

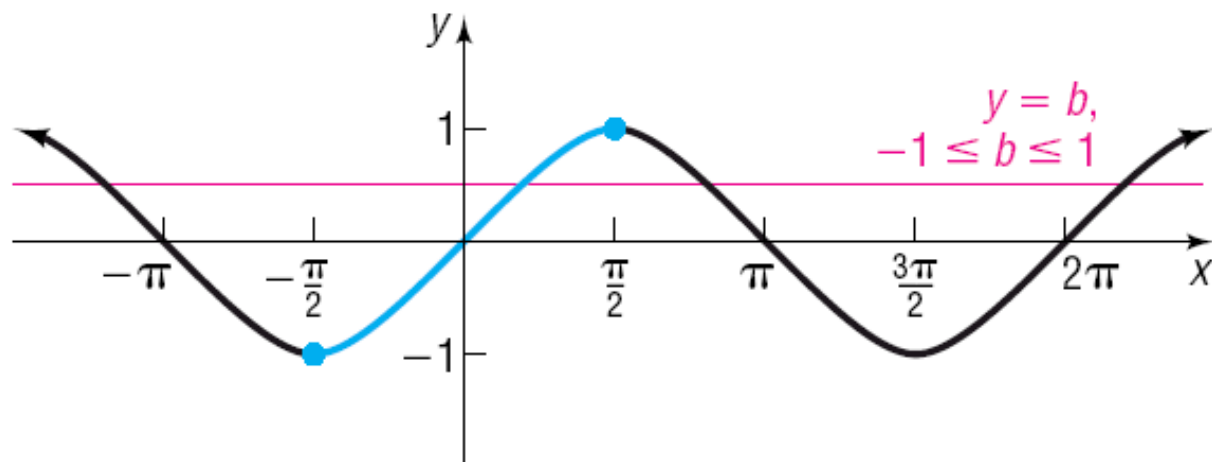
Section 6.1

The Inverse Sine, Cosine, and Tangent Functions

Review of Properties of Functions and Their Inverses

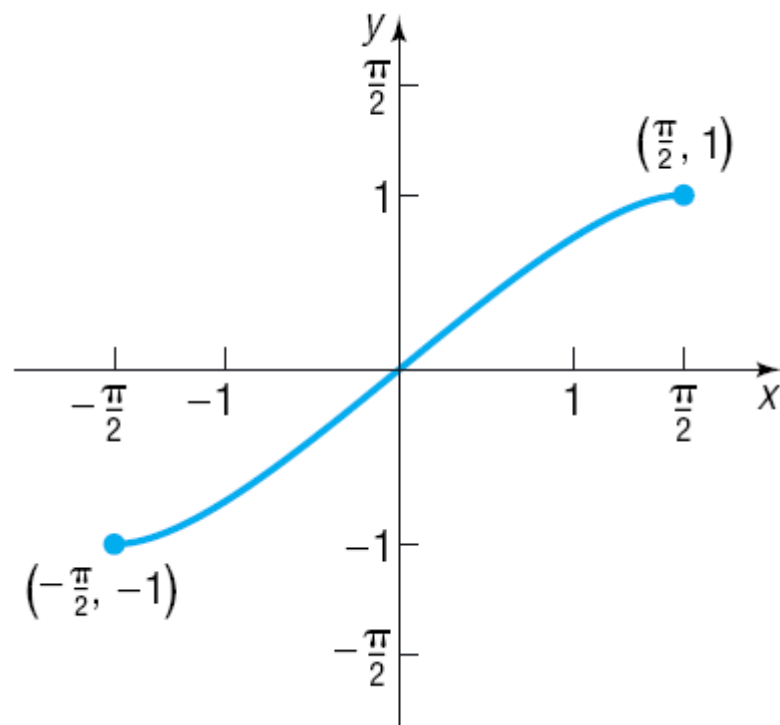
1. $f^{-1}(f(x)) = x$ for every x in the domain of f and $f(f^{-1}(x)) = x$ for every x in the domain of f^{-1} .
2. Domain of $f =$ range of f^{-1} , and range of $f =$ domain of f^{-1} .
3. The graph of f and the graph of f^{-1} are symmetric with respect to the line $y = x$.
4. If a function $y = f(x)$ has an inverse function, the equation of the inverse function is $x = f(y)$. The solution of this equation is $y = f^{-1}(x)$.

The Inverse Sine Function



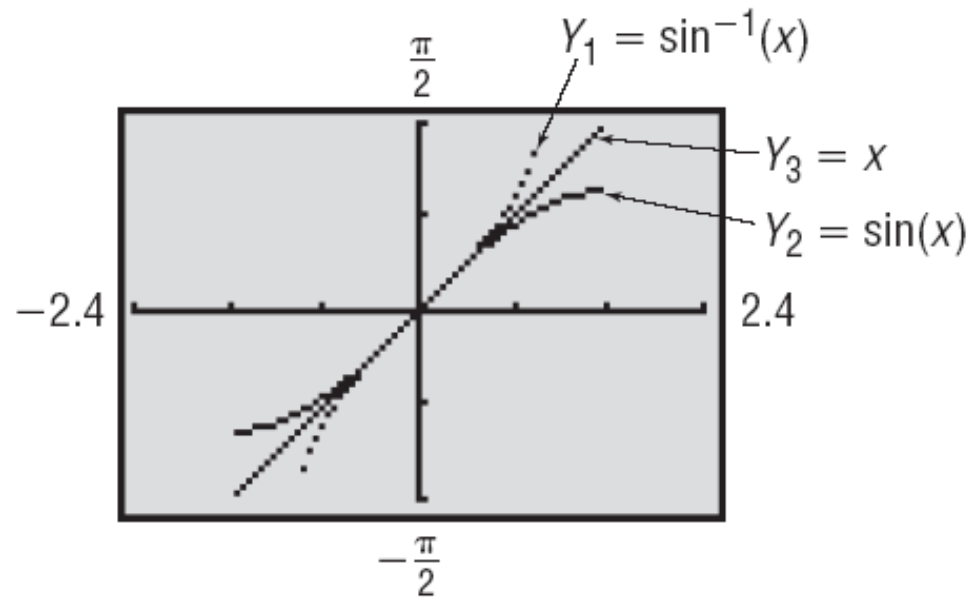
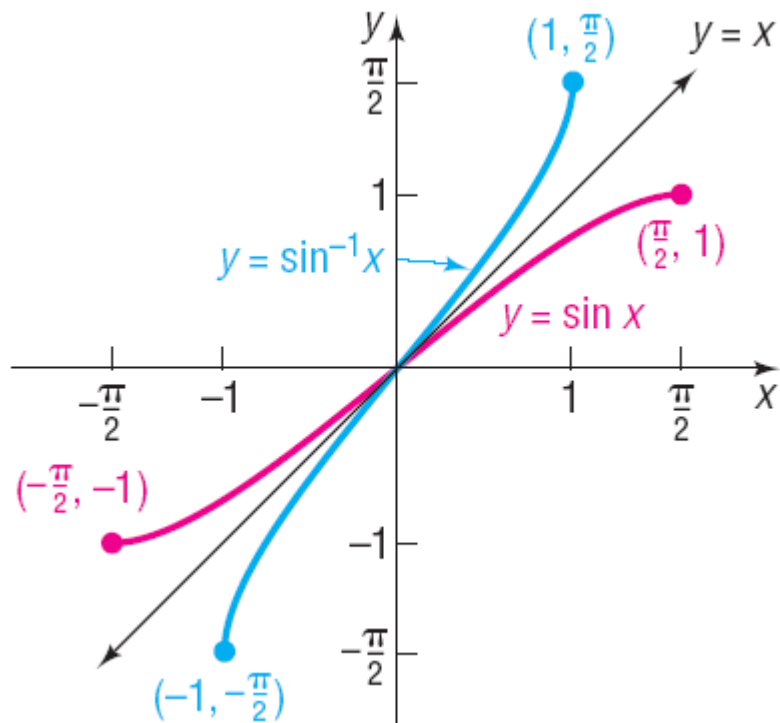
$$y = \sin x, -\infty < x < \infty, -1 \leq y \leq 1$$

$$y = \sin x, -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}, -1 \leq y \leq 1$$



$$y = \sin^{-1} x \quad \text{means} \quad x = \sin y$$

$$\text{where} \quad -1 \leq x \leq 1 \quad \text{and} \quad -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$$



$$y = \sin^{-1} x, \quad -1 \leq x \leq 1, \quad -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$$

OBJECTIVE 1

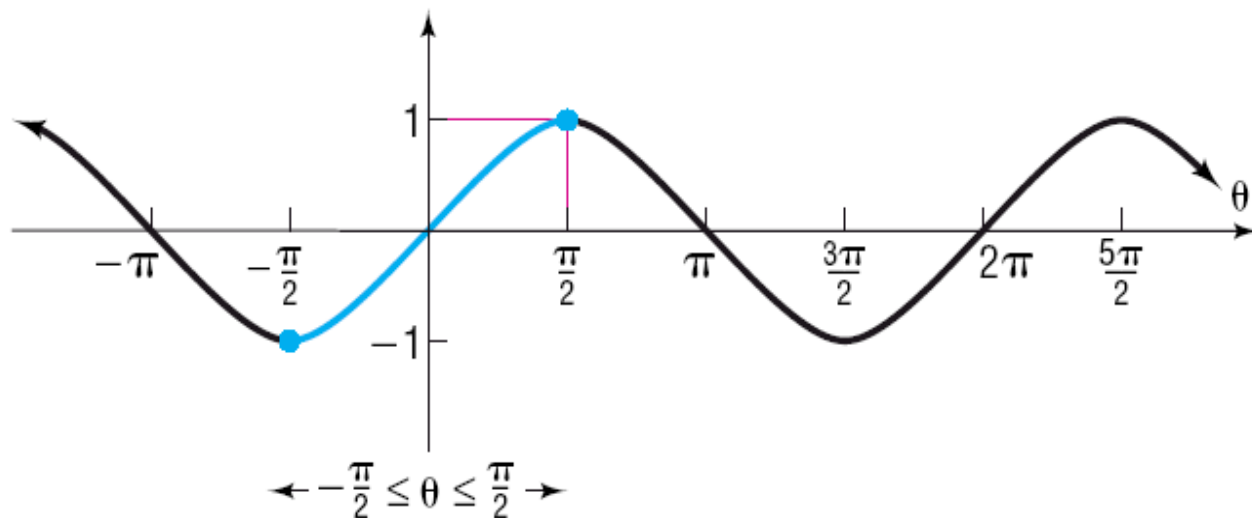
- 1 ✓ Find the Exact Value of the Inverse Sine Function

EXAMPLE

Finding the Exact Value of a Composite Function

Find the exact value of: $\sin^{-1} 1$

θ	$\sin \theta$
$-\frac{\pi}{2}$	-1
$-\frac{\pi}{3}$	$-\frac{\sqrt{3}}{2}$
$-\frac{\pi}{4}$	$-\frac{\sqrt{2}}{2}$
$-\frac{\pi}{6}$	$-\frac{1}{2}$
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	1



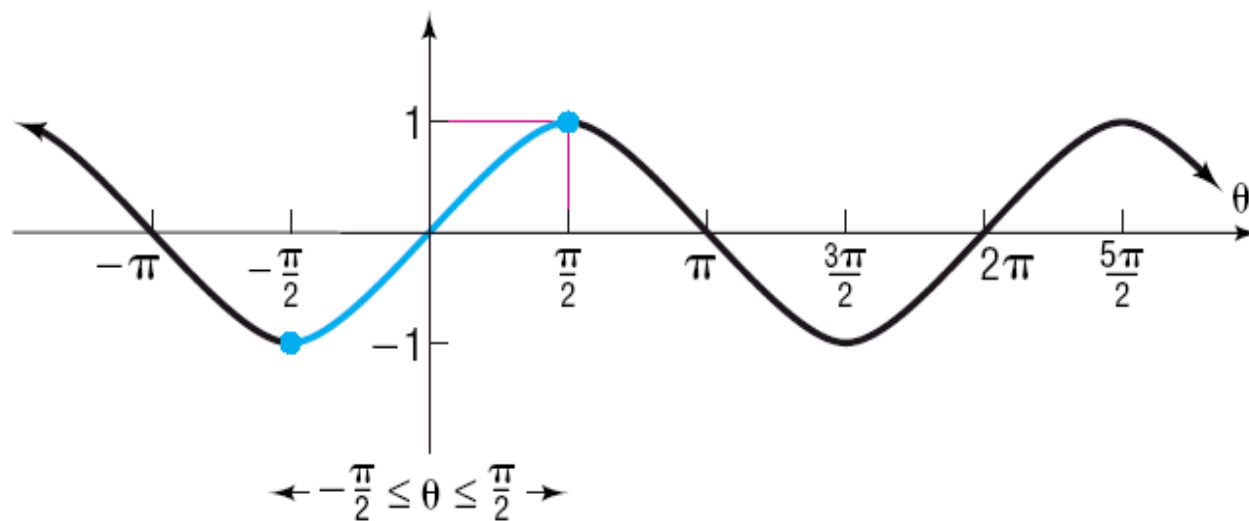
```
sin-1(1)
1.570796327
π/2
1.570796327
```

EXAMPLE

Finding the Exact Value of an Inverse Sine Function

Find the exact value of: $\sin^{-1}\left(-\frac{1}{2}\right)$

θ	$\sin \theta$
$-\frac{\pi}{2}$	-1
$-\frac{\pi}{3}$	$-\frac{\sqrt{3}}{2}$
$-\frac{\pi}{4}$	$-\frac{\sqrt{2}}{2}$
$-\frac{\pi}{6}$	$-\frac{1}{2}$
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	1



OBJECTIVE 2

- 2 Find an Approximate Value of the Inverse Sine Function

EXAMPLE

Finding an Approximate Value of an Inverse Sine Function

Find an approximate value of:

(a) $\sin^{-1} \frac{1}{3}$

(b) $\sin^{-1} \left(-\frac{1}{4} \right)$

```
sin-1(1/3)  
  .3398369095
```

```
sin-1(-1/4)  
 -.2526802551
```

$$f^{-1}(f(x)) = \sin^{-1}(\sin x) = x, \quad \text{where } -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$$

$$f(f^{-1}(x)) = \sin(\sin^{-1} x) = x, \quad \text{where } -1 \leq x \leq 1$$

```
sin-1(sin(π/8))  
      .3926990817  
sin(sin-1(0.8))  
      .8
```

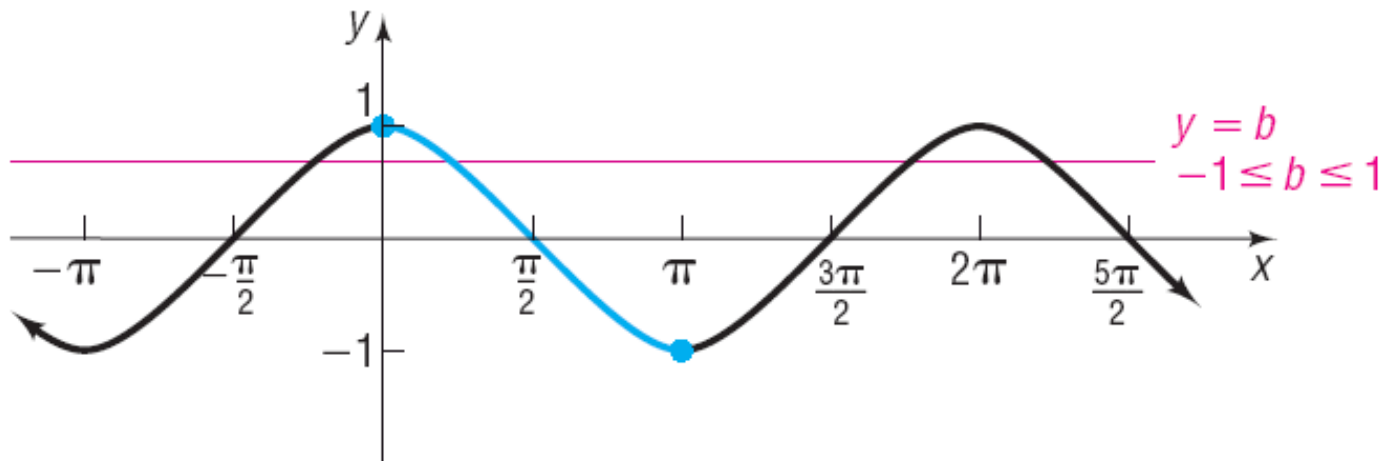
← F10

```
sin-1(sin(5π/8))  
      1.178097245  
5π/8  
      1.963495408
```

```
sin(sin-1(1.8))
```

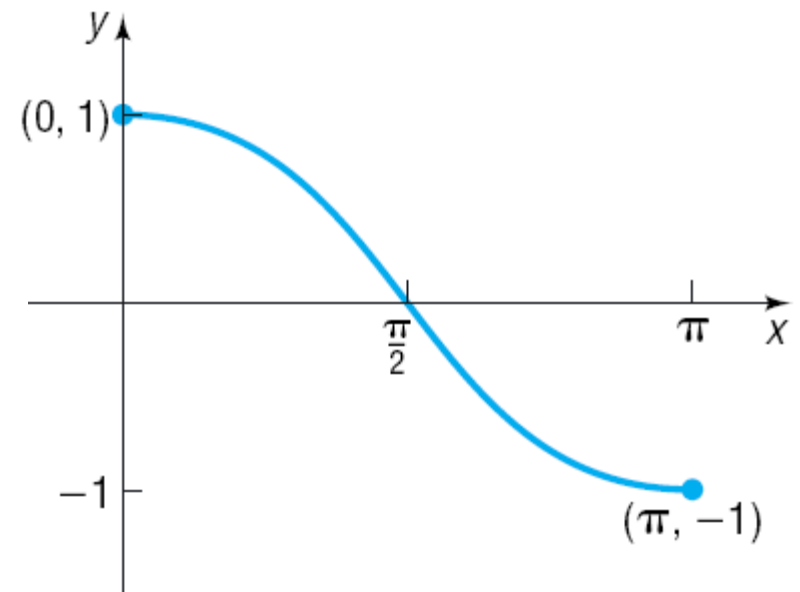
```
ERR: DOMAIN  
1: Quit  
2: Goto
```

The Inverse Cosine Function

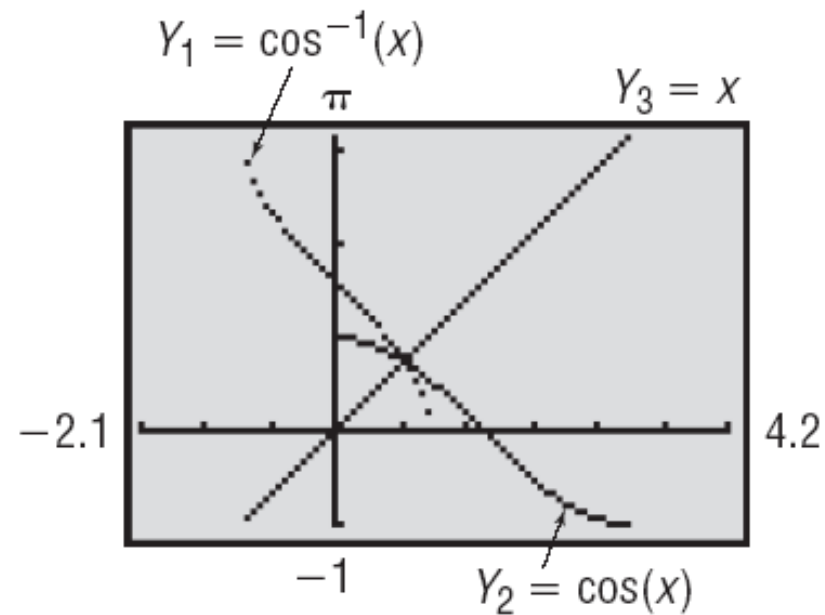
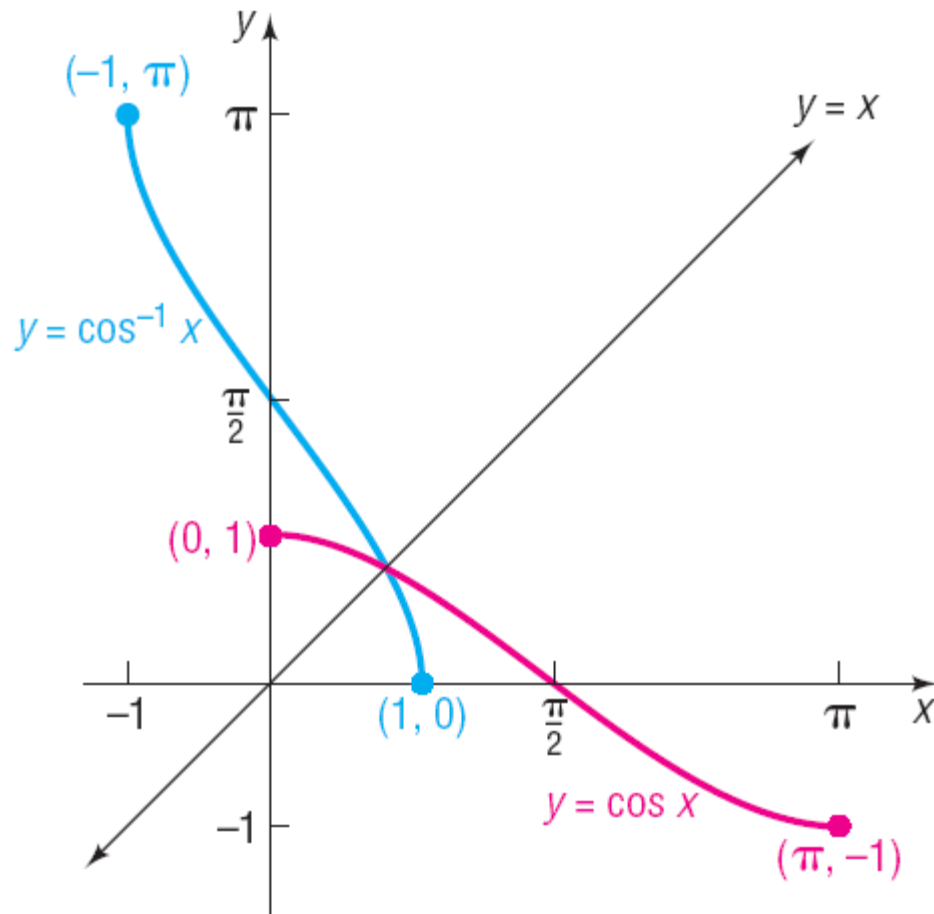


$$y = \cos x, \quad -\infty < x < \infty, \\ -1 \leq y \leq 1$$

$$y = \cos x, \quad 0 \leq x \leq \pi, \quad -1 \leq y \leq 1$$



$y = \cos^{-1} x$ means $x = \cos y$
 where $-1 \leq x \leq 1$ and $0 \leq y \leq \pi$



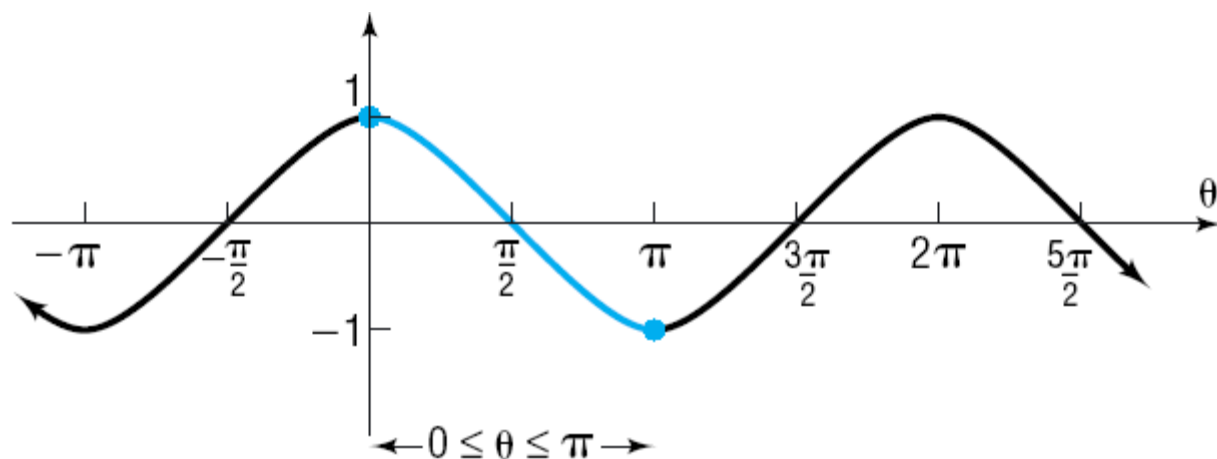
$$y = \cos^{-1} x, \quad -1 \leq x \leq 1, \quad 0 \leq y \leq \pi$$

EXAMPLE

Finding the Exact Value of an Inverse Cosine Function

Find the exact value of: $\cos^{-1} 0$

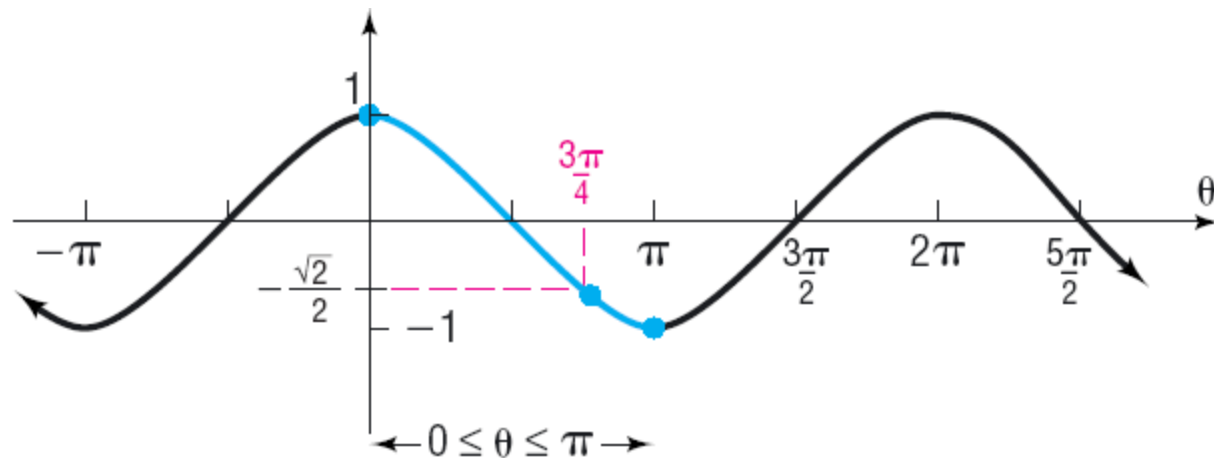
θ	$\cos \theta$
0	1
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$
$\frac{\pi}{3}$	$\frac{1}{2}$
$\frac{\pi}{2}$	0
$\frac{2\pi}{3}$	$-\frac{1}{2}$
$\frac{3\pi}{4}$	$-\frac{\sqrt{2}}{2}$
$\frac{5\pi}{6}$	$-\frac{\sqrt{3}}{2}$
π	-1



EXAMPLE

Finding the Exact Value of an Inverse Cosine Function

Find the exact value of: $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$



θ	$\cos \theta$
0	1
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$
$\frac{\pi}{3}$	$\frac{1}{2}$
$\frac{\pi}{2}$	0
$\frac{2\pi}{3}$	$-\frac{1}{2}$
$\frac{3\pi}{4}$	$-\frac{\sqrt{2}}{2}$
$\frac{5\pi}{6}$	$-\frac{\sqrt{3}}{2}$
π	-1

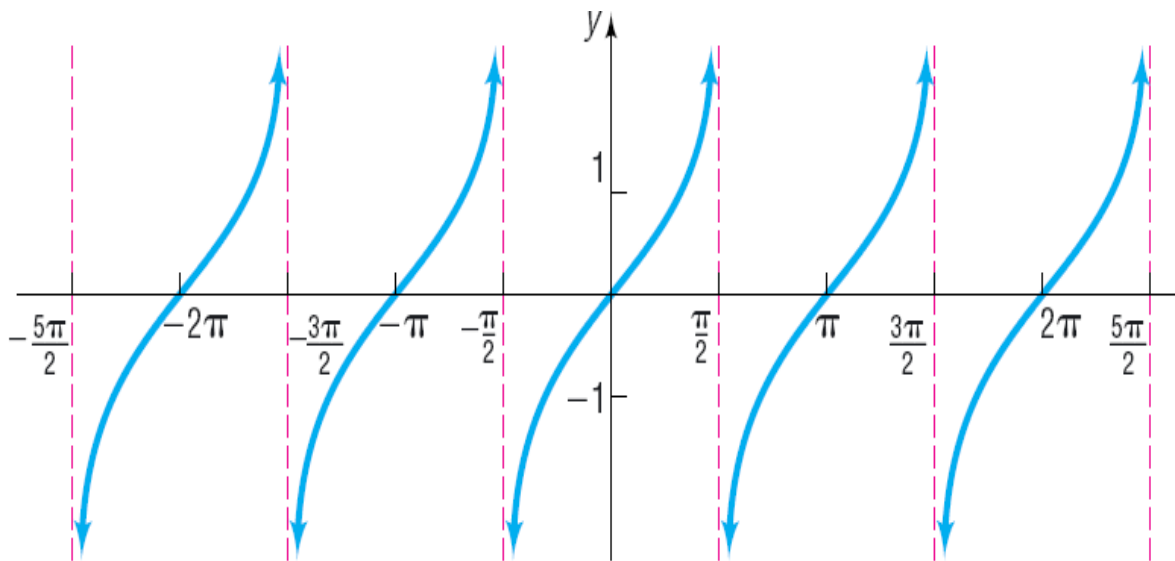
$$f^{-1}(f(x)) = \cos^{-1}(\cos x) = x, \quad \text{where } 0 \leq x \leq \pi$$
$$f(f^{-1}(x)) = \cos(\cos^{-1} x) = x, \quad \text{where } -1 \leq x \leq 1$$

EXAMPLE

Finding the Exact Value of an Inverse Cosine Function

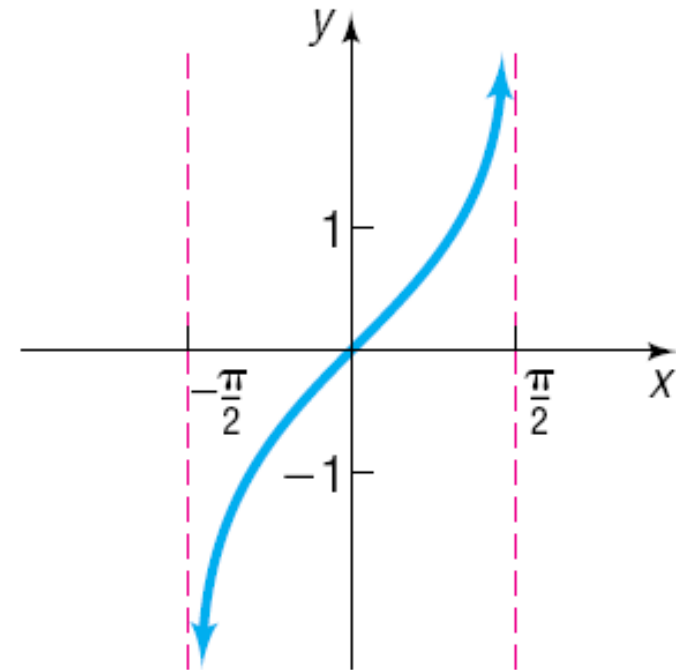
Find the exact value of: (a) $\cos^{-1}\left[\cos\left(\frac{\pi}{12}\right)\right]$ (b) $\cos[\cos^{-1}(-0.4)]$

The Inverse Tangent Function



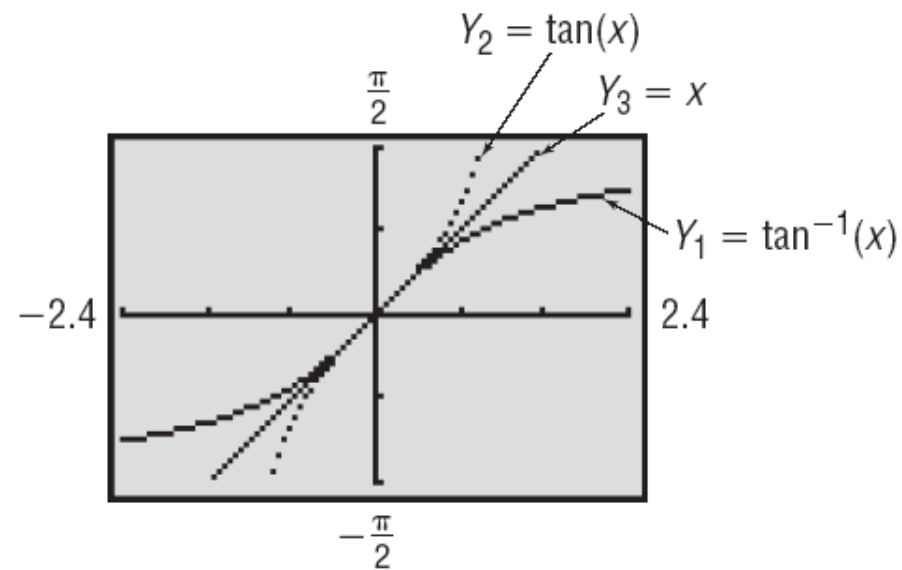
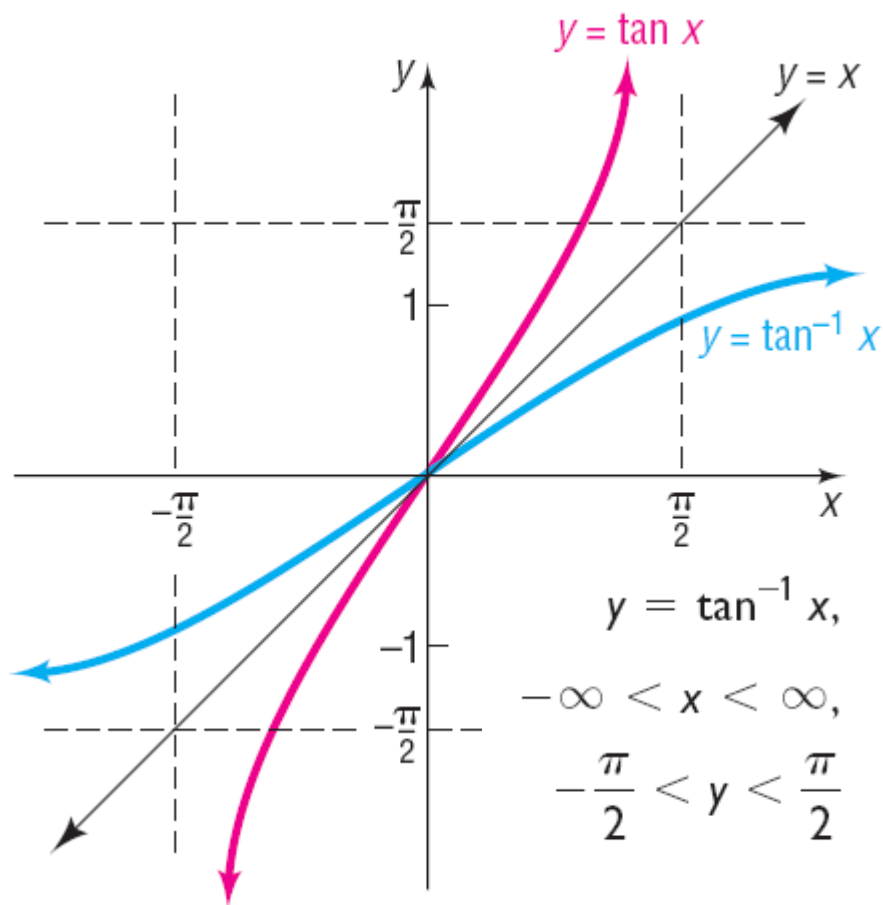
$y = \tan x, -\infty < x < \infty, x$ not equal
to odd multiples of $\frac{\pi}{2}, -\infty < y < \infty$

$$y = \tan x, -\frac{\pi}{2} < x < \frac{\pi}{2},$$
$$-\infty < y < \infty$$



$$y = \tan^{-1} x \text{ means } x = \tan y$$

where $-\infty < x < \infty$ and $-\frac{\pi}{2} < y < \frac{\pi}{2}$

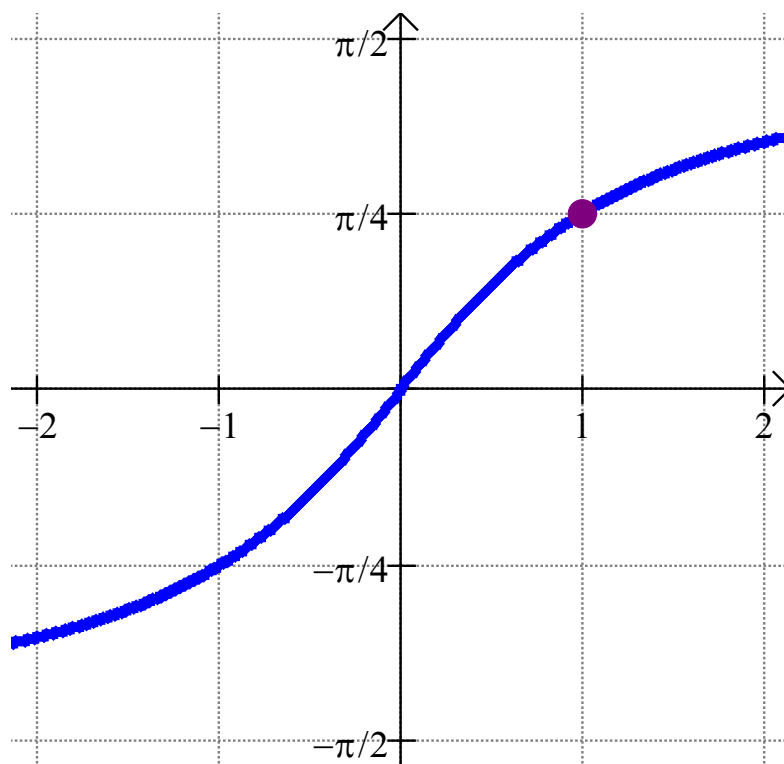


EXAMPLE

Finding the Exact Value of an Inverse Tangent Function

θ	$\tan \theta$
$-\frac{\pi}{2}$	Undefined
$-\frac{\pi}{3}$	$-\sqrt{3}$
$-\frac{\pi}{4}$	-1
$-\frac{\pi}{6}$	$-\frac{\sqrt{3}}{3}$
0	0
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{3}$
$\frac{\pi}{4}$	1
$\frac{\pi}{3}$	$\sqrt{3}$
$\frac{\pi}{2}$	Undefined

Find the exact value of: $\tan^{-1} 1$



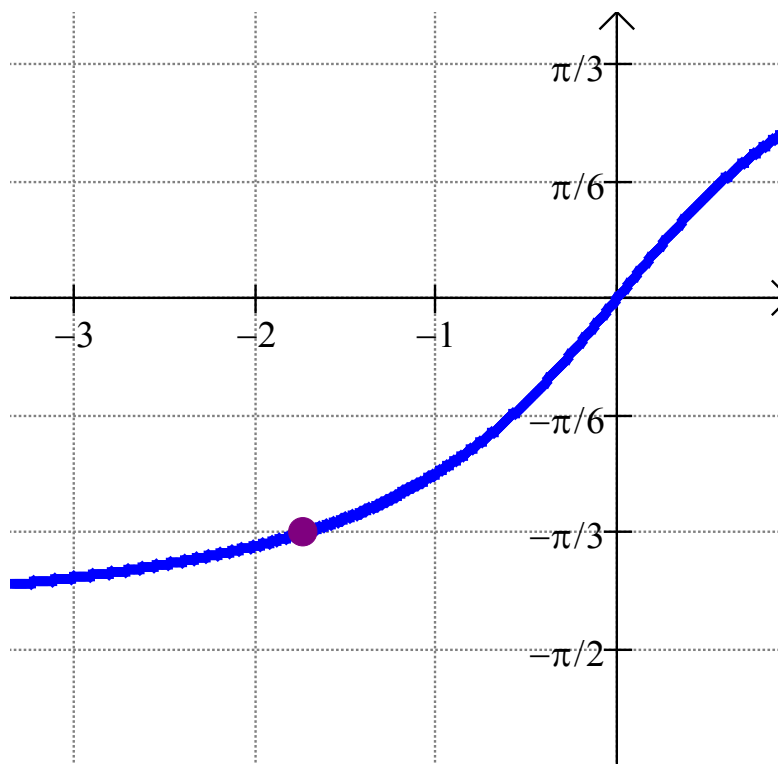
$$y = \tan^{-1} x$$

EXAMPLE

Finding the Exact Value of an Inverse Tangent Function

Find the exact value of: $\tan^{-1}(-\sqrt{3})$

θ	$\tan \theta$
$-\frac{\pi}{2}$	Undefined
$-\frac{\pi}{3}$	$-\sqrt{3}$
$-\frac{\pi}{4}$	-1
$-\frac{\pi}{6}$	$\frac{\sqrt{3}}{3}$
0	0
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{3}$
$\frac{\pi}{4}$	1
$\frac{\pi}{3}$	$\sqrt{3}$
$\frac{\pi}{2}$	Undefined



$$y = \tan^{-1} x$$