Name Solution

Solve the equation on the interval $0 \le \theta < 2\pi$.

1) $\sqrt{2}\cos(2\theta) = 1$

$\cos 2\theta = \frac{1}{\sqrt{2}} \Rightarrow \cos 2\theta = \frac{\sqrt{2}}{2}$

$2\theta = \frac{\pi}{4} + 2n\pi \qquad 2\theta = \frac{7\pi}{4} + 2n\pi$

$\theta = \frac{\pi}{8} + n\pi \qquad \theta = \frac{7\pi}{8} + n\pi$

2) $\sin(4\theta) = \frac{\sqrt{3}}{2}$

$4\theta = \frac{\pi}{3} + 2n\pi \Rightarrow \theta = \frac{\pi}{12} + \frac{n\pi}{2}$

$4\theta = \frac{2\pi}{3} + 2n\pi \Rightarrow \theta = \frac{\pi}{6} + \frac{n\pi}{2}$

$\theta = \frac{9\pi}{8}, \frac{\pi}{8}$ or $\theta = \frac{7\pi}{8}, \frac{15\pi}{8}$

3) $\cot\left(2\theta - \frac{\pi}{2}\right) = 1$

$2\theta - \frac{\pi}{2} = \frac{\pi}{4} \Rightarrow 2\theta = \frac{3\pi}{4} + K\pi \Rightarrow \theta = \frac{3\pi}{8} + \frac{K\pi}{2}$

$2\theta - \frac{\pi}{2} = \frac{5\pi}{4} \Rightarrow 2\theta = \frac{7\pi}{4} + K\pi \Rightarrow \theta = \frac{7\pi}{8} + \frac{K\pi}{2}$

Solve the equation. Give a general formula for all the solutions.

4) $\csc\frac{\theta}{3} = \frac{2\sqrt{3}}{3}$ $\qquad \sin\theta = \frac{3}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{3\sqrt{3}}{6} = \frac{\sqrt{3}}{2} \qquad \sin\frac{\theta}{3} = \frac{\sqrt{3}}{2}$

$\frac{\theta}{3} = \frac{\pi}{3} + 2n\pi \Rightarrow \theta = \pi + 6n\pi$

$\frac{\theta}{3} = \frac{2\pi}{3} + 2n\pi \Rightarrow \theta = 2\pi + 6n\pi$

Solve the equation on the interval $0 \le \theta < 2\pi$.

5) $\cos^2\theta + 2\cos\theta + 1 = 0$

$(\cos\theta + 1)(\cos\theta + 1) = 0$

$\cos\theta = -1 \qquad \theta = \pi$

6) $2\sin^2\theta = \sin\theta$

$2\sin^2\theta - \sin\theta = 0$

$\sin\theta(2\sin\theta - 1) = 0$

$\sin\theta = 0 \qquad 2\sin\theta = +1$

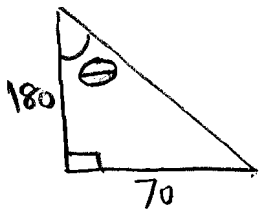
$\theta = 0, \pi \qquad \sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

Solutions $\left(0, \pi, \frac{\pi}{6}, \frac{5\pi}{6}\right)$

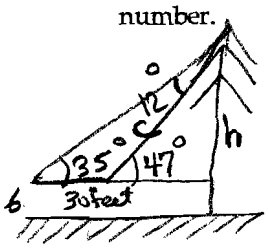
Solve the problem.

7) A building 180 feet tall casts a 70 foot long shadow. If a person looks down from the top of the building, what is the measure of the angle between the end of the shadow and the vertical side of the building (to the nearest degree)? (Assume the person's eyes are level with the top of the building.)



$\tan\theta = \frac{70}{180}$ $\quad \theta = \tan^{-1}\left(\frac{70}{180}\right) = 21.25^\circ$

8) John (whose line of sight is 6 ft above horizontal) is trying to estimate the height of a tall oak tree. He first measures the angle of elevation from where he is standing as 35°. He walks 30 feet closer to the tree and finds that the angle of elevation has increased by 12°. Estimate the height of the tree rounded to the nearest whole number.

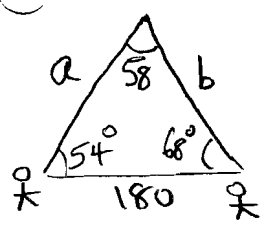


$\frac{\sin 35^\circ}{c} = \frac{\sin 12^\circ}{30} \Rightarrow c = 82.76$ feet

$\sin 47^\circ = \frac{h}{82.76} \Rightarrow h = 82.76 \sin 47^\circ = 60.53$ feet

$H = 6 + 60.53 = 66.53$ feet

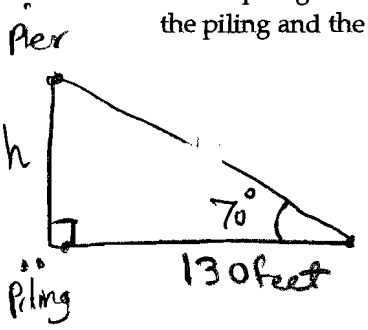
9) Two surveyors 180 meters apart on the same side of a river measure their respective angles to a point between them on the other side of the river and obtain 54° and 68°. How far from the point (line-of-sight distance) is each surveyor? Round your answer to the nearest 0.1 meter.



$\frac{\sin 58}{180} = \frac{\sin 54}{b} \Rightarrow b = 171.71$ meters

$\frac{\sin 68}{a} = \frac{\sin 58}{180} \Rightarrow a = 196.8$ meters

10) A surveyor is measuring the distance across a small lake. He has set up his transit on one side of the lake 130 feet from a piling that is directly across from a pier on the other side of the lake. From his transit, the angle between the piling and the pier is 70°. What is the distance between the piling and the pier to the nearest foot?



$\tan 70^\circ = \frac{h}{130}$

$h = 130 \tan 70^\circ = 357.17$ feet