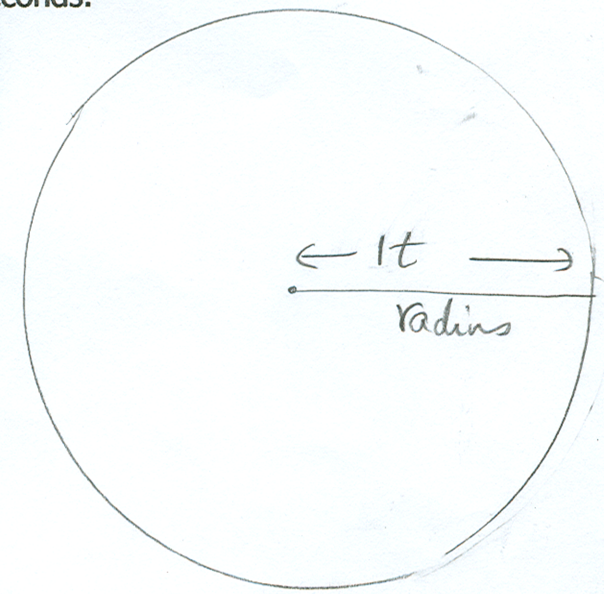
MA 180 CHAPTER 4: Exponential and LOGARITHMIC FUNCTIONS
SECTION 4.1: COMPOSITE FUNCTIONS

Example 1: Suppose we throw a rock in a pond creating a wave which is a ring (circle). It is determined that the radius of the ring is increasing by 1 foot every second. Find a formula for the area of the ring after t seconds.

$r = 1\frac{ft}{\cancel{sec}} * t\ \cancel{seconds} = 1t$

$$\boxed{(A \circ r)(t) = \pi r^2 = \pi (1t)^2 = \pi t^2 \quad \text{unit } (ft)^2}$$



Example 2 let $Y_1 = x^3 - 2x$ $\quad Y_2 = \sqrt{x-1}$

Suppose $f(x) = x^3 - 2x$ and $g(x) = \sqrt{x-1}$

a) $(f \circ g)(1) = Y_1(Y_2(1)) = 0$

b) $(g \circ f)(-1) = Y_2(Y_1(-1)) = 0$

c) $(g \circ g)(26) = 2$

d) Find $(f \circ g)(x)$ and find the domain of it.

$(f \circ g)(x) = (\sqrt{x-1})^3 - 2(\sqrt{x-1})$ $\qquad$ Domain: $x - 1 \geq 0$

$\boxed{x \geq 1}$

e)Find $(g \circ f)(x)$ and find the domain of it

$g \circ f(x) = \sqrt{x^3 - 2x - 1}$; $\quad x \geq 1.618034$

Example 3 If $f(x)=\frac{1}{x+1}$ and $g(x)=\frac{2}{x-3}$ Find the domain of $(f \circ g)(x)$

$$(f \circ g)(x) = \frac{1}{\frac{2}{x-3}+1} = \frac{1}{\frac{2+x-3}{x-3}} = \frac{x-3}{x-1}$$

Domain is all Reals except $x=1$ and $x=3$

Example 4 If $f(x)=2x-5$ and $g(x)=\frac{1}{2}(x+5)$

Show that $(f \circ g)(x)=(g \circ f)(x)=x$

$$(f \circ g)(x) = 2\left(\tfrac{1}{2}(x+5)\right)-5 = x+5-5 = x$$

$$(g \circ f)(x) = \tfrac{1}{2}(2x\cancel{-5}\cancel{+5}) = x$$

Hence, f(x) and g(x) are Inverses of each other.

Example 5 Given $H(x)=\sqrt{x^2-1}$

Find functions f and g such that $(f \circ g)(x)=H(x)$

let $g(x)=x^2$

$f(x)=\sqrt{x-1}$

then $(f \circ g)(x)=\sqrt{x^2-1}=H(x)$

MA 180 CHAPTER 4: Exponential and LOGARITHMIC FUNCTIONS
SECTION 4.2: INVERSE FUNCTIONS

The following table gives values of a function f(x) for six inputs 0, 1, 2, 3, 4, and 5.

x	f(x)
0	12
1	4
2	3
3	10
4	0
5	8

Read the table to find:

1. $f(4) =$ 0
2. $f(0) =$ 12
3. $f(2) =$ 3
4. $f(5) =$ 8

The inverse of f, written f^{-1}, and read "f inverse" sends outputs of f to inputs of f.

For example: f sends 5 to 8 and f^{-1} sends 8 to 5.
The statement $f(5) = 8$ and $f^{-1}(8) = 5$ are **equivalent**. (See bottom p.203)

Find:

5. $f^{-1}(10) =$ 3
6. $f^{-1}(0) =$ 4

7a. $f^{-1}(3) =$ 2

7b. $f^{-1}(12) =$ 0

Note: f^{-1} "undoes" f.

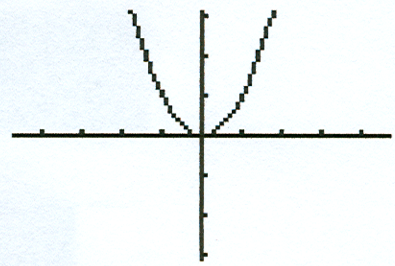
8. The inverse of a function is not necessarily a function. For example, $g(x) = x^2$ is a function. We know that $g(2) = 4$ and $g(-2) = 4$. However, how do we answer $g^{-1}(4)$?

$g^{-1}(4)$ could be 2 and -2?

Note: In order for a function to have an Inverse, it must be one-to-one (i.e) must pass the horizontal line test.

9. **Invertible functions:** When the inverse of a function, f, is also a function, we say that f is invertible. f and f^{-1} are inverse functions of each other. In general, linear functions of the form $y = mx + b$ with $m \neq 0$, are invertible. Futhermore, only functions that are **one-to-one** are invertible. A function is one-to-one if each output is used only once. A one-to-one function will pass both the vertical line test and the horizontal line test.

Recall $g(x) = x^2$, which is sketched to the right. g passes the vertical line test, but fails the horizontal line test. g is a function, but not a one-to-one function. g is not invertible.

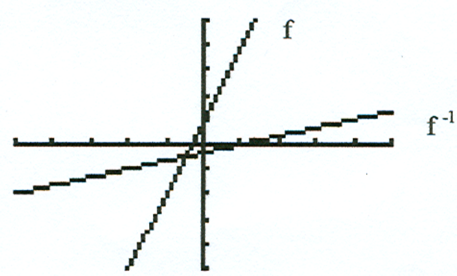


10. **Graphing an inverse function.**

For $f(x) = 3x + 2$, complete the table.

x	f(x)	x	$f^{-1}(x)$
-2	−4	-4	−2
-1	−1	-1	−1
0	2	2	0
1	5	5	1
2	8	8	2

axes scaled by 2



11a. **Reflection Property:** For an invertible function f, the graph of f^{-1} is the reflection of the graph of f across the line $y = x$.

11b. **Finding the inverse equation of a function.**

$f(x) = x - 3$ (f(x) subtracts 3 from x, **then $f^{-1}(x)$ should add 3 to x to "undo" f.)**

$$\mathbf{f^{-1}(x) = x + 3}$$

$g(x) = \frac{x}{4}$ (g(x) divides x by 4, **then $g^{-1}(x)$ should multiply x by 4 to "undo" g.)**

$$\mathbf{g^{-1}(x) = 4x}$$

Note: To check $g^{-1}(x)$, graph g and $g^{-1}(x)$ to see if they are reflections of each other about the line $y = x$. **DRAW the functions.**

FINDING THE INVERSE FUNCTION OF A LINEAR FUNCTION ALGEBRACIALLY:

(Four-Step Process, Please See page 606 of our textbook)

We Want to Find the Inverse Function of **f(x) = x - 3**	*Now You Try to Find the Inverse Function of* **g(x) = 2x + 5**
STEP 1: Replace f(x) with y $y = x - 3$	$y = 2x + 5$
STEP 2: Now, Solve for x $x - 3 = y$ $x = y + 3$	$2x = y - 5$ $x = \frac{y-5}{2}$
STEP 3: Replace x with $f^{-1}(y)$ $f^{-1}(y) = y + 3$	$g^{-1}(y) = \frac{y-5}{2}$
STEP 4: Write in terms of x $f^{-1}(x) = x + 3$	$g^{-1}(x) = \frac{x-5}{2}$

12. **Making interpretations using the inverse function.**

Let n = f(t) = .25 t – 1.67 represent the number of people (in millions) undergoing laser eye surgery in the year that is t years since 1990.

A. Find & interpret f(10) = 0.83 million people

In the year 2000; 830 000 people had laser eye surgery.

B. Find an equation for f^{-1}.

$y = 0.25t - 1.67$

$t = \frac{y + 1.67}{0.25} = 4y + 6.68 \rightarrow \boxed{f^{-1}(t) = 4t + 6.68}$

C. Find & interpret f^{-1} (3).

$f^{-1}(3) = 4(3) + 6.68 = 18.68 \approx 19$

1990 + 19 =

In the year 2009; 3 million people will Have laser eye surgery

D. What is the slope of f^{-1}? What does it mean in the context of this problem?

the slope of $f^{-1}(t) = 4$ which means every 4 years, the number of people having lasereye surgery increases by 1 million.

The Graph of a one-to-one function is given. Draw the graph of the inverse function. For convenience (and as a hint), the graph of y = x is also given.

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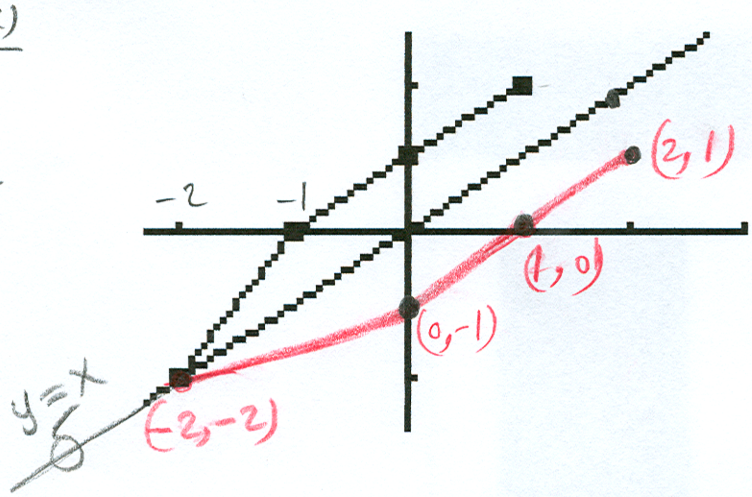
X	f(x)
−1	0
−2	−2
0	1
1	2



X	$f^{-1}(x)$
0	−1
−2	−2
1	0
2	1

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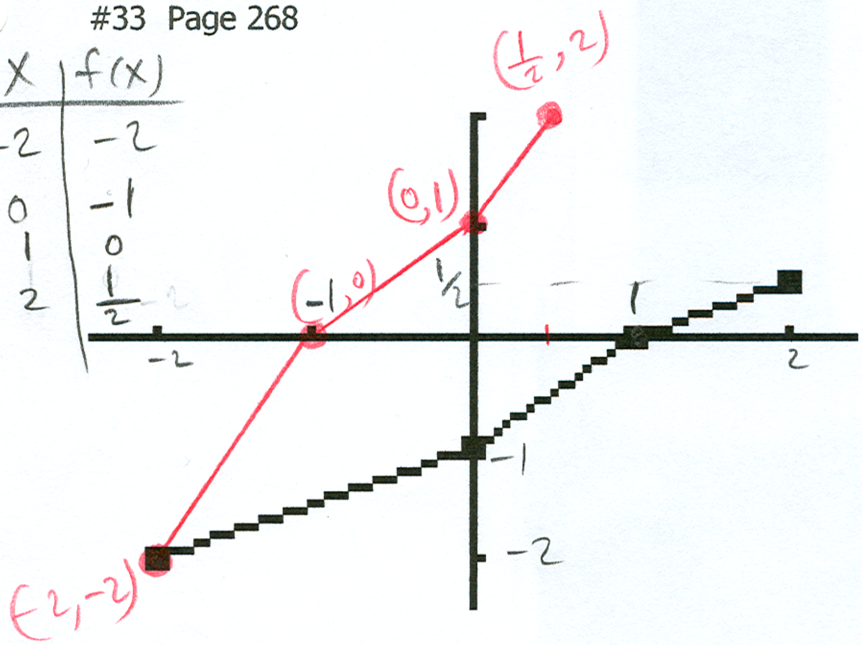
X	f(x)
−2	−2
0	−1
1	0
2	$\frac{1}{2}$



X	$f^{-1}(x)$
−2	−2
−1	0
0	1
$\frac{1}{2}$	2

MA 180 CHAPTER 4: Exponential and LOGARITHMIC FUNCTIONS
SECTION 4.3: Exponential FUNCTIONS

SKETCH THE GRAPH OF $f(x) = 3^x$

x	f(x)
-3	$\frac{1}{27}$
-2	$\frac{1}{9}$
-1	$\frac{1}{3}$
0	1
1	3
2	9
3	27

Note in the table for each unit increase in x, the y value is tripled. Compare this to how a linear function is recognized in its table form.

For linear functions we had a slope addition property. For exponential functions we have a **Base Multiplier Property:** For an exponential function of the form $y = ab^x$, for each unit increase in x, the value of y is multiplied by b.

What size window do you need to graph the points in the table above?

X_{min} = -2.5 Y_{min} = -2.5 Graph using this window.

X_{max} = 2.5 Y_{max} = 2.5

x	$g(x) = 4\left(\frac{1}{2}\right)^x$	$h(x) = 7(2)^x$	$j(x) = -4\left(\frac{1}{2}\right)^x$	$k(x) = -(2)^x$
-3	32	0.875	-32	-0.125
-2	16	1.75	-16	-0.25
-1	8	3.5	-8	-0.5
0	4	7	-4	-1
1	2	14	-2	-2
2	1	28	-1	-4
3	0.5	56	-0.5	-8

MA 180 CHAPTER 4: Exponential and LOGARITHMIC FUNCTIONS
SECTION 4.4: LOGARITHMIC FUNCTIONS

WHAT IS A LOGARITHM?

1. Solve the following equations for x:

A. $2^x = 4$

$x=2$

B. $2^x = 16$

$x=4$

C. $3^x = 9$

$x=2$

D. $3^x = 81$

$x=4$

2. Note the above problems could have been written using logarithmic notation.

A. $x = \log_2 4$

$2^x = 4$

$x=2$

B. $x = \log_2 16$

$2^x = 16$

$x=4$

C. $x = \log_3 9$

$3^x = 9$ $x=2$

D. $x = \log_3 81$

$3^x = 81 \Rightarrow x=4$

3. <u>DEFINITION</u>: For $b > 0$, $b \neq 1$, and $a > 0$

$\log_b a = k$ where k is the number such that $b^k = a$.

Note: 1. $\log_{10}$ is often written as log.
2. When you evaluate a log – you are finding a number that will be used as an exponent.

4. Evaluate each of the following. You can always check your answer using an exponential expression.

A. $\log_6 (36) =$ 2

B. $\log_4 (64) =$ 3

C. $\log_5 (125) =$ 3

D. $\log_{10} (100{,}000) =$ 5

E. $\log_2 \left(\frac{1}{2}\right) =$ -1

F. $\log_3 \left(\frac{1}{9}\right) =$ -2

G. $\log_7(\sqrt{7}) = \underline{\frac{1}{2}}$ H. $\log_4 8 = \underline{\frac{\log 8}{\log 4} = 1.5}$

PROPERTIES OF LOGARITHMS:

5. A. $\log_3(3) = 1$

   B. $\log_7(7) = 1$

   C. $\log_{12}(12) = 1$

   D. Use your answers above to guess the rule for $\boxed{\log_b(b). = 1}$

6. A. $\log_5 1 = 0$

   B. $\log_6 1 = 0$

   C. $\log_{14} 1 = 0$

   D. Use your answers above to guess the rule for $\boxed{\log_b 1 = 0}$

7. PROPERTIES OF LOGARITHMIC FUNCTIONS: [redacted]

   For $b > 0$, and $b \neq 1$,

   $\log_b(b) = 1$
   $\log_b(1) = 0$

8. RELATIONSHIP BETWEEN LOGARITHM & EXPONENTIAL FUNCTIONS: [redacted]

   For the exponential function $f(x) = b^x$, $f^{-1}(x) = \log_b(x)$.

   For the logarithmic function $g(x) = \log_b(x)$, $g^{-1}(x) = b^x$.

   $f(x) = b^x$ and $g(x) = \log_b(x)$ are inverse functions of each other.

9. For the functions listed below, find a formula for the inverse function.

   A. $f(x) = 7^x$ B. $g(x) = \log x$

   $f^{-1}(x) = \log_7(x)$ $g^{-1}(x) = 10^x$

$h(x) = 3^x \Longleftrightarrow$ then $h^{-1}(x) = \log_3 x$

10. $h(x) = 3^x$

A. Find $h^{-1}(1)$

$\log_3 1 = 0$

B. Find $h^{-1}(3)$

$h^{-1}(3) = \log_3 3 = 1$

11. <u>THE GRAPH OF A LOGARITHMIC FUNCTION</u>:

Fill in the table and plot points to graph f and g.

x	f(x) = $\log_4 x$ $Y_1 = \frac{\log x}{\log 4}$	g(x) = $\log_{(1/4)} x$ $= \frac{\log X}{\log(1/4)}$
-2	Undefined	Undefined
-1	Undefined	Undefined
0	Undefined	Undefined
1	0	0
2	0.5	-0.5
3	0.7928	-0.7925