MA 180 Professor Fred Katiraie Pre Calculus Quiz 3A

Name: Solution **Date:**

1) Change logarithmic expression to exponential expression, and solve:

1pt

a) Given $\ln 2x = 2$, **Solve for x** $e^2 = 2x \Rightarrow x = \frac{e^2}{2} = 3.69$

1pt

b) Given $\log_{10} N = 5$, **Solve for N**

$N = 10^5 = \boxed{100000}$

2pts

2) Find the Domain of the following function $g(x) = \ln\left(\frac{x+2}{x-1}\right)$

$\frac{x+2}{x-1} > 0$ $\quad + \quad -2 \quad - \quad +1 \quad +$

$\boxed{(-\infty, -2) \cup (1, \infty)}$

2pts

3) Verify that the following functions are inverses of each other by showing that

$f(g(x)) = x$, and $g(f(x)) = x$

$f(x) = 2x + 6$

$g(x) = \frac{x}{2} - 3$

$f(g(x)) = 2\left(\frac{x}{2} - 3\right) + 6 = \frac{2x}{2} - 6 + 6 = \boxed{x}$

$g(f(x)) = \frac{2x+6}{2} - 3$

$= x + 3 - 3 = \boxed{x}$

3pts

4) Find the inverse of the following functions.

a) $f(x)=\frac{2x-3}{x+4}$

$y=\frac{2x-3}{x+4}$

$xy+4y=2x-3$

$xy-2x=-4y-3$

$x(y-2)=-4y-3$

$x=\frac{-4y-3}{y-2} \Rightarrow f^{-1}(x)=\frac{-4x-3}{x-2}=\boxed{\frac{4x+3}{2-x}}$

3pts

b) $y=\sqrt[3]{x+5}-6$

$y+6=\sqrt[3]{x+5}$

$(y+6)^3=x+5$

$x=(y+6)^3-5$

$\boxed{f^{-1}(x)=(x+6)^3-5}$

3 pts

5) Solve the following algebraically:

a) $e^{x^2}=\left(e^{5x}\right)\bullet\frac{1}{e^{-6}}$

$e^{x^2}=e^{5x+6}$

$x^2=5x+6$

$x^2-5x-6=0$

$(x-6)(x+1)=0$ $\quad x=6$ $\quad x=-1$

2pts

b) If $3^x=49$, what does 3^{-2x} equal?

$3^{-2x}=(3^x)^{-2}=49^{-2}=\frac{1}{49^2}$

$=\boxed{\frac{1}{2401}}$

3pts

6) Fine the exponential function whose table of x and y values is given below.

X	Y1	
-2	-.2222	
-1	-.6667	
0	-2	
1	-6	
2	-18	
3	-54	
4	-162	

X= -2

$y=ab^x$

$-2=ab^0 \Rightarrow \boxed{a=-2}$

$(1,-6) \Rightarrow -6=(-2)(b)^1 \Rightarrow \boxed{b=3}$

$\boxed{y=(-2)(3)^x}$

MA 180 Professor Fred Katiraie Pre Calculus Quiz 3B

Name: Solution **Date:**

(1pt) 1) Change logarithmic expression to exponential expression, and solve:

(1pt) a) Given $\ln 5x = 10$, **Solve for x**

$5x = e^{10} \Rightarrow x = \frac{e^{10}}{5} = \boxed{4405.29}$

(2pts) b) Given $\log_{10} N = -3$, **Solve for N**

$10^{-3} = N \Rightarrow \boxed{N = 0.001}$

2) Find the Domain of the following function $g(x) = \ln\left(\frac{x-3}{x+5}\right)$

$\frac{x-3}{x+5} > 0$

$+ \quad -5 \quad - \quad 3 \quad +$

$\boxed{(-\infty, -5) \cup (3, \infty)}$

(2pts) 3) Verify that the following functions are inverses of each other by showing that

$f(g(x)) = x,$ *and* $g(f(x)) = x$

$f(x) = 3x + 6$

$g(x) = \frac{x}{3} - 2$

$f(g(x)) = 3\left(\frac{x}{3} - 2\right) + 6 = x - 6 + 6 = \boxed{x}$

$g(f(x)) = \frac{3x+6}{3} - 2 = x + 2 - 2 = \boxed{x}$

4) Find the inverse of the following functions.

a) $f(x)=\frac{3x-4}{x-5}$

3pts $y=\frac{3x-4}{x-5}$

$xy-5y=3x-4$

$xy-3x=5y-4$

$x(y-3)=5y-4$

$x=\frac{5y-4}{y-3} \Rightarrow \boxed{f^{-1}(x)=\frac{5x-4}{x-3}}$

3pts b) $y=\sqrt[3]{x-5}+6$

$y-6=\sqrt[3]{x-5}$

$(y-6)^3=x-5$

$x=(y-6)^3+5$

$\boxed{f^{-1}(x)=(x-6)^3+5}$

5) Solve the following algebraically:

a) $e^{x^2}=\left(e^{5x}\right)\bullet\frac{1}{e^{-6}}$

3pts

$e^{x^2}=e^{5x+6}$

$x^2=5x+6$

$x^2-5x-6=0$

$(x-6)(x+1)=0$

$\boxed{x=6 \quad x=-1}$

2pts b) If $3^x=4$, what does 3^{-2x} equal?

$\boxed{3^{-2x}=(3^x)^{-2}=4^{-2}=\frac{1}{16}}$

6) Fine the exponential function whose table of x and y values is given below.

3pts

X	Y1	
-2	-1.25	
-1	-2.5	
0	-5	
1	-10	
2	-20	
3	-40	
4	-80	

X=-2

$y=ab^x$

$\boxed{a=-5}$ and

$(+1,-10) \Rightarrow -10=(-5)(b)^{+1}$

$\boxed{2=b}$

$\therefore \boxed{y=(-5)(2)^x}$

Professor Katiraie Precalculus Fall 2007 Quiz Four Form B (Sections 4.6--4.9)

Name Solution

The year will be $1990 + 103 = 2093$

olve the problem.

1) In 1990, the population of a country was estimated at 4 million. For any subsequent year the population, P(t) (in millions), can be modeled by the equation $P(t) = \frac{240}{5 + 54.99e^{-0.0208t}}$, where t is the number of years since 1990. Estimate the year when the population will be 21 million.

$$21 = \frac{240}{5+54.99e^{-0.0208t}}$$

$$21(5+54.99e^{-0.0208t}) = 240$$

$$5+54.99e^{-0.0208t} = \frac{240}{21}$$

$$t = \ln\left(\frac{\frac{240}{21} - 5}{54.99}\right) \div -0.0208$$

$$t = 103$$

2) The half-life of radium is 1690 years. If 150 grams is present now, how long (to the nearest year) till only 100 grams are present?

$$\frac{1}{2} = e^{1690K}$$

$$\frac{\ln\frac{1}{2}}{1690} = K$$

$$100 = 150e^{\frac{\ln 1/2}{1690}t}$$

$$\ln\left(\frac{100}{150}\right) \div \left(\frac{\ln\frac{1}{2}}{1690}\right) = t$$

$$t \cong 989 \text{ years}$$

3) The temperature (in degrees Fahrenheit) of a dead body that has been cooling in a room set at 70° is measured as 88°. One hour later, the body temperature is 87.5°. How long (to the nearest hour) before the first measurement was the time of death, assuming that the body temperature of the deceased at the time of death was 98.6°. Assume the cooling follows Newton's Law of Cooling:

$U = T + (U_0 - T)e^{kt}$.

$$87.5 = 70 + (88-70)e^{1K}$$

$$17.5 = 18e^{1K}$$

$$\ln\left(\frac{17.5}{18}\right) = K$$

$$-0.0282 = K$$

$$88 = 70 + (98.6-70)e^{-0.0282t}$$

$$18 = 28.6e^{-0.0282t}$$

$$\ln\left(\frac{18}{28.6}\right) = -0.0282t$$

$$t = 16.4 \text{ HRS}$$

4) A culture of bacteria obeys the law of uninhibited growth. If 140,000 bacteria are present initially and there are 609,000 after 6 hours, how long will it take for the population to reach one million?

$$609000 = 140000e^{6K}$$

$$K = \frac{\ln\left(\frac{609000}{140000}\right)}{6}$$

$$\cancel{1000000} = \cancel{140000}e^{\frac{\ln\left(\frac{609000}{140000}\right)}{6} * t}$$

$$\frac{\ln\left(\frac{100}{14}\right)}{\left(\frac{\ln\left(\frac{609000}{140000}\right)}{6}\right)} = t \Rightarrow t = 8.024 \text{ HRS}$$

5) The value of a particular investment follows a pattern of exponential growth. In the year 2000, you invested money in a money market account. The value of your investment t years after 2000 is given by the exponential growth model $A = 2700e^{0.057t}$. When will the account be worth \$3801?

$$3801 = 2700\, e^{0.057t}$$

$$t = \frac{\ln\left(\frac{3801}{2700}\right)}{0.057} \cong 6$$

$$2000 + 6 = \boxed{2006}$$

Solve the exponential equation. Use a calculator to obtain a decimal approximation, correct to two decimal places, for the solution.

6) $e^{5x} = 8$

$\ln e^{5x} = \ln 8 \Rightarrow 5x = \ln 8 \Rightarrow x = \frac{\ln 8}{5}$

$\boxed{x = 0.42}$

7) $e^{x+4} = 2$

$\cancel{\ln e}^{x+4} = \ln 2$

$x + 4 = \ln 2 \Rightarrow \boxed{x = -3.31}$

Solve the equation.

8) $\log_{15}(x - 30) = 3 - \log_{15} x$

$\log_{15}(x-30) + \log_{15} x = 3$

$\log_{15} x(x-30) = 3$

$x^2 - 30x = 3375$

$x^2 - 30x - 3375 = 0$

$(x - 75)(x + 45) = 0$

$\boxed{x = 75}$ $\cancel{x = -45}$

9) $\log_4(x + 5) + \log_4(x - 1) = 2$

$(x+5)(x-1) = 16$

$x^2 + 4x - 5 = 16$

$x^2 + 4x - 21 = 0$

$(x+7)(x-3) = 0$

$\cancel{x = -7}$ $\boxed{x = 3}$

10) $3^{2x} + 3^x - 6 = 0$

$(3^x + 3)(3^x - 2) = 0$

$3^x = -3$ $\quad 3^x = 2 \Rightarrow x\ln 3 = \ln 2$

No sln

$\boxed{x = \frac{\ln 2}{\ln 3}}$

Professor Katiraie MA180 Quiz 5 Form B Fall 2007

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Name Solution

Use a calculator to find the approximate value of the expression rounded to two decimal places.

1) $\cot \frac{\pi}{10}$ $= \frac{1}{\tan(\frac{\pi}{10})} = \boxed{3.08}$

Find the exact value of the indicated trigonometric function of θ.

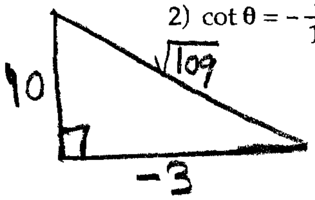
2) $\cot \theta = -\frac{3}{10}$, $\cos \theta < 0$ Find csc θ.

Quad II

$a^2 + b^2 = c^2$

$(-3)^2 + 10^2 = 109$

$c = \sqrt{109}$



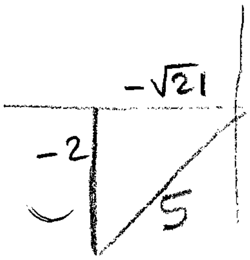
$\boxed{\csc\theta = \frac{\sqrt{109}}{10}}$

3) $\csc \theta = -\frac{5}{2}$, θ in quadrant III Find cot θ.

$x^2 + (-2)^2 = 25$

$x^2 = 21$

$x = -\sqrt{21}$



$\cot\theta = \frac{-\sqrt{21}}{-2}$

$\boxed{\cot\theta = \frac{\sqrt{21}}{2}}$

Solve the problem.

4) If f(θ) = sin θ and f(a) = $-\frac{1}{9}$, find the exact value of f(a) + f(a − 4π) + f(a − 2π).

$-\frac{1}{9} + -\frac{1}{9} + -\frac{1}{9} = -\frac{1}{3}$

5) A pendulum swings though an angle of 30° each second. If the pendulum is 60 inches long, how far does its tip move each second? If necessary, round the answer to two decimal places.

$30^\circ * \frac{1\pi}{180^\circ} = \frac{\pi}{6}$ Radians

$\boxed{S = r\theta = (60)(\frac{\pi}{6}) = 10\pi \text{ inches}}$

$\boxed{= 31.42 \text{ inches}}$

6) The force acting on a pendulum to bring it to its perpendicular resting point is called the restoring force. The restoring force F, in Newtons, acting on a string pendulum is given by the formula

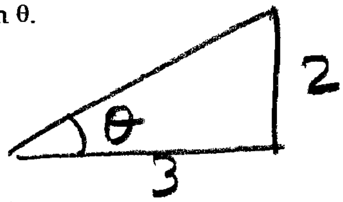
$$F = mg \sin\theta$$

where m is the mass in kilograms of the pendulum's bob, g = 9.8 meters per second per second is the acceleration due to gravity, and θ is angle at which the pendulum is displaced from the perpendicular. What is the value of the restoring force when m = 0.6 kilogram and $\theta = 45°$? If necessary, round the answer to the nearest tenth of a Newton.

$$F = (0.6)(9.8)\sin 45° = (0.6)(9.8)\frac{\sqrt{2}}{2} = \boxed{4.2 \text{ Newton}}$$

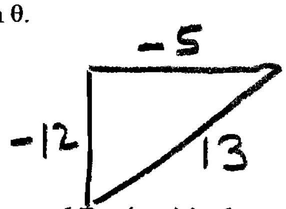
A point on the terminal side of an angle θ is given. Find the exact value of the indicated trigonometric function of θ.

7) (3, 2)   Find tan θ.



$$\boxed{\tan\theta = \frac{2}{3}}$$

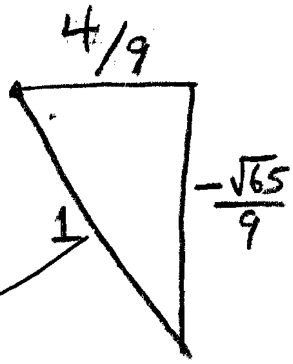
8) (−5, −12)   Find sin θ.



$$\boxed{\sin\theta = \frac{-12}{13}}$$

In the problem, t is a real number and P = (x, y) is the point on the unit circle that corresponds to t. Find the exact value of the indicated trigonometric function of t.

9) $\left(\frac{4}{9}, -\frac{\sqrt{65}}{9}\right)$ Find csc t.



$$\csc t = \frac{1}{-\frac{\sqrt{65}}{9}} = \frac{-9}{\sqrt{65}} = \boxed{\frac{-9\sqrt{65}}{65}}$$

Find the exact value. Do not use a calculator.

10) sec (−π)

$$\frac{1}{\cos(-\pi)} = \frac{1}{-1} = \boxed{-1}$$