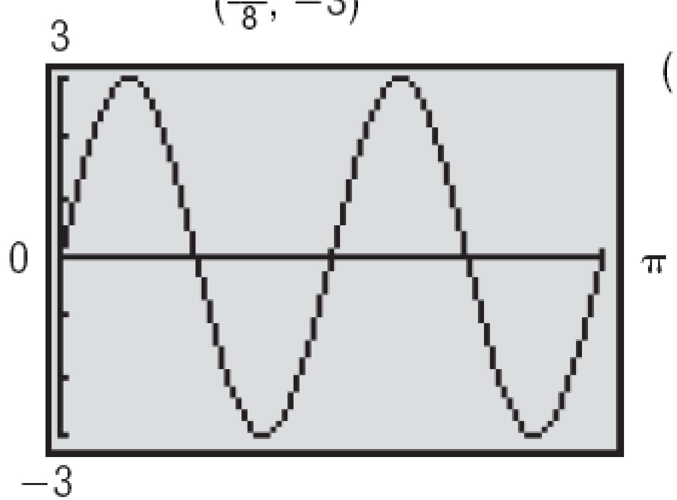
y
(0, 1)
(2π, 1)
1
(π/2, 0)
(3π/2, 0)
x
−1
(π, −1)

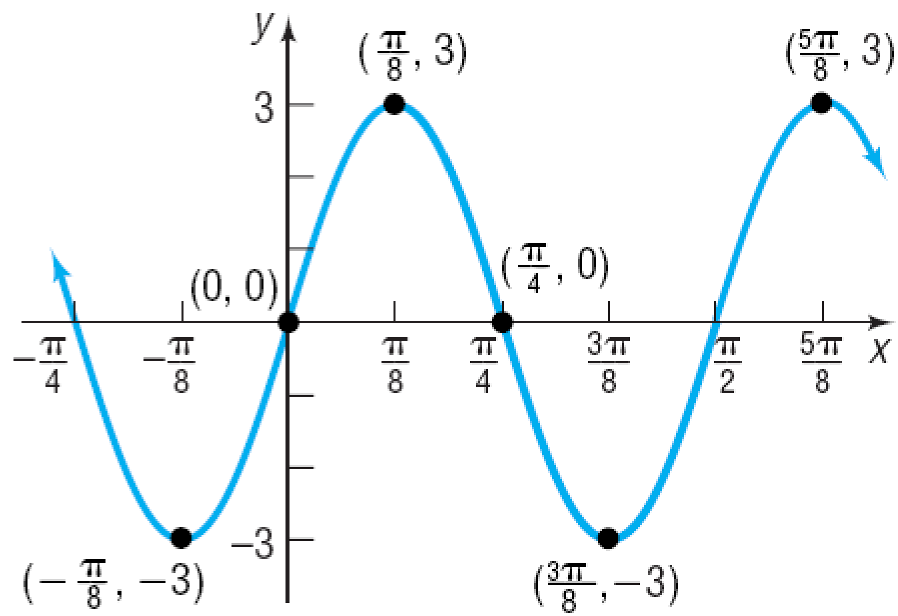
EXAMPLE

# Graphing a Sinusoidal Function Using Key Points

Graph: $y = 3\sin(4x)$



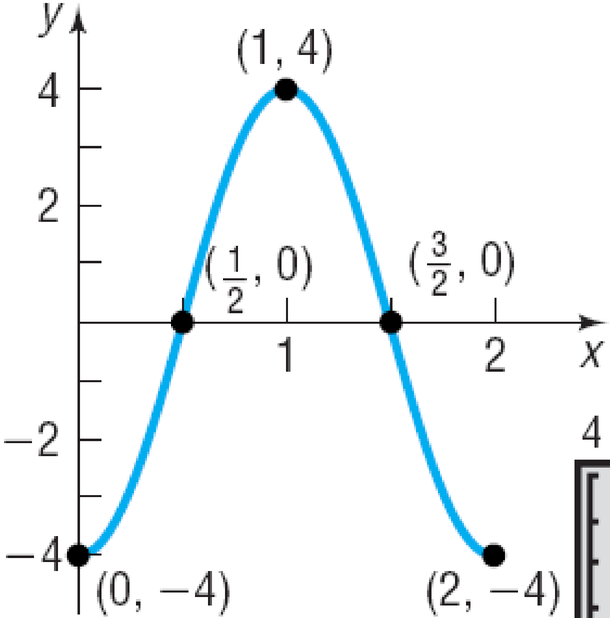



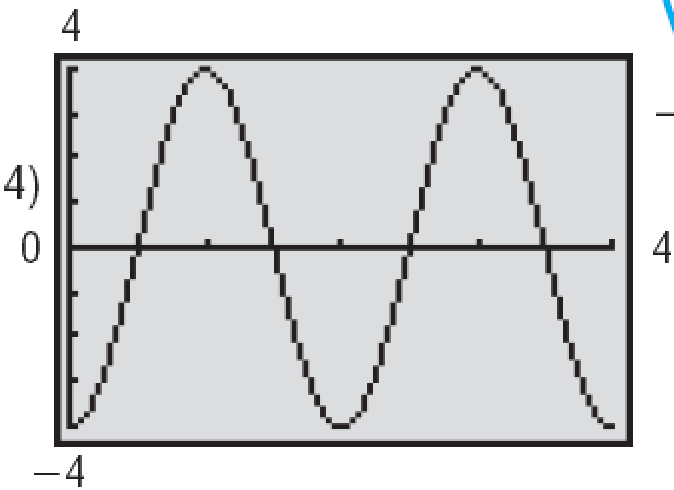



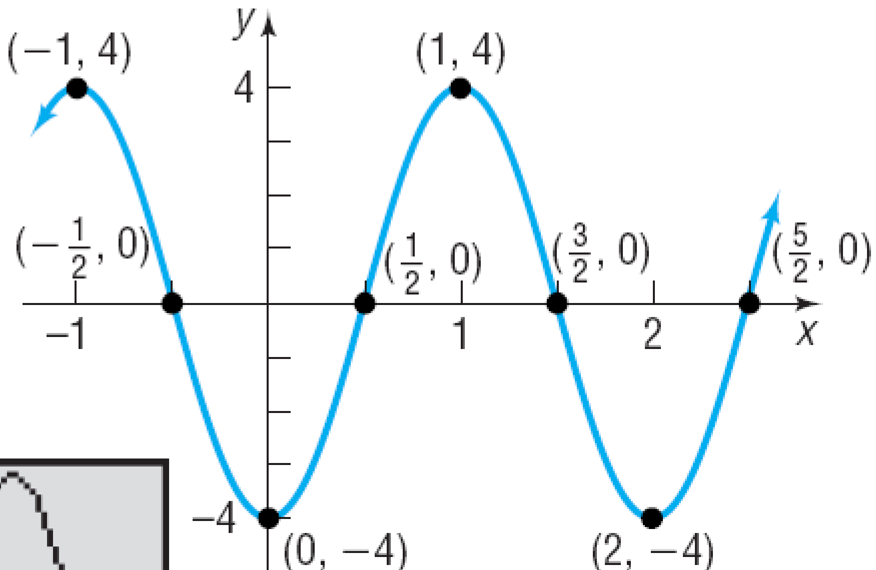
EXAMPLE

## Finding the Amplitude and Period of a Sinusoidal Function and Graphing It Using Key Points

Determine the amplitude and period of $y = -4\cos(\pi x)$, and graph the function.



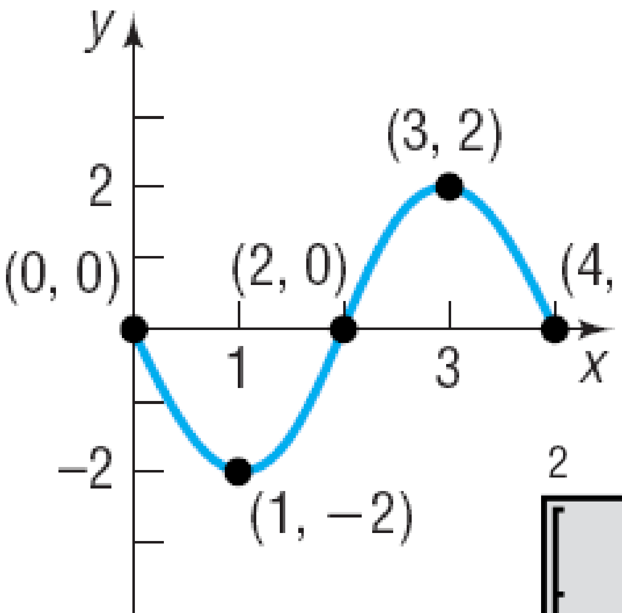



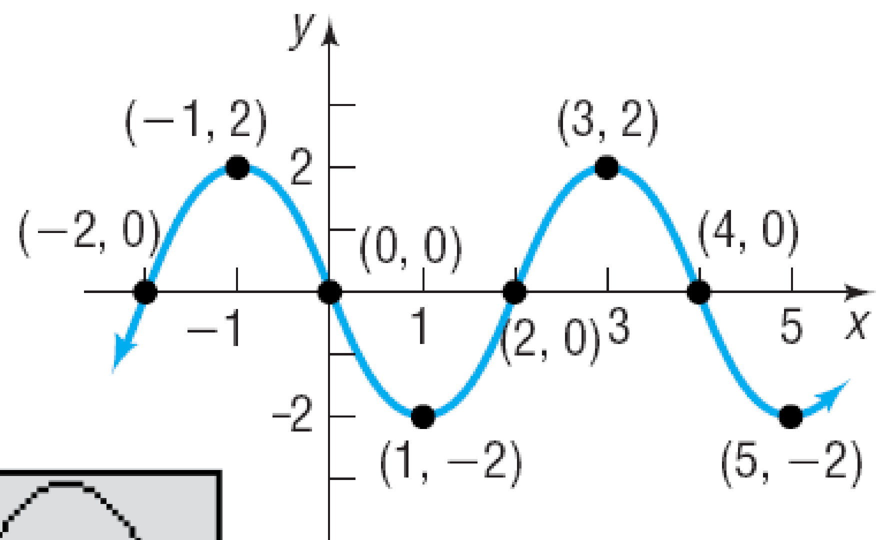



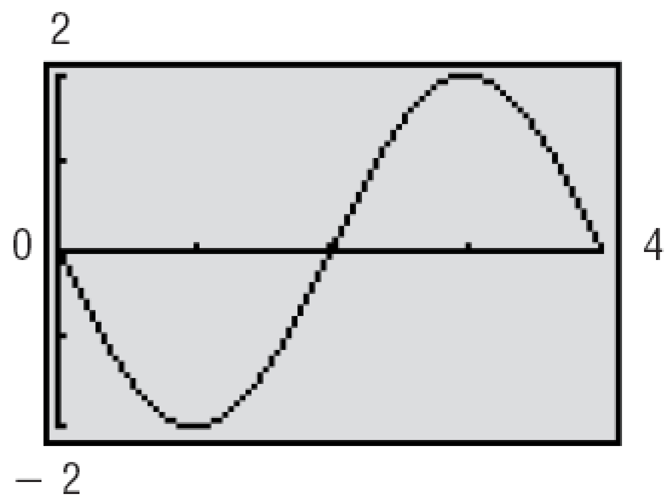
EXAMPLE

## Finding the Amplitude and Period of a Sinusoidal Function and Graphing It Using Key Points

Determine the amplitude and period of $y = 2\sin\left(-\frac{\pi}{2}x\right)$, and graph the function.



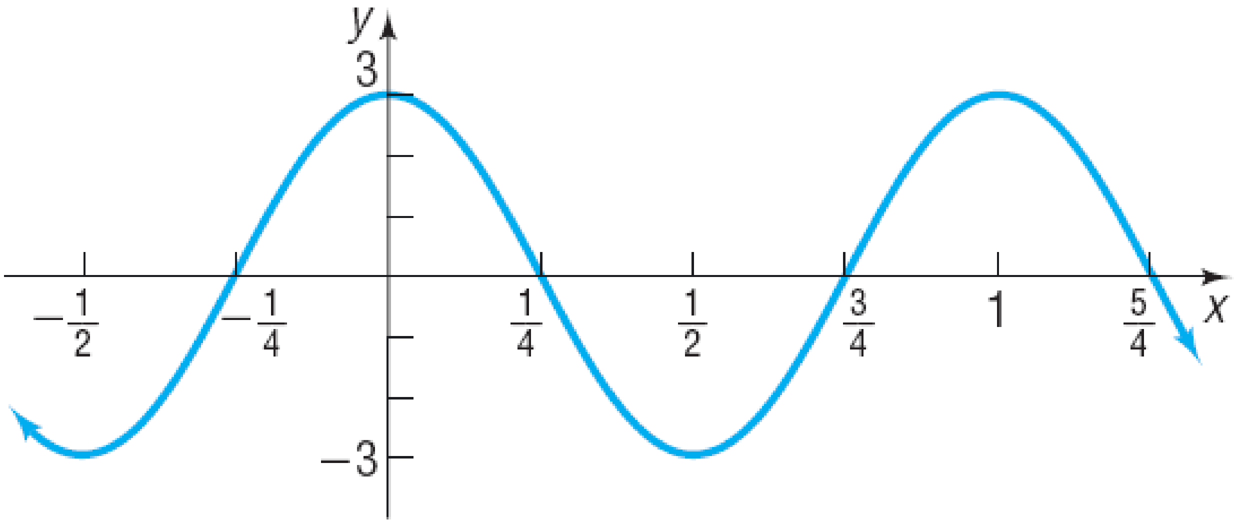
# OBJECTIVE 5

5 Find an Equation for a Sinusoidal Graph

EXAMPLE

## Finding an Equation for a Sinusoidal Graph

Find an equation for the graph shown



EXAMPLE

## Finding an Equation for a Sinusoidal Graph

Find an equation for the graph shown