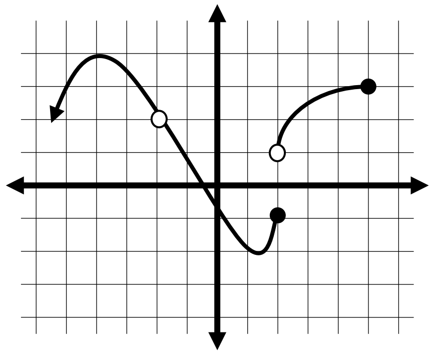
MATH 165 Precalculus **Spring 2018** **MATH 165 FINAL REVIEW**

General Directions: When asked for EXACT SOLUTIONS, leave answers in fractional or radical form - not decimal form. That is, leave numbers like $\frac{2}{3}$, $\sqrt{3}$, π, and *e* as part of your answer.

1. State the domain of each of the following functions.

(a) $f(x)=\dfrac{\sqrt{7x+3}}{x-1}$ (b) $g(x)=\dfrac{2}{\sqrt{4-x^2}}$ (c) $h(x)=\dfrac{x^2+4x+5}{2x^2+13x-7}$

(d) $k(x)=\ln\left(\dfrac{3x-2}{x+4}\right)$ (e)  (f) $m(x)=\dfrac{\sin x}{e^x}$

Assume x-scale and y-scale equal 1

2. The table on the right, generated on a graphing calculator, shows some points on the graphs of two functions.

Use the table to answer these questions:

(a) Where is y_1's *x*-intercept?

(b) Where is y_2's *y*-intercept?

(c) Give the coordinates of each point where y1 and y2 intersect.

x	y_1	y_2
–2	0	6
–1	1	1
0	2	–1
1	3	–3
2	4	–2
3	5	1
4	6	6

3. Use the graph of function *f* at the right to answer the followi

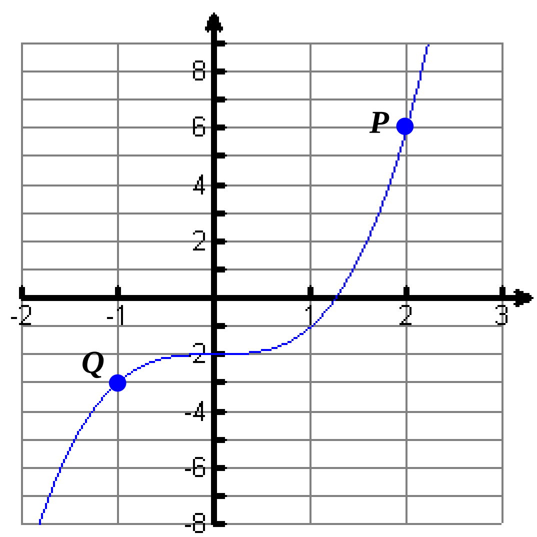
(a) What are the coordinates of the points P and Q?

(b) Evaluate *f(1)*.

(c) Evaluate *f*(0).

(d) Solve $f(x)=6$ for *x*

(e) Estimate the value *x* when *f(x)* = 0.



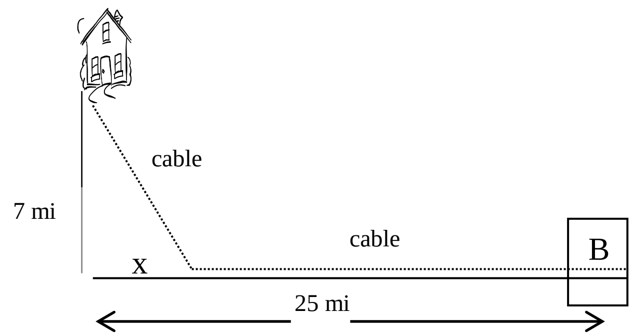
4. If $f(x) = 3x^2 + 5x - 8$, evaluate and simplify $\dfrac{f(x+h) - f(x)}{h}$, $h \neq 0$.

5. Given $f(x) = 35x^4 - 25x^2$, use your graphing calculator to approximate the following. Round your answers to three decimal places.

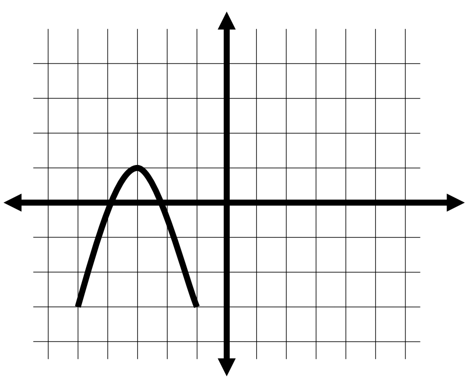
   (a) Find any local minima/maxima.

   (b) Find intervals where *f* is increasing and/or decreasing. Use interval notation.

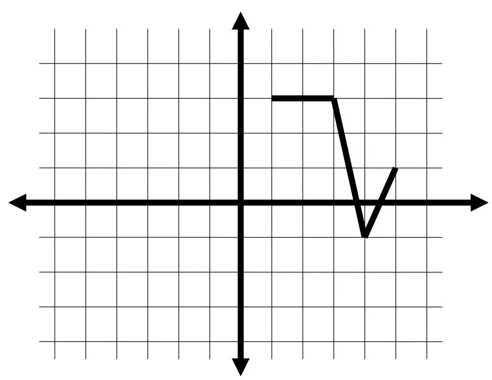
6. A media company is going to install cable from a house to their connection box B. The house is located at one end of a driveway 7 miles back from a road (see diagram). The other end of the driveway and the nearest connection box are on the same road, 25 miles apart. The cost of installing the cable is \$656 per mile off the road and \$375 per mile along the road. Let *x* be the distance from where the driveway meets the road to where the cable comes to the road. Develop a function C(*x*) that expresses the total installation cost as a function of *x*. Now use your calculator to graph C. Use the graph to determine the value of *x* that will produce the minimum cost. Round to the nearest thousandth of a mile. (*Tip: use a window [0, 20, 1] x [12,000 , 20,000 , 1000]*.) State the minimum cost for that installation, rounded to the nearest cent.



7. (a) A partial graph of of *f* is shown. Complete the graph if $f(-x) = f(x)$



   (b) A partial graph of of *g* is shown. Complete the graph if $g(-x) = -g(x)$.



8. On the right is a table of some values of a function *f*.
Answer these questions about *f*:

(a) Tell whether the function is even, odd, or neither. Explain how you know this.

(b) List each *x*-intercept.

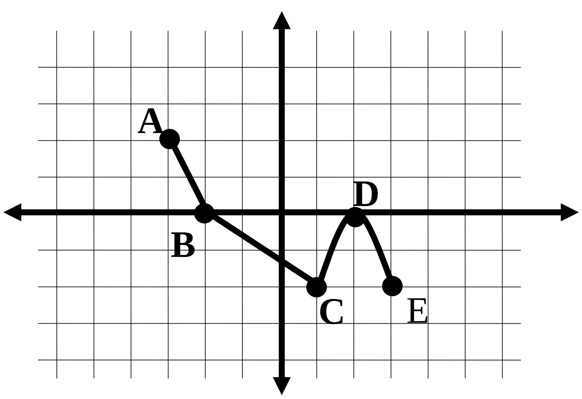
(c) List each *y*-intercept.

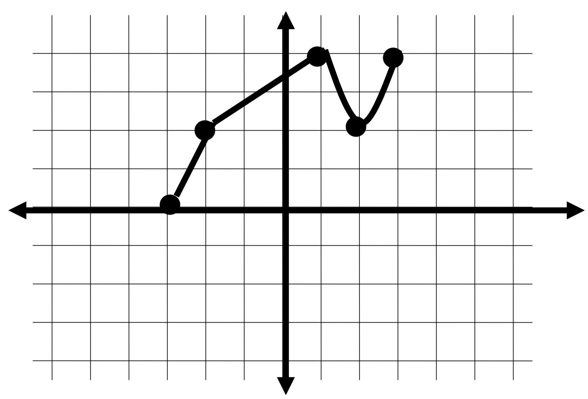
x	y
–3	–7
–2	0
0	3
1	5
3	9
5	0

9. The complete graph of *f(x)* is shown at the left. On the graphs, xscl=1.

(a) **Sketch** the graph of $-\frac{1}{2}f(x-3)$.

(b) In terms of *f(x)*, what is the formula for the graph shown below?

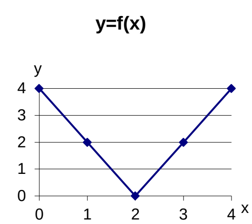




10. Use the functions *f* and *g* to evaluate the following.

(a) $(f+g)(3)$ (b) $(f-g)(1)$ (c) $\left(\frac{f}{g}\right)(2)$

(d) $(fg)(4)$ (e) $(f\circ g)(2)$ (f) $(g\circ f)(2)$



x	$y=g(x)$
0	1
1	-2
2	0
3	-4
4	-3

11. If $f(x)=x^2+1$, and $g(x)=\sqrt{x-5}$, find and simplify

(a) $f(g(f(3)))$ (b) $f(g(x))$ and state the domain (c) $(g\circ f)(x)$ and state the domain

(d) $g^{-1}(x)$ and state its domain (e) $(f\circ g^{-1})(2)$

12. Describe the behavior of the power function $f(x) = ax^{88}$, for a < 0 as regards each of (a)-(d) below

If there is insufficient information to decide, then answer "*cannot be determined*"

(a) symmetries (b) domain (c) range

(d) end behaviors: As $x \to \infty$, $f(x) \to$ _______ Which end is this? *left/right*

As $x \to -\infty$, $f(x) \to$ ______ Which end is this? *left/right*

13. For each of these functions: I. $f(x) = 5x^3 - 17x^2 + 4x + 14$

II. $g(x) = 3x^4 - 20x^3 + 51x^2 - 86x + 40$.

(a) Find the zeros and identify them as rational, irrational, or non-real complex.

(b) Factor the polynomial completely over the complex numbers.

14. The data below represents the number of students enrolled in grades 9 through 12 in private institutions.

Year	**Enrollment in 1000's**
1975	1300
1980	1339
1981	1400
1982	1400
1983	1400
1984	1400
1985	1362
1986	1336
1987	1247
1988	1206
1989	1193
1990	1137
1991	1125
1992	1163
1993	1191
1994	1236
1995	1260
1996	1297

(a) Using a graphing utility, draw a scatter diagram of the data. Determine if the data appears to be linear, quadratic, cubic, exponential, logarithmic or sinusoidal. (Let 1975 correspond to $t = 0$)

(b) Using a graphing utility, find the cubic function of best fit and graph.

(c) Use the function found in part (b) to predict the number of students enrolled in private institutions for grades 9 to 12 in the year 2000. Round your answer to the nearest thousand students.

(d) Use the function found in part (b) to predict the year in which enrollment in private institutions for grades 9 through 12 will reach 2 ½ million students.

(e) The function found in part (b) does approximate the data quite nicely for the years that we have information. Explain why, for this situation, a cubic function might not be a good model over the long term. Include the phrase "end behavior" in your discussion.

15. Following are some tables for the same rational function *f*.

TABLE A	
x	*f(x)*
-4	8.25
-3.1	3104.25
-3.01	3010004.25
-3.001	3001000004

TABLE B	
x	*f(x)*
-2	2.25
-2.9	-2895.75
-2.99	-2989995.75
-2.999	-2998999996

TABLE C	
x	*f(x)*
40	4.2505031
400	4.250006111
4000	4.250000062
40000	4.250000001

TABLE D	
x	*f(x)*
-30	4.251524158
-300	4.250011451
-3000	4.250000111
-30000	4.250000001

(a) Name each table that suggests the existence of a horizontal asymptote (end behavior) .

(b) Name each table that suggests the existence of a vertical asymptote (local behavior).

(c) Use the tables to complete each of the following statements:

i. As $x \to \infty$, $f(x) \to$ _______

ii. As $x \to -\infty$, $f(x) \to$ _______

iii. As $x \to -3$ from the right side, $f(x) \to$ _______

iv. As $x \to -3$ from the left side, $f(x) \to$ ___________

v. Write the equation of any horizontal asymptotes.____________
(*Write "none" for your answer if there isn't one.)*

vi. Write the equation of any vertical asymptotes.______________
(*Write "none" for your answer if there isn't one.)*

16. Consider the rational function $h(x) = \dfrac{(3x+2)(x-2)}{x(x-2)}$:

(a) State the domain of the function.

(b) Complete this statement: As $x \to -\infty$, $h(x) \to$ ______ .

(c) Write the equation of any vertical asymptotes.

(d) Write the equation of any horizontal asymptotes.

(e) Write the coordinates of any holes.

(f) List any *x*-intercepts.

(g) Graph and label any asymptotes, intercepts, and holes.

(h) State the range of this function.

17. The Parks and Wildlife commission introduces 80,000 fish into a large man-made lake. The population of the fish, in thousands, is given by $N(t) = \frac{20(4+3t)}{1+0.05t}, 0 \le t$, where t is the time in years.

    (a) Approximate the population when t is 5, 10, and 25.

    (b) What is the limiting number of fish in the lake as time increase?

18. Find the exact solution of $f(x) = \frac{x(3x-5)}{(7x+3)(x+4)} \ge 0$ .Give the exact values for answer(s) using interval notation.

19. (a) Solve for x: $\frac{1}{32} = 2^{1-4x}$ (b) Simplify: $e^{2\ln(x)+\ln 5}$

20. The value of an automobile depreciates. It is originally worth $40,000, but then it loses one-tenth of its value every year.
    (a) How much is it worth at the end of the first year?
    (b) How much is it worth after 3 years?
    (c) How much is it worth after t years?
    (d) How many years would pass before it was worth only half of its original value? *(This would be its "half-life".)* Round this value to the nearest tenth of a year.

21. A function f is defined by the table below:
    Complete a table for f^{-1}

x	-2	-1	0	1	2	3
$f(x)$	-3	-1	0	6	5	7

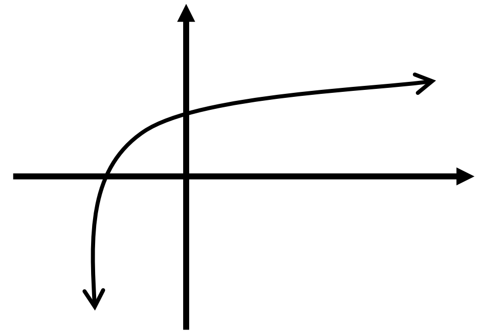
x						
f^{-1}						

22. Graph $g(x) = 1 - 5e^{x-3}$. (a) Find all intercepts and asymptotes. Round to two decimal places.

    (b) State the domain and range of the function.

23. If $y = f(x) = \frac{1}{3}\log(5x)$, find the inverse function $f^{-1}(x)$

24. The unlabeled sketch at right shows the graph of $y = \log_3(x+3)$. The sketch correctly shows the shape of the graph, but it is not to scale.



(a) Find the equation of any asymptotes and sketch.
(b) Find the *x*-intercept and label.
(c) Find the *y*-intercept and label.

25. Solve for *x*: $\log_5(8-18x) - 2\log_5 x = 1$

26. Approximate the value of y to three decimal places: $y = \log_5 40$

27. Exponential Growth Applications:
Assume the function $P(t) = Ce^{kt}$ describes the population P of a certain country where the time is measured in *t* years.

(a) What is the growth rate constant (*k*) if the population has tripled in 23 years? Approximate *k* to a tenth of a percent.
(b) Use the *exact* value of *k* to find the population in 50 years if the initial population was 10,000,

28. Given $\sin\theta = 4/7$ and $\pi/2 < \theta < \pi$, find the EXACT values of the following:

(a) $\tan\theta$ (b) $\sec\theta$ (c) $\cos\left(\theta + \frac{\pi}{2}\right)$ (d) $\csc(2\theta)$

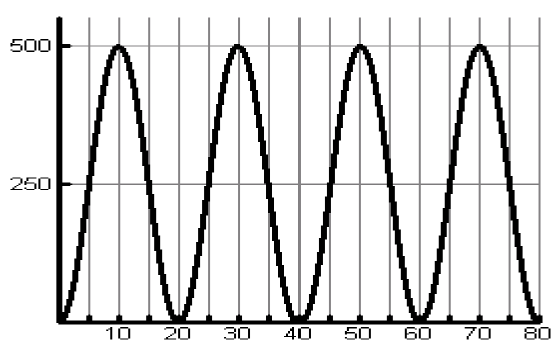
29. Graph two full periods of each of the following functions.

$F(x) = -3\cos(4x) + 5$ $G(t) = \frac{1}{3}\sin(2\pi t - \pi)$

For each function determine the following:

(a) period (b) range (c) amplitude (d) phase shift

30. Find a function of the form $f(x) = A\cos(Bx) + D$ for the graph below.



31. Household electrical power in the US is provided in the form of alternating current. Typically voltage fluctuates smoothly between +155.6 volts and –155.6 volts. The voltage cycle 60 times per second. Use a cosine function to model the alternating voltage.

32. Verify that $\frac{sin\emptyset}{1+sin\emptyset} - \frac{sin\emptyset}{1-sin\emptyset} = -2\tan^2\emptyset$

33. Use trigonometric identities to find the exact value of $\sin(\alpha+\beta)$ and $\cos(\alpha+\beta)$, given:

$$\tan\alpha = \frac{5}{12},\ \pi < \alpha < \frac{3\pi}{2}; \sin\beta = -\frac{1}{2}, \pi < \beta < \frac{3\pi}{2}$$

34. Use trigonometric identities to find the exact value of tan(2θ), where (–3, 4) is a point on the terminal side of θ.

35. Find, to the nearest degree, all angles $\theta,\ 0 \le \theta \le 360°$, such that $\sin(\theta) = -0.81$

36. Find an algebraic expression in *x* (not involving trig functions) that is equivalent to $\csc\left(\tan^{-1}\left(3/x\right)\right)$ for *x* >0.

37. Given $\cos 2x = -\sin x$:

(a) Solve the equation algebraically for the exact value of the solution(s) on the interval [0, 2π).

(b) Verify the answer(s) in part (a) using the ZERO or INTERSECT features of your graphing calculator. Round answers to the nearest tenth of a radian.

38. Write TRUE or FALSE in the blank before each statement. [*Recall that if a statement in mathematics is not always true, then it is considered false*.]
t
If it is false, write a *corrected* true statement in the blank.
If it is already true, write “Identity” in the blank.

(a)____ sin (A + B) = sin(A) + sin (B) ____

(b)____ $\sin^2(A) = (\sin A)^2$ ____

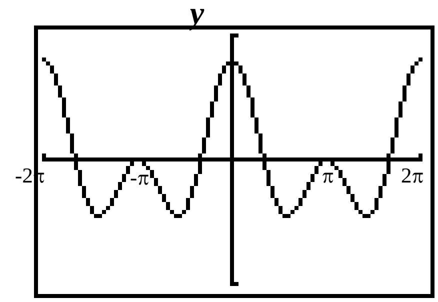
(c)____ (sin A + cos A) ^{2} = $\sin^2$A + $\cos^2$A ____

(d)____ cos (A – B) = cos A – cos B ____

(e)____ sin (2A) = 2 sin A ____

(f)____ $\tan^{-1}(A) = \frac{1}{\tan(A)}$ ____

39. The graph at right shows two cycles of the graph of
$y = 2\cos^2 t + \cos t - 1$.
Algebraically find all zeros to the equation over the domain of all reals. Express your answer in terms of π.



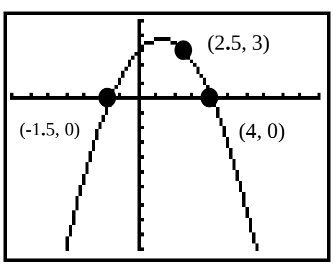
40. In traveling across flat land, you notice a mountain directly in front of you. The angle of elevation [to the peak] is 3.5°. After you drive 13 miles closer to the mountain, the angle of elevation is 9°. Approximate the height of the mountain to the nearest foot. (5280 feet = 1 mile)

41. Sketch the curve described by the parametric equations by hand. Verify using a graphing calculator :
$x = \cos^3 t$, $y = \sin^3 t$; $0 \le t \le 2\pi$

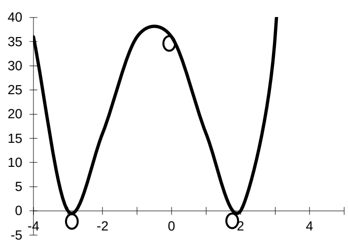
Does this curve represent *y* as a function of *x*?

42. This question concerns the three graphs.
(a) Find possible formulas for the three functions.

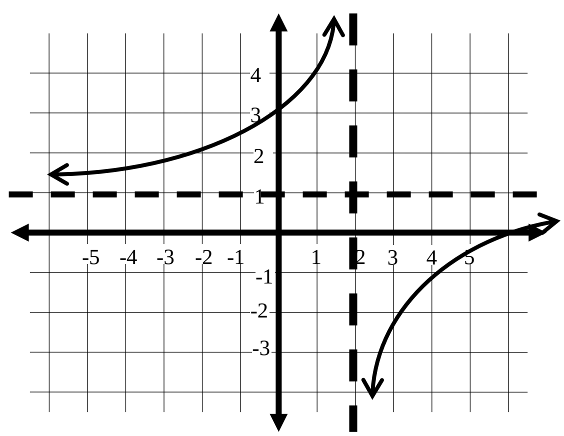
(i) *f*



(ii) *g*



(iii) *h*



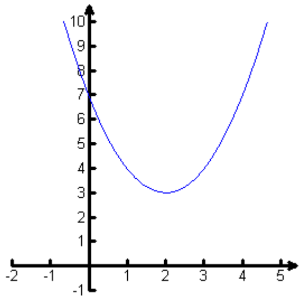
(b) Estimate the average rate of change for function *h* between $x = -2$ and $x = 0$?

(c) Where is function *h* increasing faster, between $x = -4$ and $x = -2$, or between $x = -2$ and $x = 0$?

43. For each function, select its most specific function type from the name bank below.
Name Bank = {Exponential, linear, logarithmic, polynomial, quadratic, rational, sinusoidal}

a. $f(x) = \dfrac{5}{x+3}$

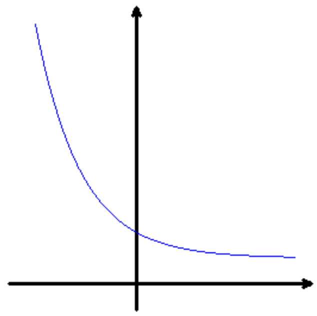
b.



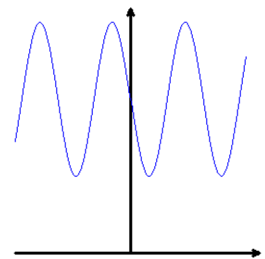
c. $f(x) = 5x^3$

d. $f(x) = \dfrac{\sqrt{x}}{x+2}$

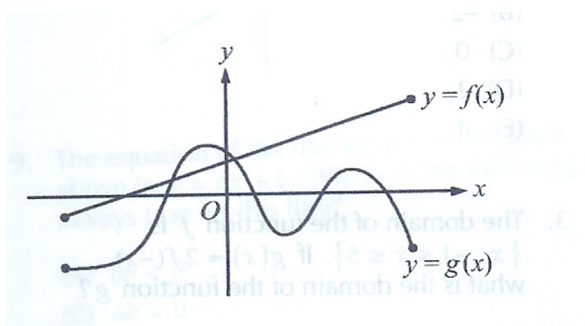
e.



f.



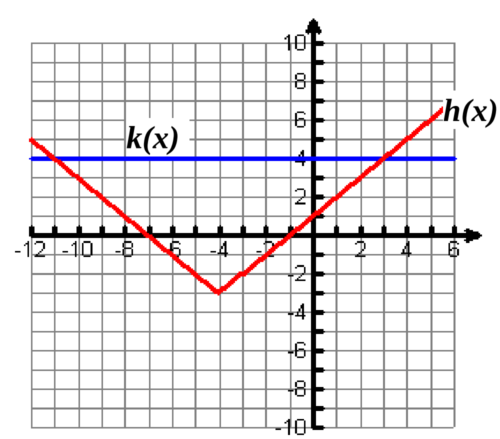
44. The figure below shows the complete graphs of the functions *f* and *g*. Based on the graphs, give the number of solutions to each of the following equations
a) $f(x) - g(x) = 0$ b) *f(x) = 0* c) *g(x) = 0*



45. Use the graphs of the functions *h* and *k* below to answer the followin

a. For what value(s) of *x* is $h(x) \geq k(x)$?

b. Solve the inequality $h(x) < k(x)$



46. Give the coordinates of the center and radius of the following circle $x^2 + y^2 - 16x + 4y - 3 = 0$

47. a) Find the average rate of change of the function $f(x) = 3x^2 + 6x - 5$ from -1 to 1.
    b) Find an equation of the secant line containing (-1, *f(-1)) and (1, f(1))*

48. Write as a sum or difference of logarithms. Express powers as factors. $\ln(\frac{x^2e^3}{\sqrt{1+x^2}})$

49. Express *y* as a function of *x*. The constant C is a positive number. $\ln(y-3) = -4x + lnC$

50. Graph the function $f(x) = \begin{pmatrix} (x-1)^2\ if & 0 \le x < 2 \\ -2x + 10\ if & 2 \le x \le 6 \end{pmatrix}$

MA 165 COURSE REVIEW - Answer Key

1. a) $\{x | x \ge -\frac{3}{7} \text{ and } x \ne 1\}$
   b) $\{x | -2 < x < 2\}$
   c) $\left\{x \middle| x \ne -7 \text{ or } x \ne \frac{1}{2}\right\}$
   d) $(-\infty, -4) \cup (\frac{2}{3}, \infty)$
   e) $\{x | x < -2 \text{ or } -2 < x \le 5\}$
   f) $(-\infty, \infty)$

2. a) at $x = -2$ b) at $y = -1$ c) (-1,1) and (4,6)

3. a) P(2,6) Q(-1,-3)
   b) -1
   c) -2
   d) 2
   e) $x \approx 1.25$

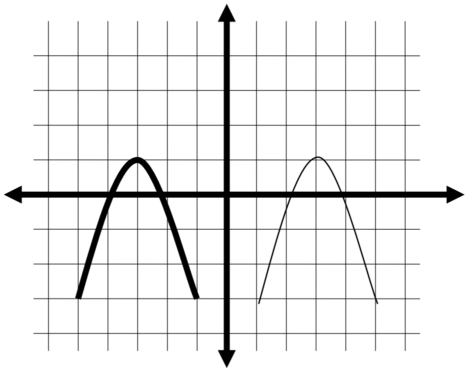
4. $6x + 3h + 5$

5. a) local minima: (-0.598, -4.464) and (0.598, -4.464)
   local maximum: (0, 0)

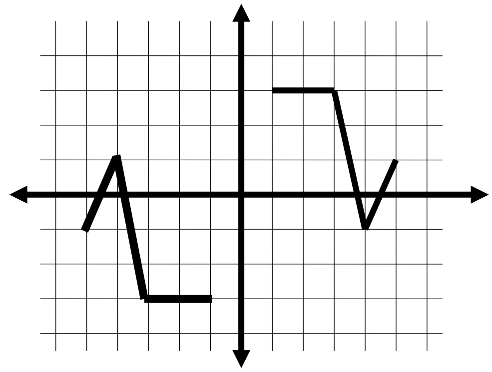
   b) Decreasing: $(-\infty, -0.598) \cup (0, 0.598)$
   Increasing: $(-0.598, 0) \cup (0.598, \infty)$

6. $C(x) = 656\sqrt{x^2 + 49} + 375(25 - x)$
   $13,142.74

7. a)



b)



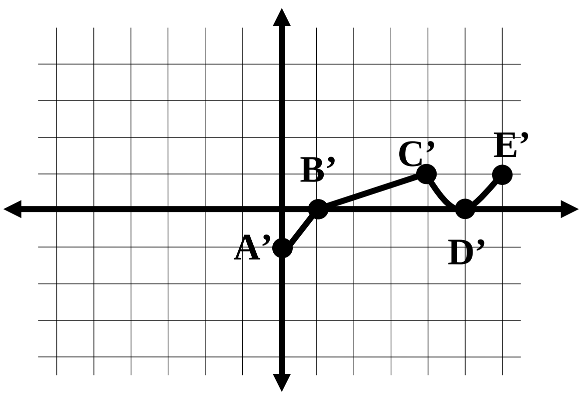
8. a) f is not odd because $f(-x) \neq -f(x)$ for all x

   f is not even because $f(-x) \neq f(x)$ for all x

   b) (-2,0) (5,0)

   c) (0,3)

9. a)



Translated points

A' (0,-1)
B' (1,0)
C' (4,1)
D' (5,0)
E' (6,1)

   b) $-f(x)+2$

10. a) -2 b) 4 c) undefined d) -12 e) 4 f) 1

11. a) 6

   b) $x-4, \quad x \geq 5$

   c) $\sqrt{x^2-4}, \quad x \leq -2$ or $x \geq 2$

   d) $g^{-1}(x) = x^2 + 5$ domain: $\{x | x \geq 0\}$

   e) 82

12. a) symmetric with respect to the *y*-axis

   b) all reals

   c) $\{y | y \leq 0\}$

   d) as $x \to \infty$, $f(x) \to -\infty$ right end

   as $x \to -\infty$, $f(x) \to -\infty$ left end

13\. I. a) rational: $x = \frac{7}{5}$; irrational: $x = 1 \pm \sqrt{3}$

b) $f(x) = 5\left(x - \frac{7}{5}\right)\left(x - 1 + \sqrt{3}\right)\left(x - 1 - \sqrt{3}\right)$

II. a) rational: $x = \frac{2}{3}, 4$; nonreal complex: $x = 1 \pm 2i$

b) $g(x) = 3\left(x - \frac{2}{3}\right)(x-4)(x-1-2i)(x-1+2i)$

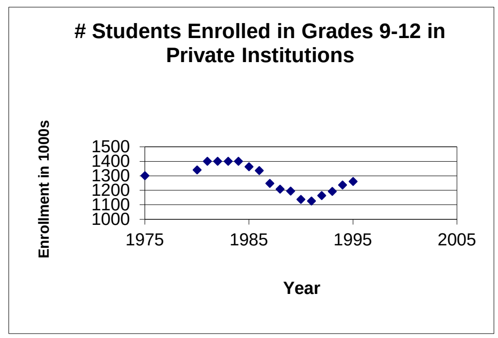
14\. a) The data appears to fit a cubic or a sinusoidal model.

b) If you elect to use year 1975 as $x = 0$, and rounding coefficients to 6 decimal places, the regression function is: $f(x) = 0.307723x^3 - 9.790235x^2 + 71.198866x + 1277.177287$.

c) 1,746,000 students

d) The closest year is 2004.

e) Because the end behavior of this function is increasing as x→∞, the enrollment at these institutions would increase without bounds as time goes by. Realistically, this is unlikely to happen.



15\. a) C, D

b) A, B

c) (i) as $x \to \infty$, $f(x) \to 4.25$

(ii) as $x \to -\infty$, $f(x) \to 4.25$

(iii) as $x \to -3^+$, $f(x) \to -\infty$

(iv) as $x \to -3^-$, $f(x) \to \infty$

(v) $y = 4.25$

(vi) $x = -3$

16\. a) $\{x \mid x \neq 0 \text{ and } x \neq 2\}$

b) as $x \to -\infty$, $h(x) \to 3$

c) $x = 0$

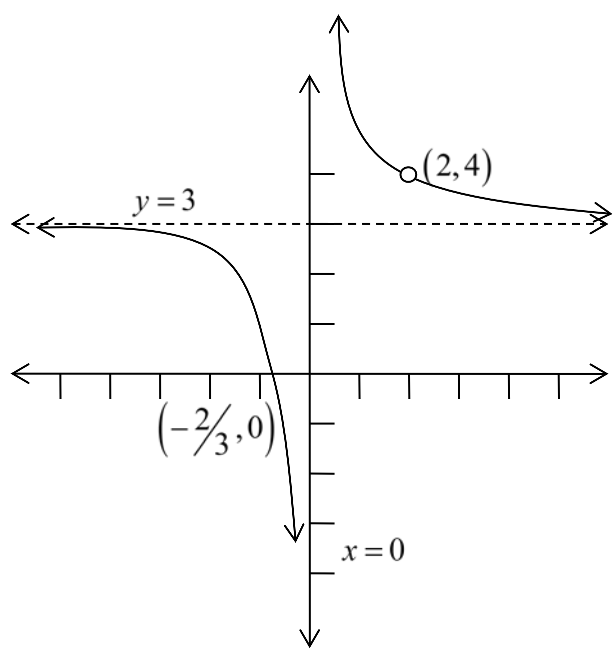
d) $y = 3$

e) $(2,4)$

f) $\left(-\frac{2}{3}, 0\right)$

g) See graph at right

h) $\{y \mid y \neq 3 \text{ and } y \neq 4\}$



17. a) $t = 5$ 304,000 fish
    $t = 10$ 453,333 fish
    $t = 25$ 702,222 fish

    b) 1,200,000 fish

18. $(-\infty, -4) \cup \left(\frac{-3}{7},\ 0\right] \cup \left[\frac{5}{3},\ \infty\right)$

19. a) $x = \frac{3}{2}$ b) $5x^2$

20. a) \$36,000 c) $40,000(0.9)^t$ dollars
    b) \$29,160 d) 6.6 years

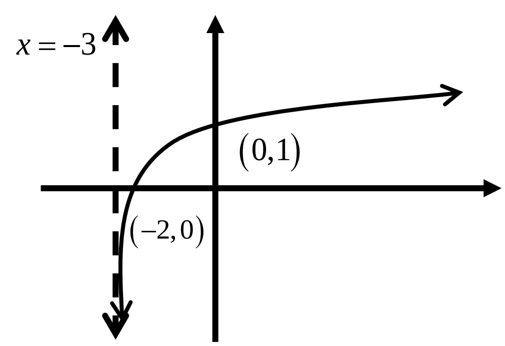
21.

x	-3	-1	0	6	5	7
$f^{-1}(x)$	-2	-1	0	1	2	3

22. a) (1.39,0), (0,0.75) $y = 1$ b) domain: all reals
    range: $\{y \mid y < 1\}$

23. $f^{-1}(x) = \frac{10^{3x}}{5}$ or $f^{-1}(x) = 0.2(10)^{3x}$

24. a) No horizontal asymptote
    vertical asymptote at $x = -3$
    b) x-intercept at -2
    c) y-intercept at 1



25. $x = 0.4 = \frac{2}{5}$

26. 2.292

27. a) 4.8% b) 108,948

28. a) $\frac{-4}{\sqrt{33}}$ b) $\frac{-7}{\sqrt{33}}$ c) $-\frac{4}{7}$ d) $-\frac{49}{8\sqrt{33}} = -\frac{49\sqrt{33}}{264}$

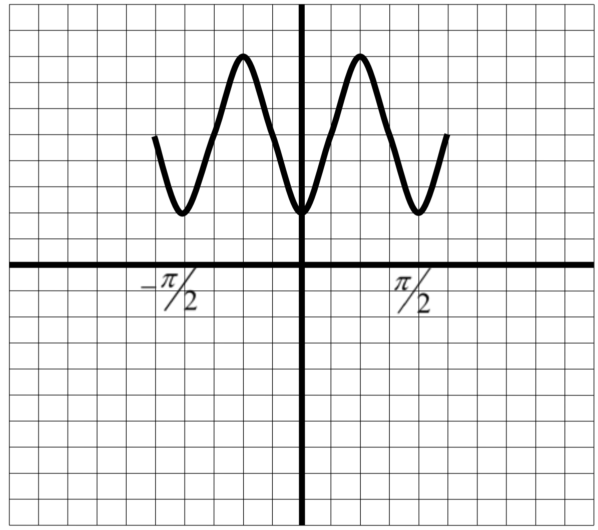
29. F

a) $\dfrac{\pi}{2}$

b) $[2,8]$

c) 3

d) none



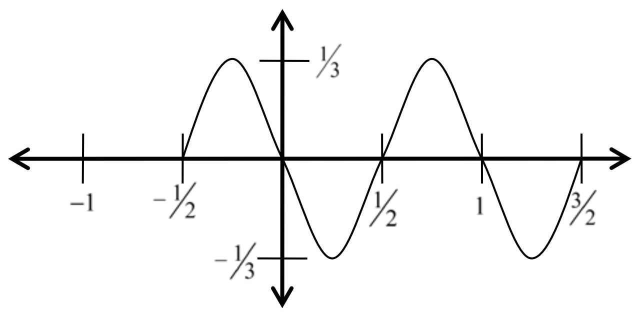
G

a) 1

b) $\left[-\dfrac{1}{3},\dfrac{1}{3}\right]$

c) $\dfrac{1}{3}$

d) $\dfrac{1}{2}$



30. $f(x) = -250\cos\left(\dfrac{\pi}{10}x\right) + 250$

31. $f(t) = 155.6\cos(120\pi t)$

32. solutions vary

33. a) $\dfrac{5\sqrt{3}+12}{26}$, $\dfrac{12\sqrt{3}-5}{26}$

34. $\dfrac{24}{7}$

35. 234°, 306°

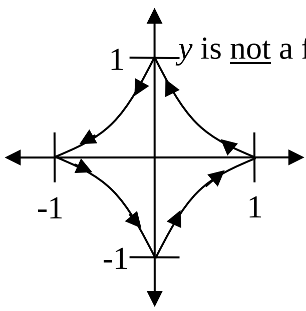
36. $\dfrac{\sqrt{x^2+9}}{3}$

37. a) $\dfrac{\pi}{2}$, $\dfrac{7\pi}{6}$, $\dfrac{11\pi}{6}$ b) 1.6, 3.7, 5.8

38. a) False $\sin(A+B)=\sin A\cos B+\cos A\sin B$
    b) True
    c) False $(\sin A+\cos A)^2=\sin^2 A+2\sin A\cos A+\cos^2 A=1+2\sin A\cos A$
    d) False $\cos(A-B)=\cos A\cos B+\sin A\sin B$
    e) False $\sin(2A)=2\sin A\cos A$
    f) False $\dfrac{1}{\tan A}=\cot A$

39. $\dfrac{\pi}{3}+2n\pi,\quad \dfrac{5\pi}{3}+2n\pi,\quad \pi+2n\pi$

40. 6839 feet

41. 

y is <u>not</u> a function of x

42. a.i.) $f(x)=-\dfrac{1}{2}\left(x+\dfrac{3}{2}\right)(x-4)$

a.ii.) $g(x)=\dfrac{35}{36}(x+3)^2(x-2)^2$

a.iii.) $h(x)=\dfrac{x-6}{x-2}$ OR $h(x)=\dfrac{-4}{x-2}+1$

42. b) The rate of change is ½ c) Function h is increasing faster between x = –2 and x =0 .

43. a) rational b) polynomial c) polynomial d) none e) exponential f) sinusoidal

44. a) two, b) one, c) four

45. a) $(-\infty,-11]\cup[3,\infty)$ b. $(-11,3)$ or $-11<x<3$

46. C(8, -2), radius = $\sqrt{71}=8.4$

47. a) 6; b) $y = 6x - 2$

48. $3+2lnx-\frac{1}{2}\ln(1+x^2)$

49. $y=Ce^{-4x}+3$