

MA180 Spring 2007 Extra Credit Quiz 1A to be Added to Quiz 1

Name: Solution

- 1) An open box with a square base is required to have a volume of 27 cubic feet. Express the amount A of material used to make such a box as a function of the length x of a side of the base.



$$y = \frac{27}{x^2}$$

$$\text{Volume} = L \cdot W \cdot H$$

$$27 = x \cdot x \cdot y \implies y = \frac{27}{x^2}$$

$$\text{Surface Area} = \text{Bottom} + 4 \cdot \text{sides}$$

$$= x^2 + 4 \left(x \cdot \frac{27}{x^2} \right)$$

$$A = x^2 + \frac{108}{x} \quad 0 < x$$

- 2) Find the average rate of change for the function $f(x) = \frac{3}{x-2}$ between $x = 4$ to $x = 7$.

x	$f(x) = \frac{3}{x-2}$
4	$\frac{3}{2}$
7	$\frac{3}{5}$

$$\begin{aligned} \text{Avg Rate of Change} &= \frac{\frac{3}{5} - \frac{3}{2}}{7-4} = \frac{\frac{6-15}{10}}{\frac{3}{1}} = \frac{-9}{10} \cdot \frac{1}{3} \\ &= \frac{-3}{10} \end{aligned}$$

3) Given $f(x) = 3x^2 - 5x + 2$ Evaluate $\frac{f(x+h) - f(x)}{h}$, where $h \neq 0$

$$f(x+h) = 3(x+h)^2 - 5(x+h) + 2 = 3(x^2 + 2xh + h^2) - 5x - 5h + 2$$

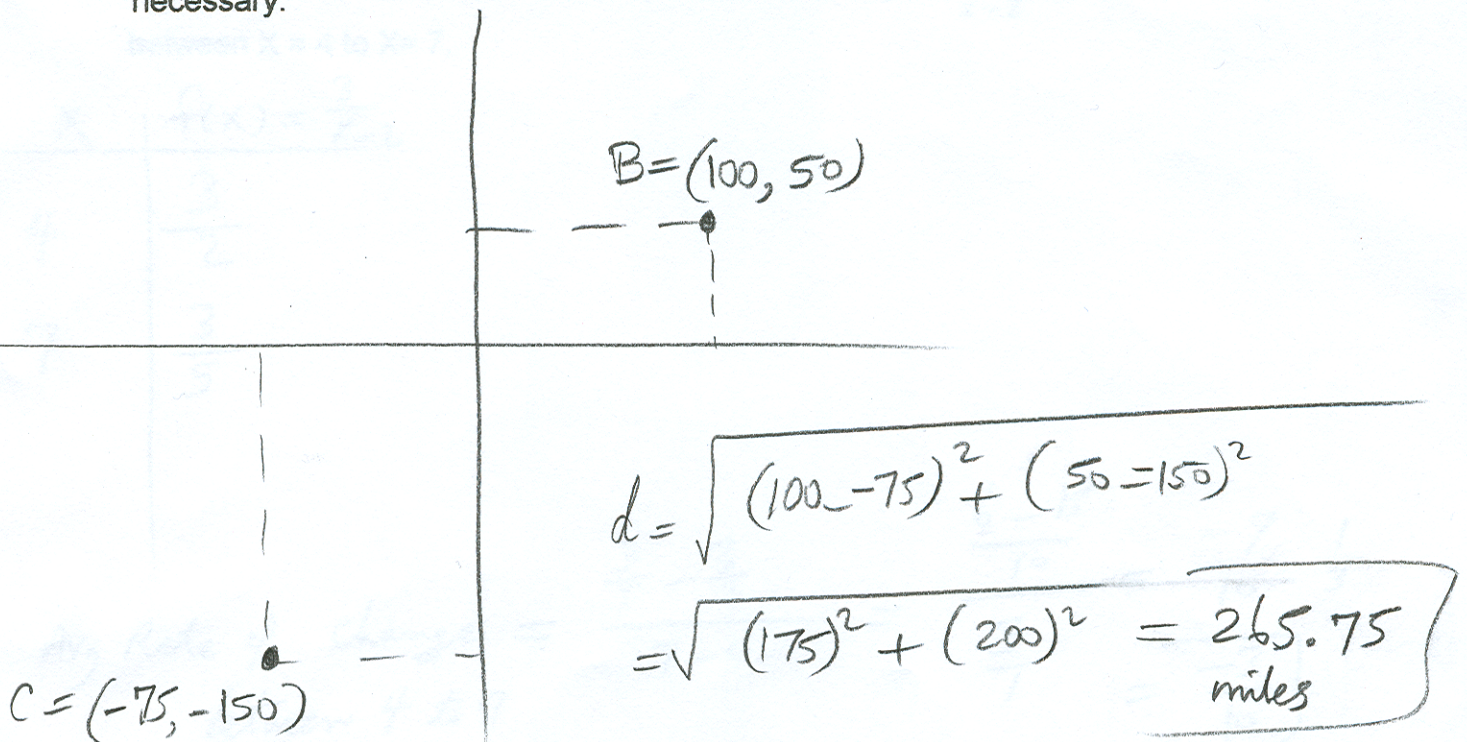
$$= 3x^2 + 6xh + 3h^2 - 5x - 5h + 2$$

$$f(x+h) - f(x) = 3x^2 + 6xh + 3h^2 - 5x - 5h + 2 - (3x^2 - 5x + 2)$$

$$= \cancel{3x^2} + 6xh + 3h^2 - \cancel{5x} - 5h + \cancel{2} - \cancel{3x^2} + \cancel{5x} - \cancel{2}$$

$$\frac{f(x+h) - f(x)}{h} = \frac{6xh + 3h^2 - 5h}{h} = \frac{h(6x + 3h - 5)}{h} = \boxed{6x + 3h - 5}$$

4) City B is located at 100 miles east and 50 miles north of city A. City C is located at 75 miles west and 150 miles south of city A. Find the distance between city B and city C. You can choose city A as the origin of the rectangular coordinate system. Write your answer rounded to two decimal places, if necessary.



MA180 Spring 2007 Extra Credit Quiz 1B to be Added to Quiz 1

Name: Solution

- 1) Find the center (h, k) and radius r of the following circle and graph the circle

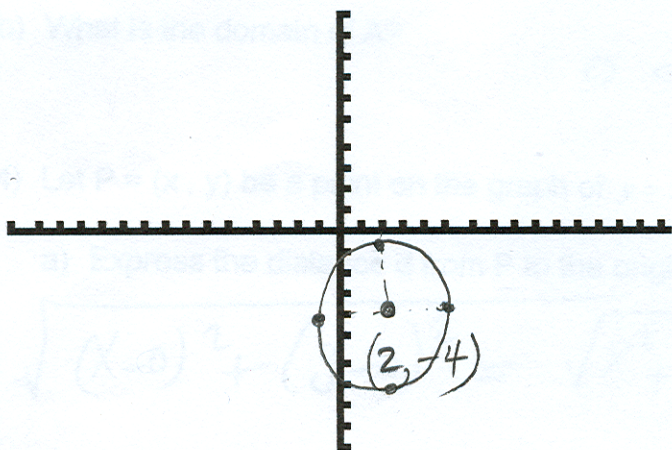
$$x^2 + y^2 - 4x + 8y + 11 = 0$$

$$x^2 - 4x + 4 + y^2 + 8y + 16 = -11 + 4 + 16$$

$$(x - 2)^2 + (y + 4)^2 = 9$$

$$\text{Center} = (2, -4)$$

$$\text{Radius} = 3$$



- 2) Solve $\sqrt{2x+3} - \sqrt{x+1} = 1$

$$(\sqrt{2x+3})^2 = (1 + \sqrt{x+1})^2$$

$$2x+3 = (1 + \sqrt{x+1})(1 + \sqrt{x+1})$$

$$2x+3 = 1 + 2\sqrt{x+1} + x+1$$

$$2x+3 = 2 + 2\sqrt{x+1} + x$$

$$-x - 2 = 2\sqrt{x+1} - x$$

$$x+1 = 2\sqrt{x+1} \quad \text{Square both sides}$$

$$(x+1)^2 = 4(x+1)$$

$$x^2 + 2x + 1 = 4(x+1)$$

$$x^2 + 2x + 1 = 4x + 4$$

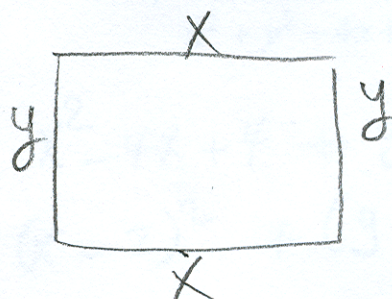
$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0$$

$$\boxed{x=3} \quad \boxed{x=-1}$$

3) David has available 400 yards of fencing and wishes to enclose a rectangular area.

a) Express the area A of the rectangle as a function of the width x of the rectangle.



$$2x + 2y = 400 \Rightarrow x + y = 200$$

$$y = 200 - x$$

$$A = x \cdot y$$

$$= x(200 - x) = 200x - x^2$$

b) What is the domain of A ?

$$0 < x < 200$$

4) Let $P = (x, y)$ be a point on the graph of $y = x^2 - 8$

a) Express the distance d from P to the origin as a function of x .

$$d = \sqrt{(x-0)^2 + (y-0)^2} = \sqrt{x^2 + (x^2 - 8)^2}$$

b) What is d if $x = 0$?

$$d = \sqrt{(-8)^2} = 8$$

c) What is d if $x = 1$?

$$d = \sqrt{1 + (1-8)^2} = \sqrt{50}$$

d) Use your calculator to graph $d(x)$

e) For what values of x is d smallest?

$$x = \pm 2.77$$