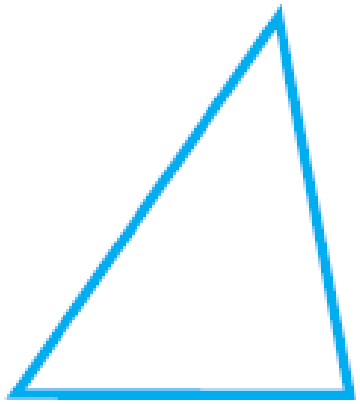
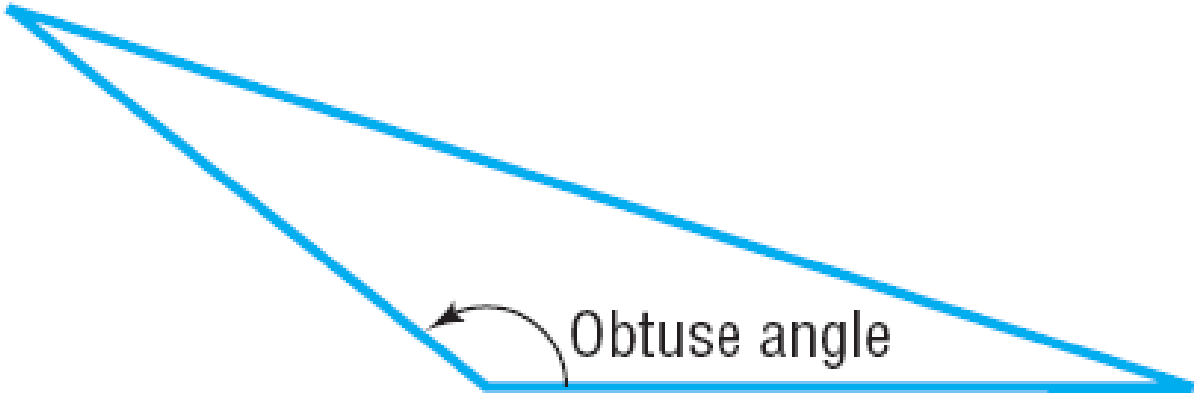
Section 7.2

The Law of Sines



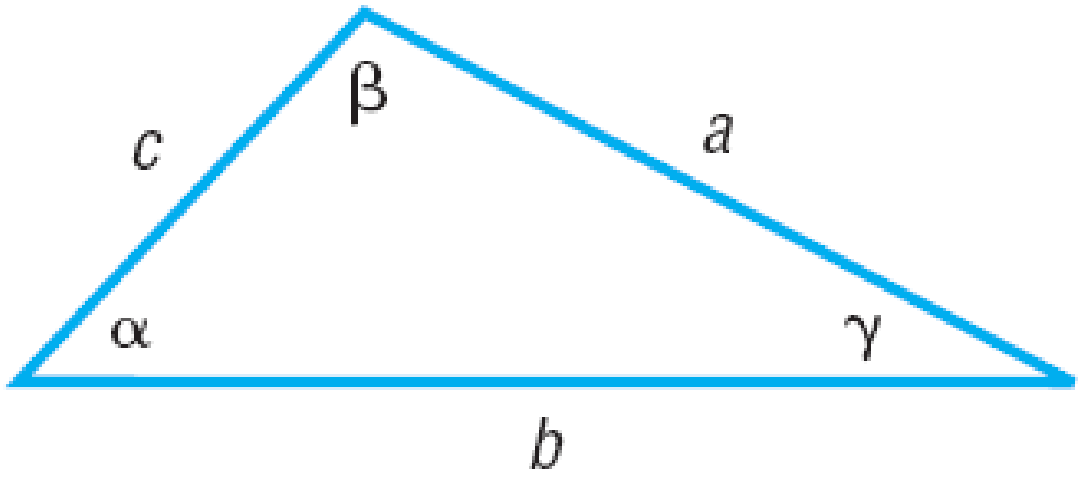
(a) All angles are acute



(b) Two acute angles and one obtuse angle

Oblique Triangle

(None of the angles is a right angle)

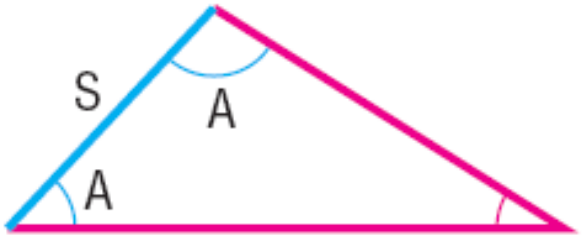


CASE 1: One side and two angles are known (ASA or SAA).

CASE 2: Two sides and the angle opposite one of them are known (SSA).

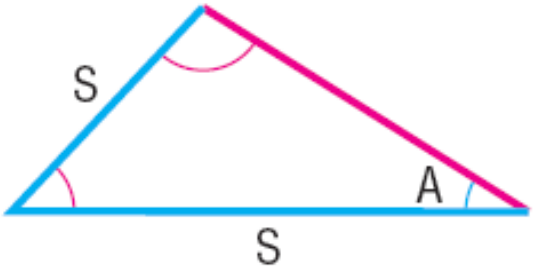
CASE 3: Two sides and the included angle are known (SAS).

CASE 4: Three sides are known (SSS).

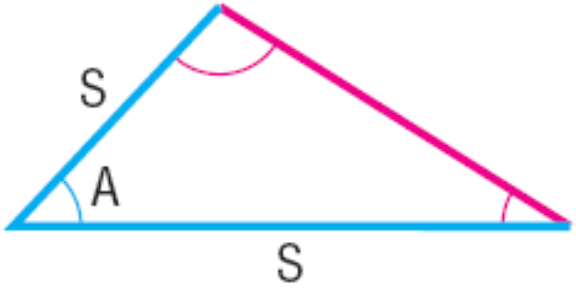


Case 1: ASA

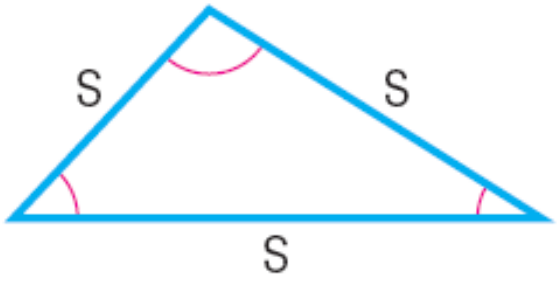
Case 1: SAA



Case 2: SSA



Case 3: SAS



Case 4: SSS

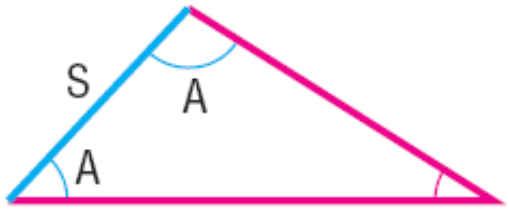
The **Law of Sines** is used to solve triangles for which Case 1 or 2 holds.

Theorem

Law of Sines

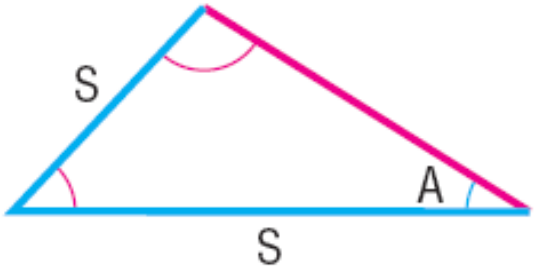
For a triangle with sides a, b, c and opposite angles α, β, γ, respectively,

$$\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c}$$

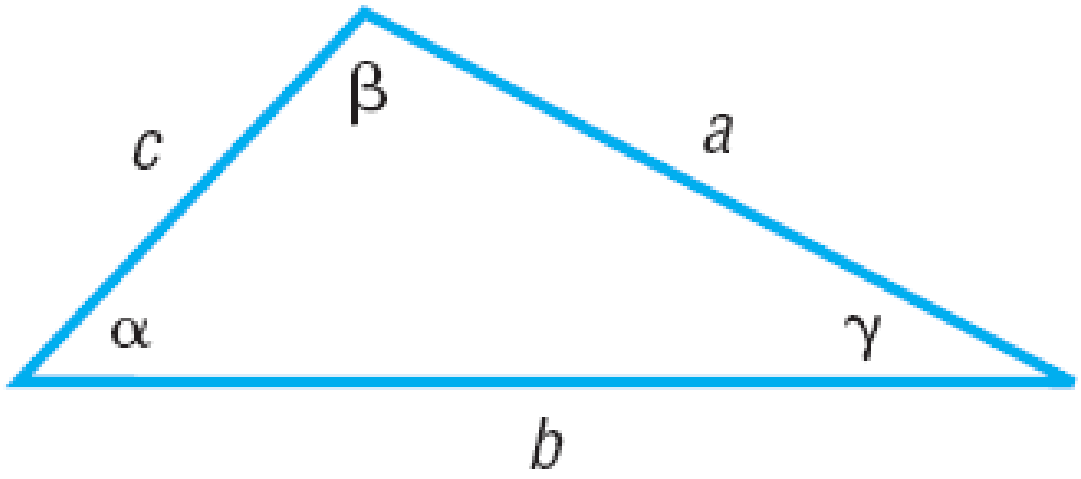


Case 1: ASA

Case 1: SAA



Case 2: SSA



$$\alpha + \beta + \gamma = 180^\circ$$

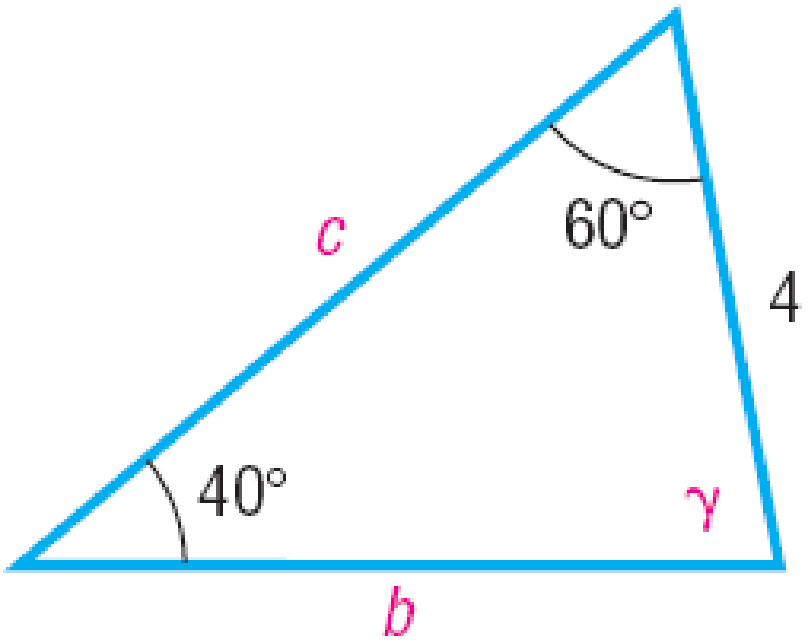
OBJECTIVE 1

Solve SAA or ASA Triangles

EXAMPLE

Using the Law of Sines to Solve a SAA Triangle

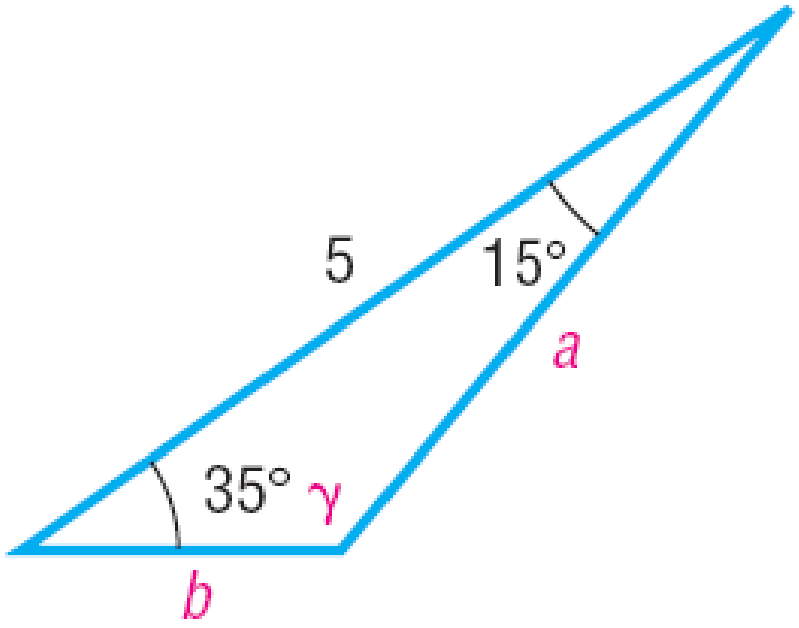
Solve the triangle: $\alpha = 40°, \beta = 60°, a = 4$



EXAMPLE

Using the Law of Sines to Solve an ASA Triangle

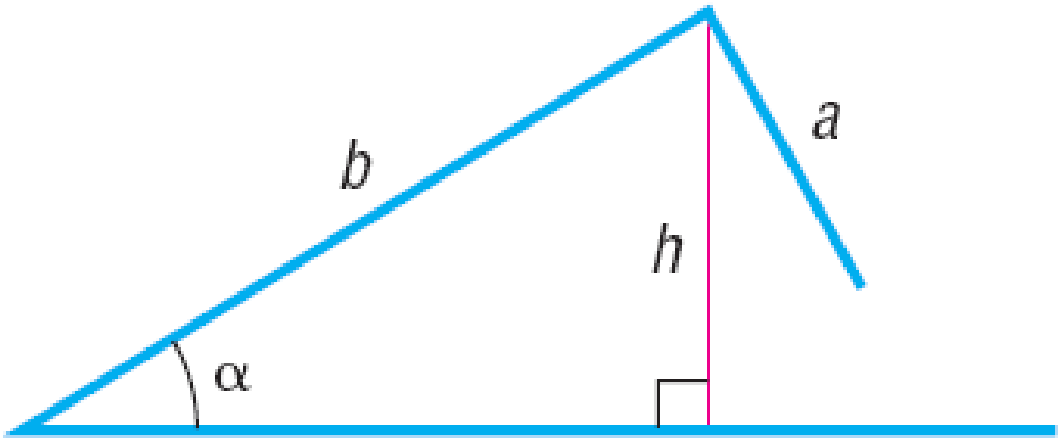
Solve the triangle: $\alpha = 35°, \beta = 15°, c = 5$



OBJECTIVE 2

Solve SSA Triangles

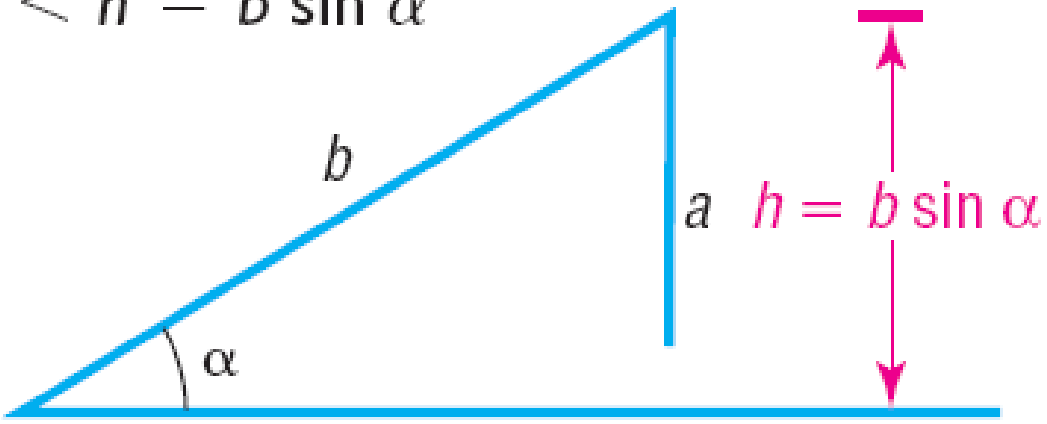
SSA --- The Ambiguous Case



No Triangle If $a < h = b \sin \alpha$, then side a is not sufficiently long to form a triangle. See Figure 14.

Figure 14

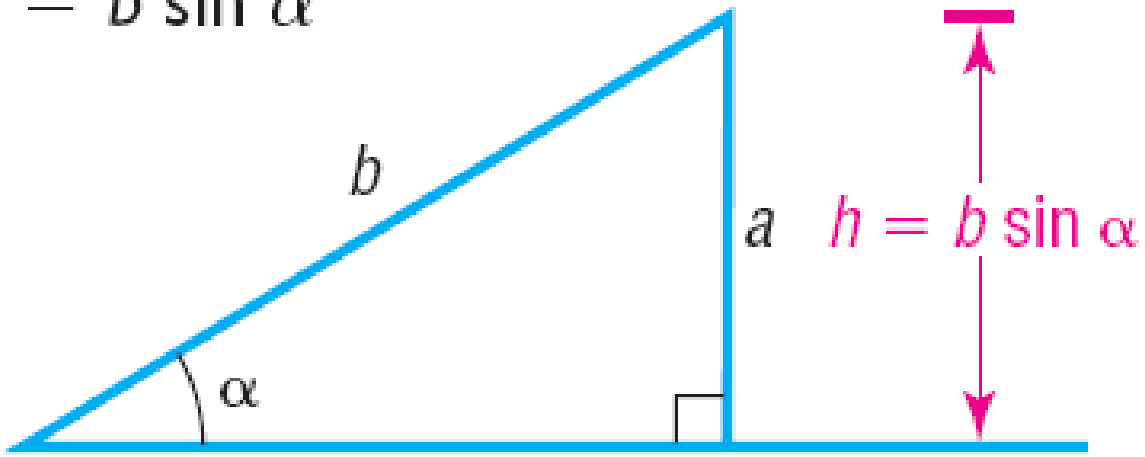
$a < h = b \sin \alpha$



One Right Triangle If $a = h = b \sin \alpha$, then side a is just long enough to form a right triangle. See Figure 15.

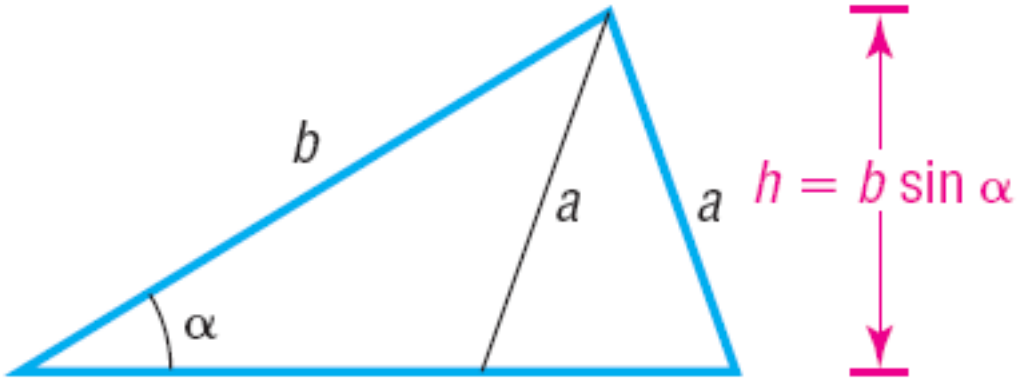
Figure 15

$a = b \sin \alpha$



Two Triangles If $a < b$ and $h = b \sin \alpha < a$, then two distinct triangles can be formed from the given information. See Figure 16.

Figure 16
$b \sin \alpha < a$ and $a < b$



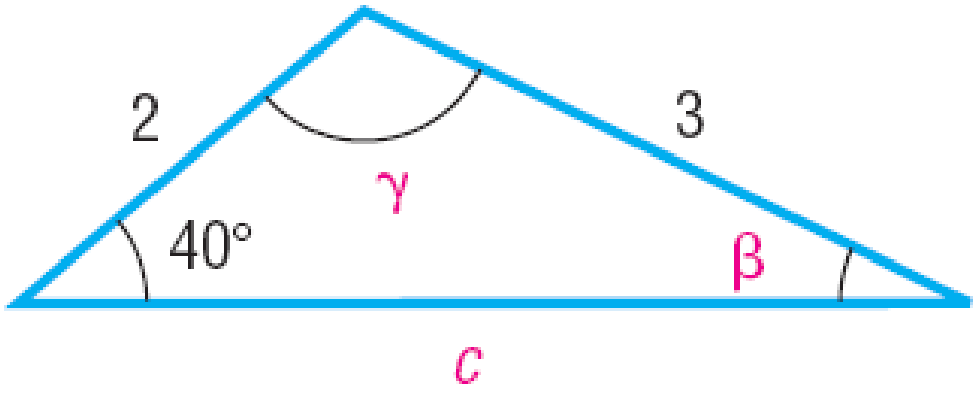
One Triangle If $a \geq b$, then only one triangle can be formed. See Figure 17.

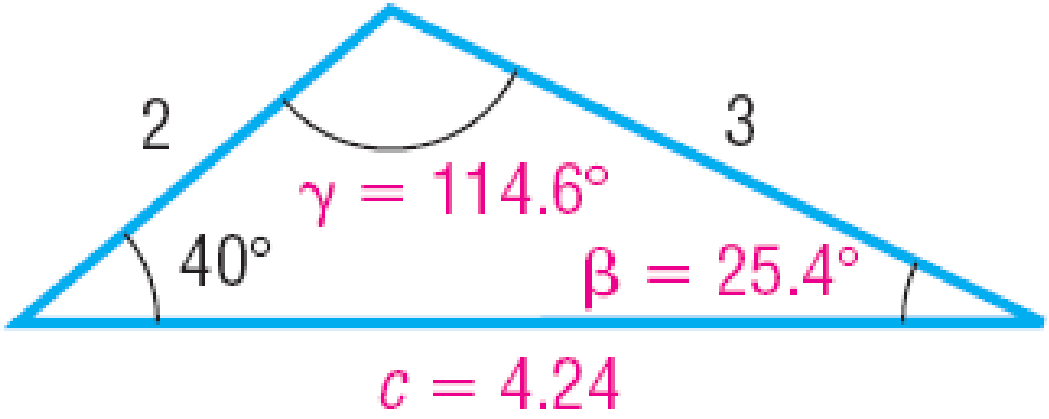
Figure 17
$a \geq b$

EXAMPLE

Using the Law of Sines to Solve a SSA Triangle (One Solution)

Solve the triangle: $a = 3, b = 2, \alpha = 40°$

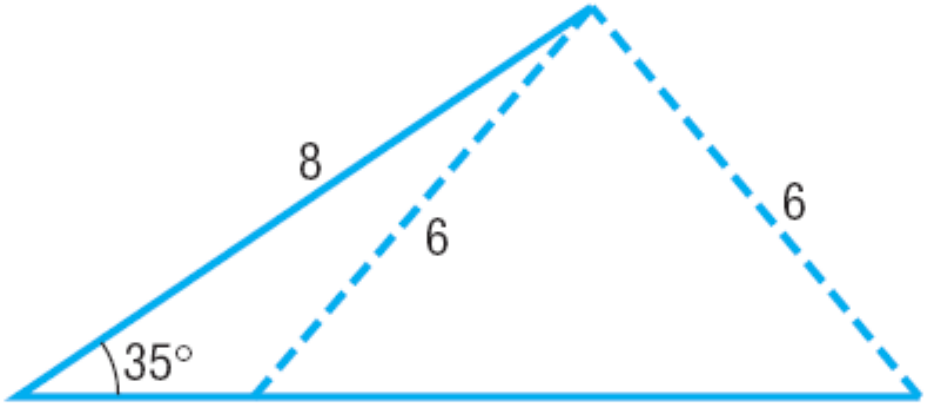


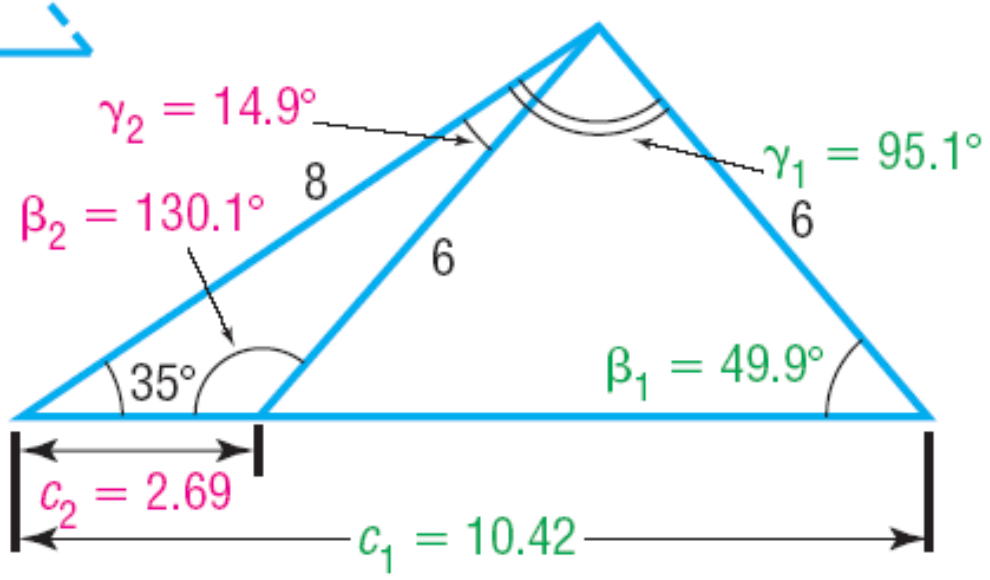


EXAMPLE

Using the Law of Sines to Solve a SSA Triangle (Two Solutions)

Solve the triangle: $a = 6, b = 8, \alpha = 35°$

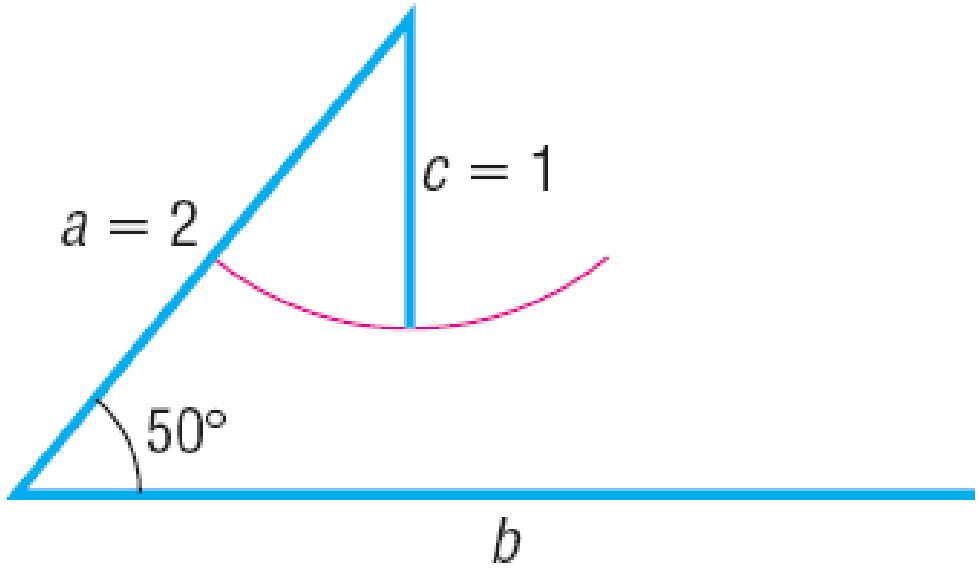




EXAMPLE

Using the Law of Sines to Solve a SSA Triangle (No Solution)

Solve the triangle: $a = 2, c = 1, \gamma = 50°$



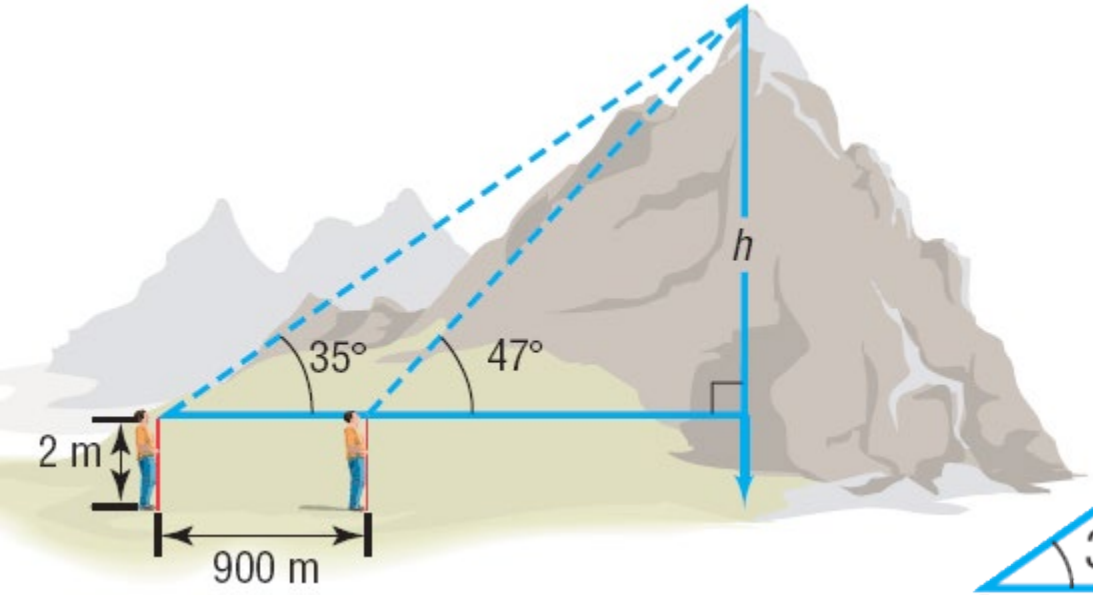
OBJECTIVE 3

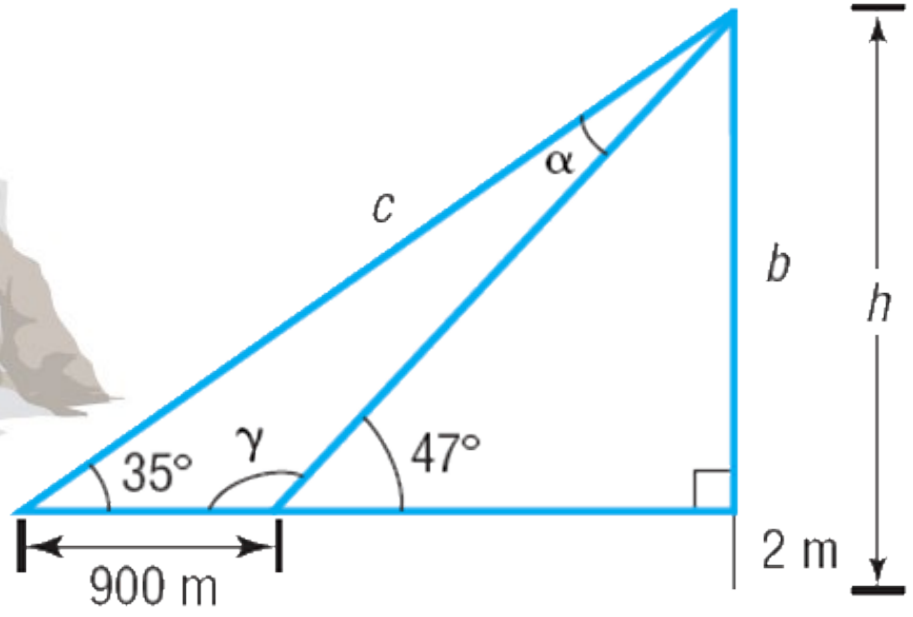
Solve Applied Problems

EXAMPLE

Finding the Height of a Mountain

To measure the height of a mountain, a surveyor takes two sightings of the peak at a distance 900 meters apart on a direct line to the mountain.* See Figure 31(a). The first observation results in an angle of elevation of 47° and the second results in an angle of elevation of 35°. If the transit is 2 meters high, what is the height h of the mountain?



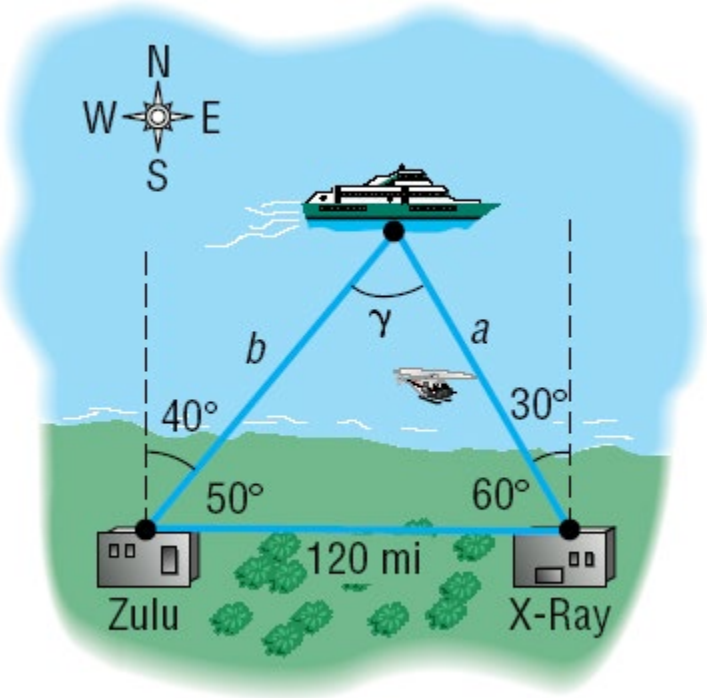


EXAMPLE Rescue at Sea

Coast Guard Station Zulu is located 120 miles due west of Station X-ray. A ship at sea sends an SOS call that is received by each station. The call to Station Zulu indicates that the bearing of the ship from Zulu is N40°E (40° east of north). The call to Station X-ray indicates that the bearing of the ship from X-ray is N30°W (30° west of north).

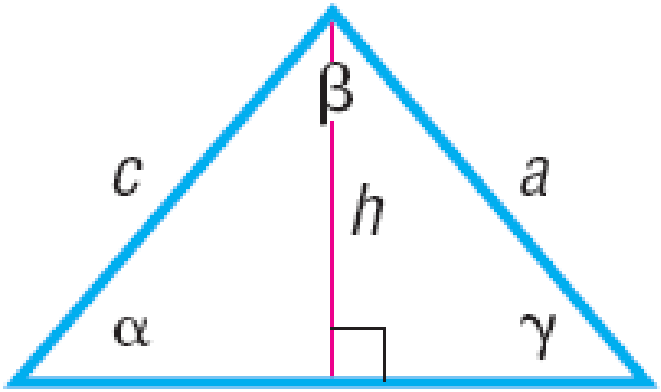
(a) How far is each station from the ship?

(b) If a helicopter capable of flying 200 miles per hour is dispatched from the nearest station to the ship, how long will it take to reach the ship?

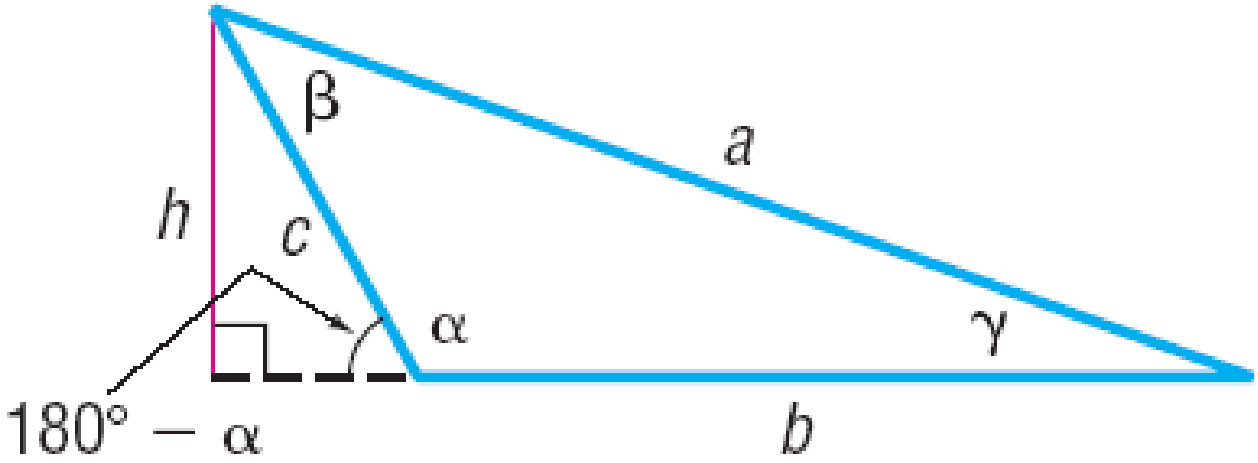


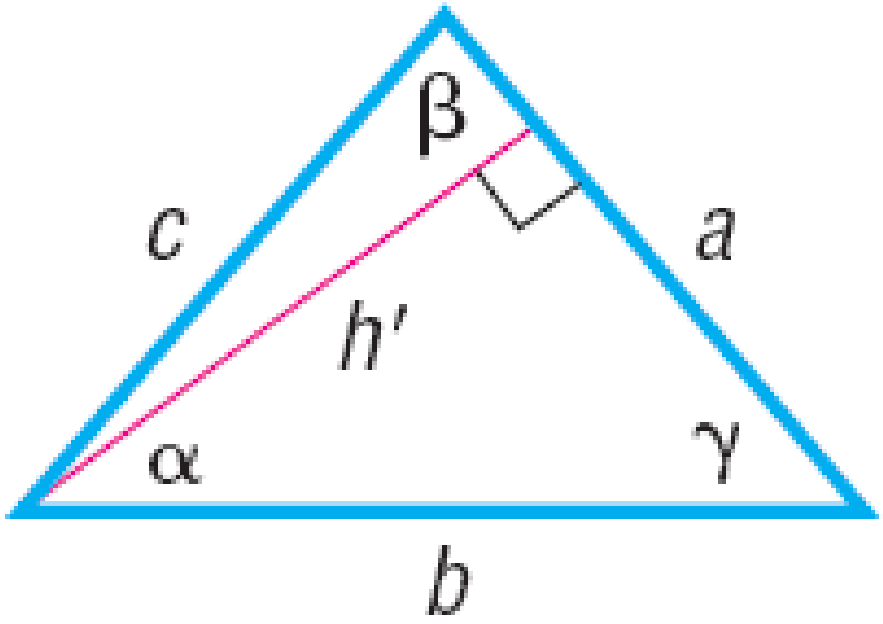
Proof of the Law of Sines

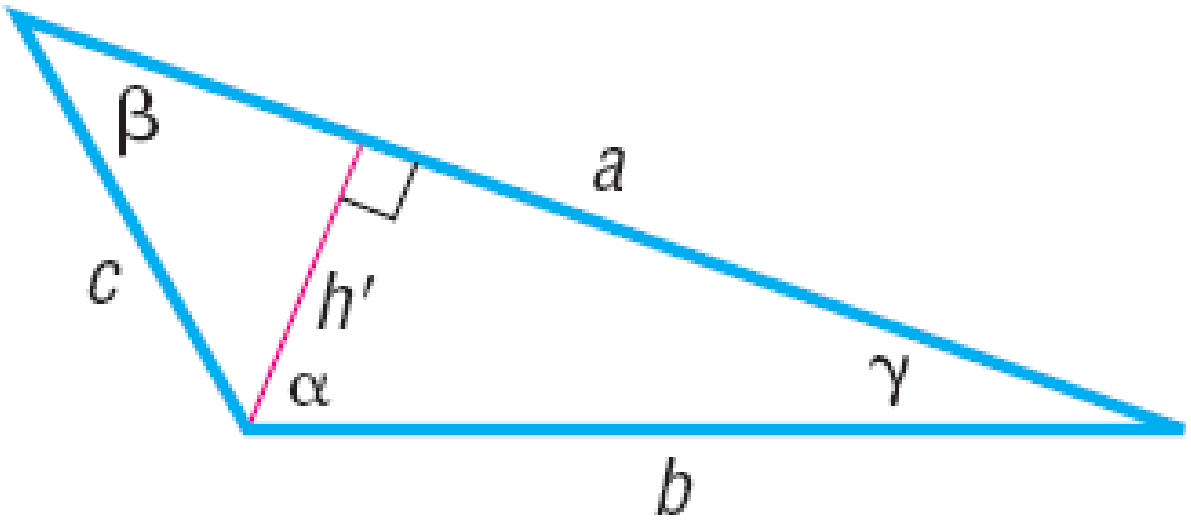
$$\sin \gamma = \frac{h}{a}$$



$$h = a \sin \gamma$$




β
c
a
h′
α
γ
b


β
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h′
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γ
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