#32 $\dfrac{\csc\theta}{1+\csc\theta} - \dfrac{\csc\theta}{1-\csc\theta} = 2\sec^2\theta$

$$\frac{\csc\theta(1-\csc\theta) - \csc\theta(1+\csc\theta)}{1^2 - \csc^2\theta} =$$

$$= \frac{\cancel{\csc\theta} - \csc^2\theta - \cancel{\csc\theta} - \csc^2\theta}{1-\csc^2\theta}$$

$$= \frac{-2\csc^2\theta}{1-\csc^2\theta}$$

$$= \frac{-2\csc^2\theta}{-\cot^2\theta}$$

$$= 2\,\frac{1}{\cancel{\sin^2\theta}} \cdot \frac{\cancel{\sin^2\theta}}{\cos^2\theta}$$

$$= \frac{2}{\cos^2\theta} = \boxed{2\sec^2\theta}$$

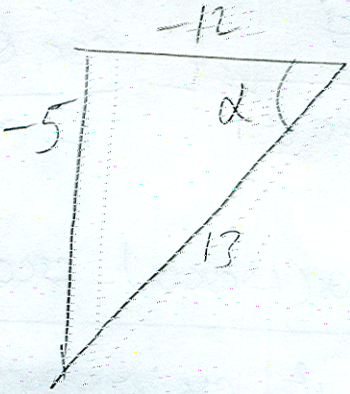
$\sin^2\theta + \cos^2\theta = 1$

$\dfrac{\sin^2\theta}{\sin^2\theta} + \dfrac{\cos^2\theta}{\sin^2\theta} = \dfrac{1}{\sin^2\theta}$

$1 + \cot^2\theta = \csc^2\theta$

$1 - \csc^2\theta = -\cot^2\theta$

#33 a) $\sin(\alpha+\beta)$



$\pi < \alpha < \frac{3\pi}{2}$

$\pi < \alpha < \frac{3\pi}{2}$

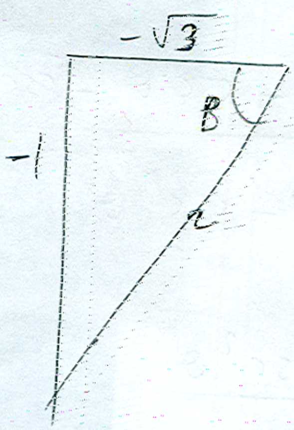
Quad 3

a) $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$= \frac{-5}{13}\,\frac{-\sqrt{3}}{2} + \frac{-12}{13}\,\frac{-1}{2}$

$= \frac{5\sqrt{13}}{26} + \frac{12}{26}$

$= \boxed{\frac{5\sqrt{13}+12}{26}}$

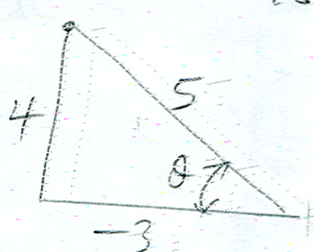


b) $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$= \frac{-12}{13}\,\frac{-\sqrt{3}}{2} - \frac{-5}{13}\,\frac{-1}{2}$

$= \frac{12\sqrt{3}}{26} - \frac{5}{26} = \boxed{\frac{12\sqrt{3}-5}{26}}$

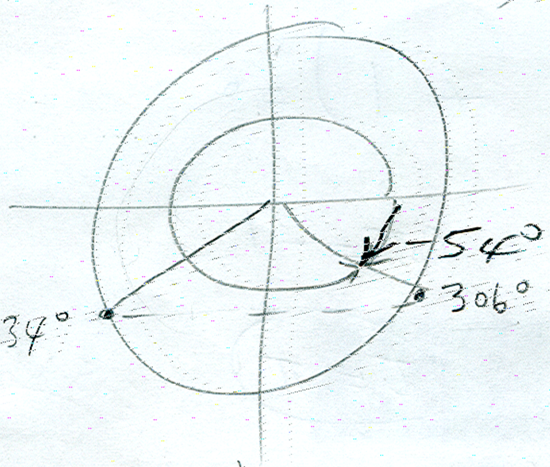
34) $\tan 2\theta$



$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\left(\frac{4}{-3}\right)}{1-\left(\frac{4}{-3}\right)^2} = \frac{\frac{8}{-3}}{1-\frac{16}{9}} = \frac{\frac{8}{-3}}{\frac{9-16}{9}} = \boxed{\frac{24}{7}}$

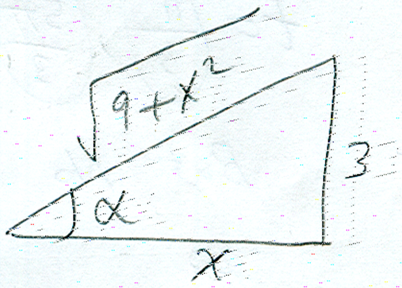
(#35) $\sin\theta = {}^{-}0.81$ $\qquad \theta = \cancel{-54^\circ}\ 306^\circ$

$\boxed{\theta = 306^\circ \text{ and } 234^\circ}$



$180^\circ + 54^\circ = 234^\circ$

(#36) $\csc\left(\tan^{-1}\left(\frac{3}{x}\right)\right)$ for $x > 0$



$$\Downarrow$$

$$\csc(\alpha) = \frac{1}{\sin\alpha} = \boxed{\frac{\sqrt{9+x^2}}{3}}$$

(#37) $\cos 2x = -\sin x$

$$\Downarrow$$

$$1 - 2\sin^2 x = -\sin x$$

$$2\sin^2 x - \sin x - 1 = 0$$

a) $(2\sin x + 1)(\sin x - 1) = 0$

$2\sin x = -1$ | $\sin x = 1$

$\sin x = \frac{-1}{2}$ | $\boxed{x = \frac{\pi}{2}}$ ✓

$\boxed{x = \frac{7\pi}{6}, \frac{11\pi}{6}}$ ✓

(b) 1.6, 3.7, 5.8 radians using graphing calculator

#39 $y = 2\cos^2 t + \cos t - 1$

$(2\cos t - 1)(\cos t + 1) = 0$

$\cos t = \frac{1}{2}$ $\qquad$ $\cos t = -1$

$t = \frac{\pi}{3}, \frac{5\pi}{3}$ $\qquad$ $t = \pi$

But No Restrictions

so

$$\begin{cases} \frac{\pi}{3} + 2n\pi \\ \frac{5\pi}{3} + 2n\pi \\ \pi + 2n\pi \end{cases} \qquad n = \ldots, -2, -1, 0, 1, 2, \ldots$$

#38

a) $\sin(A+B) = \sin A \cos B + \cos A \sin B$ False

b) TRUE

c) False $(\sin A + \cos A)^2 = \sin^2 A + \cos^2 A$

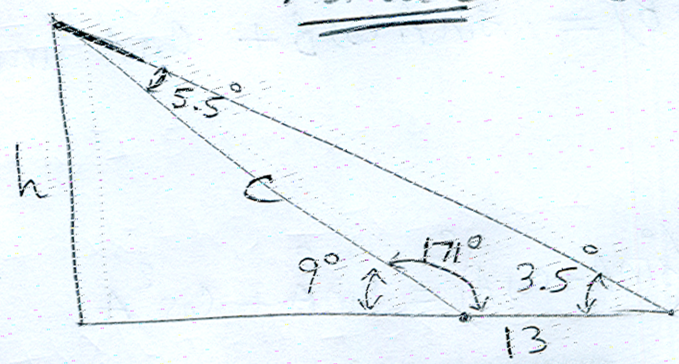
d) False $\cos(A-B) = \cos A \cos B + \sin A \sin B$

e) False $\sin(2A) = 2\sin A \cos A$

f) False $\frac{1}{\tan A} = \cot A$

Method I : by use of law of sines

#40



$$\frac{\sin 3.5°}{c} = \frac{\sin 5.5°}{13} \Rightarrow c = \frac{13 \sin 3.5°}{\sin 5.5°} = 8.28$$

$$\sin 9° = \frac{h}{c}$$

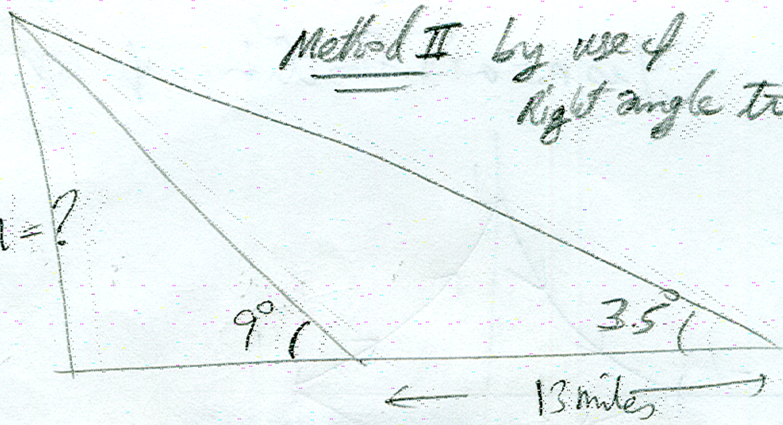
$$h = c \sin 9° = 8.28 \sin 9° = 1.2953 \text{ miles}$$

$$= 1.2953 \text{ miles} \times \frac{5280 \text{ ft}}{1 \text{ mile}}$$

$$\cong 6839 \text{ feet}$$

#40

Method II by use of Right angle trigonometry.



$$\tan 9^\circ = \frac{h}{x} \Rightarrow x = \frac{h}{\tan 9^\circ}$$

$$\tan 3.5 = \frac{h}{x+13}$$

$$\tan 3.5^\circ = \frac{h}{\frac{h}{\tan 9^\circ} + 13}$$

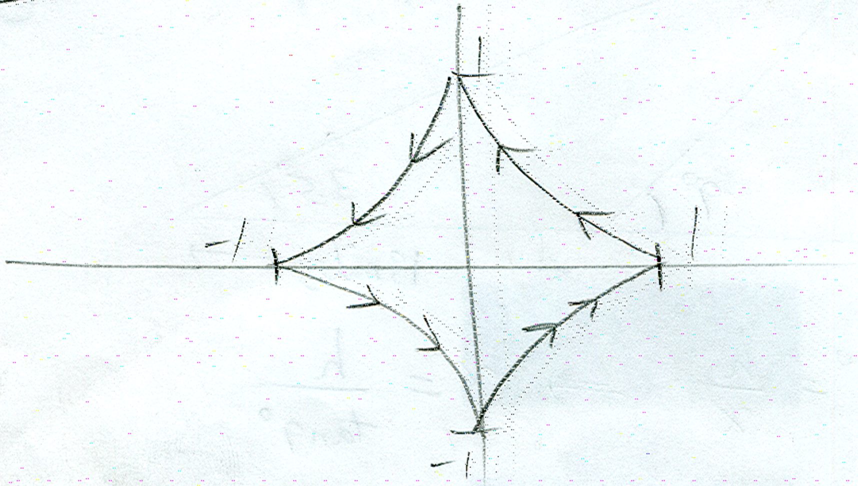
$$\tan 3.5^\circ\left(\frac{h}{\tan 9^\circ}\right) + \tan 3.5^\circ(13) = h \quad -h\left(\frac{\tan 3.5^\circ}{\tan 9^\circ}\right)$$

$$h\left(1 - \frac{\tan 3.5^\circ}{\tan 9^\circ}\right) = \tan(3.5^\circ)(13)$$

$$h = \frac{(\tan 3.5^\circ)(13)}{\left(1 - \left(\frac{\tan 3.5^\circ}{\tan 9^\circ}\right)\right)} = 1.2953 \text{ miles} \times \frac{5280 \text{ feet}}{1 \text{ miles}}$$

$$= \boxed{6839 \text{ feet}}$$

#41 y is not a function of x



#42 a) $f(x) = a\left(x+\frac{3}{2}\right)(x-4)$

$3 = a(2.5+1.5)(2.5-4)$

$a = -\frac{1}{2}$

(ai) $\boxed{f(x) = -\frac{1}{2}\left(x+\frac{3}{2}\right)(x-4)}$

(a iii) V.A $x=2$
H.A $y=1$

$g(x) = \frac{x-6}{x-2}$ OR $g(x) = \frac{-4}{x-2}+1$

(a ii) $h(x) = A(x+3)^2(x-2)^2$ (Touch at 2 even touch at -3 even)

y int: $x=0$ $y=35$

$35 = A(0+3)^2(0-2)^2$

$35 = A(9)(4)$ $\Longrightarrow$ $\boxed{h(x) = \frac{35}{36}(x+3)^2(x-2)^2}$

$\frac{35}{36} = A$

Continued

#42 b) The rate of change is $\frac{1}{2}$

c) Function h is increasing faster between $x = -2$ and $x = 0$

#43 a) $f(x) = \frac{5}{x+3}$ Rational function

b) Polynomial (quadratic)

c) $f(x) = 5x^3$ (Polynomial)

d) None $f(x) = \frac{\sqrt{x}}{x+2}$

e) Exponential

f) Sinusoidal

#44 $f(x) - g(x) = 0 \Rightarrow f(x) = g(x)$

We Have 2 Solutions (Base on the graph)

#45 a) $h(x) \geq K(x)$ $x \leq -11$ and $x \geq 3$

$(-\infty, -11] \cup [3, \infty)$

b) $h(x) < K(x)$ $-11 < x < 3$

$(-11, 3)$