Section 6.4

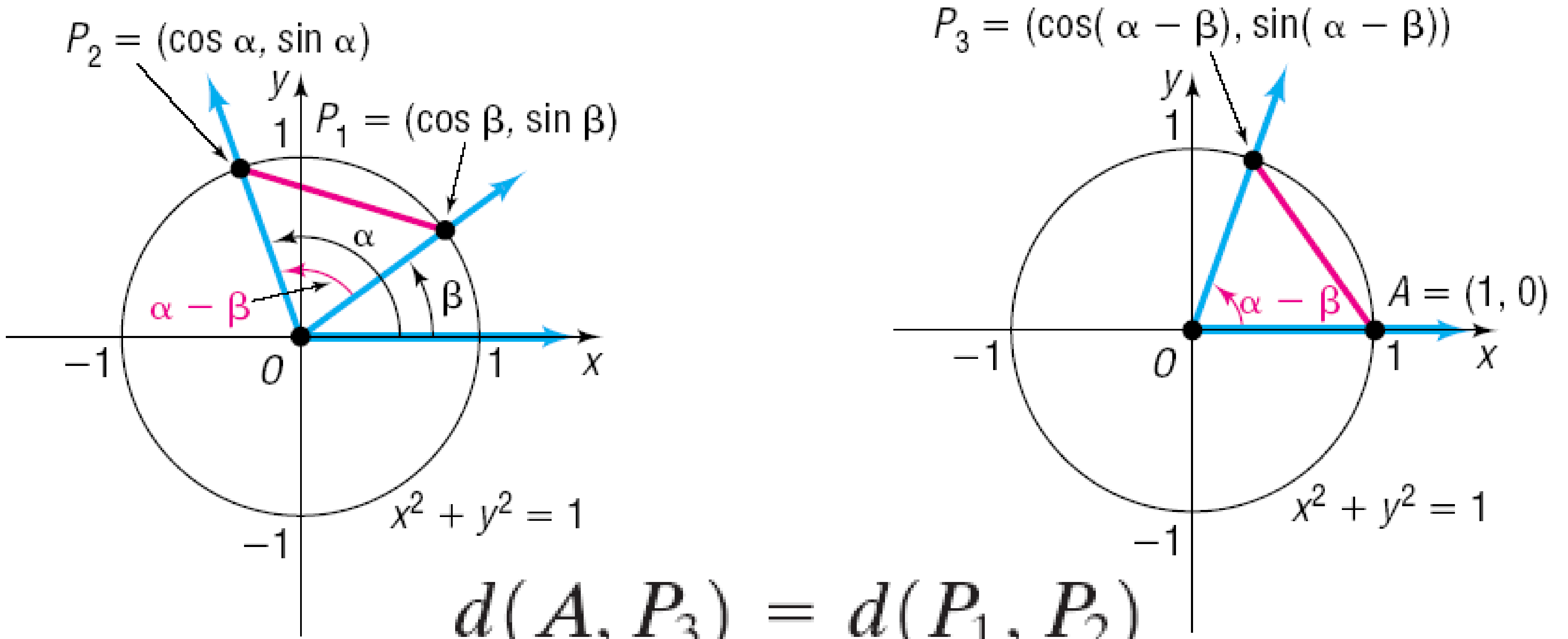
Sum and Difference Formulas

Theorem

Sum and Difference Formulas for Cosines

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$



$$d(A, P_3) = d(P_1, P_2)$$

OBJECTIVE 1

1 **Use Sum and Difference Formulas to Find Exact Values**

EXAMPLE

Using the Sum Formula to Find Exact Values

Find the exact value of $\cos 75°$.

Find the exact value of $\cos \frac{\pi}{12}$.

OBJECTIVE 2

2 Use Sum and Difference Formula to Establish Identities

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$$

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$$

Seeing the Concept

Graph $Y_1 = \cos\left(\frac{\pi}{2} - \theta\right)$ and $Y_2 = \sin\theta$ on the same screen. Does this demonstrate the result 3(a)? How would you demonstrate the result 3(b)?

Theorem

Sum and Difference Formulas for Sines

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

EXAMPLE

Using the Sum Formula to Find Exact Values

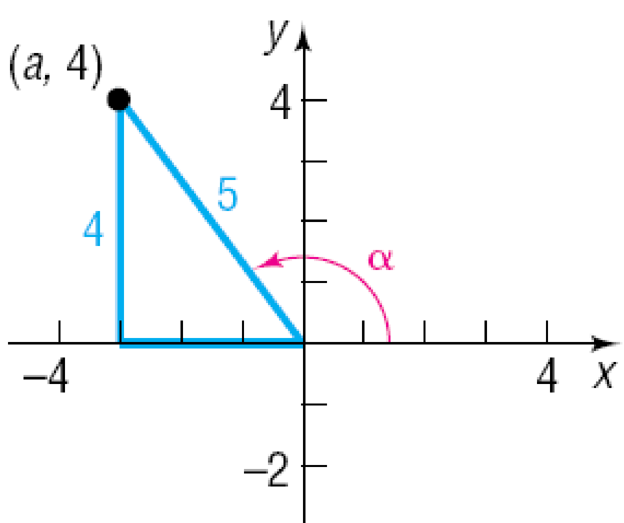
Find the exact value of $\sin \frac{7\pi}{12}$.

Find the exact value of $\sin 80^\circ \cos 20^\circ - \cos 80^\circ \sin 20^\circ$.

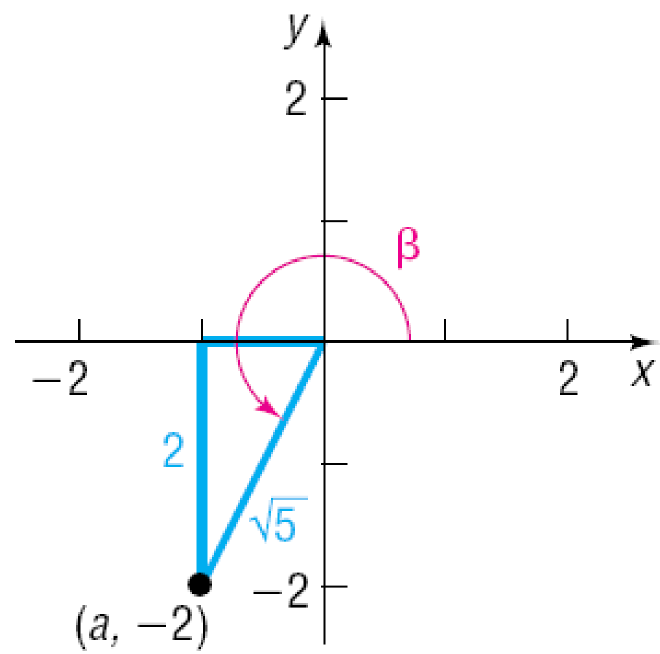
EXAMPLE Finding Exact Values

If it is known that $\sin\alpha = \frac{4}{5}$, $\frac{\pi}{2} < \alpha < \pi$, and that $\sin\beta = -\frac{2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$, $\pi < \beta < \frac{3\pi}{2}$, find the exact value of

(a) $\cos\alpha$ (b) $\cos\beta$ (c) $\cos(\alpha + \beta)$ (d) $\sin(\alpha + \beta)$



$\sin\alpha = \frac{4}{5}, \frac{\pi}{2} < \alpha < \pi$



$\sin\beta = \frac{-2}{\sqrt{5}}, \pi < \beta < \frac{3\pi}{2}$

EXAMPLE

Establishing an Identity

Establish the identity: $\dfrac{\cos(\alpha - \beta)}{\sin\alpha \sin\beta} = \cot\alpha \cot\beta + 1$

Theorem

Sum and Difference Formulas for Tangents

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

EXAMPLE **Establishing an Identity**

Prove the identity: $\tan(\theta + \pi) = \tan\theta$

Prove the identity: $\tan\left(\theta + \frac{\pi}{2}\right) = -\cot\theta$

OBJECTIVE 3

3 Use Sum and Difference Formulas Involving Inverse Trigonometric Functions

EXAMPLE

Finding the Exact Value of an Expression Involving Inverse Trigonometric Functions

Find the exact value of: $\sin\left(\cos^{-1}\frac{1}{2} + \sin^{-1}\frac{3}{5}\right)$

$$\cos\alpha = \frac{1}{2}, \quad 0 \le \alpha \le \pi, \quad \text{and} \quad \sin\beta = \frac{3}{5}, \quad -\frac{\pi}{2} \le \beta \le \frac{\pi}{2}$$

EXAMPLE

Writing a Trigonometric Expression as an Algebraic Expression

Write $\sin(\sin^{-1} u + \cos^{-1} v)$ as an algebraic expression containing u and v (that is, without any trigonometric functions).

Let $\alpha = \sin^{-1} u$ and $\beta = \cos^{-1} v$. Then

$$\sin \alpha = u, \quad -\frac{\pi}{2} \leq \alpha \leq \frac{\pi}{2}, \quad \text{and} \quad \cos \beta = v, \quad 0 \leq \beta \leq \pi$$