

MA 180 CHAPTER 4: Exponential and LOGARITHMIC FUNCTIONS
SECTION 4.2: INVERSE FUNCTIONS

The following table gives values of a function $f(x)$ for six inputs 0, 1, 2, 3, 4, and 5.

x	f(x)
0	12
1	4
2	3
3	10
4	0
5	8

Read the table to find:

1. $f(4) =$ _____ 2. $f(0) =$ _____
3. $f(2) =$ _____ 4. $f(5) =$ _____

The inverse of f , written f^{-1} , and read "f inverse" sends outputs of f to inputs of f .

For example: f sends 5 to 8 and f^{-1} sends 8 to 5.

The statement $f(5) = 8$ and $f^{-1}(8) = 5$ are **equivalent**. (See bottom p.203)

Find:

5. $f^{-1}(10) =$ _____ 6. $f^{-1}(0) =$ _____
7a. $f^{-1}(3) =$ _____ 7b. $f^{-1}(12) =$ _____

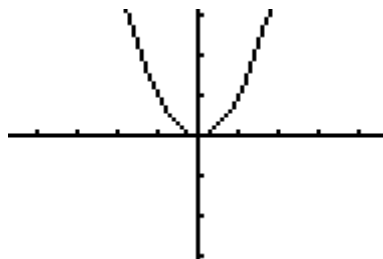
Note: f^{-1} "undoes" f .

8. The inverse of a function is not necessarily a function. For example, $g(x) = x^2$ is a function. We know that $g(2) = 4$ and $g(-2) = 4$. However, how do we answer $g^{-1}(4)$?

Note:

9. **Invertible functions:** When the inverse of a function, f , is also a function, we say that f is invertible. f and f^{-1} are inverse functions of each other. In general, linear functions of the form $y = mx + b$ with $m \neq 0$, are invertible. Furthermore, only functions that are **one-to-one** are invertible. A function is one-to-one if each output is used only once. A one-to-one function will pass both the vertical line test and the horizontal line test.

Recall $g(x) = x^2$, which is sketched to the right. g passes the vertical line test, but fails the horizontal line test. g is a function, but not a one-to-one function. g is not invertible.

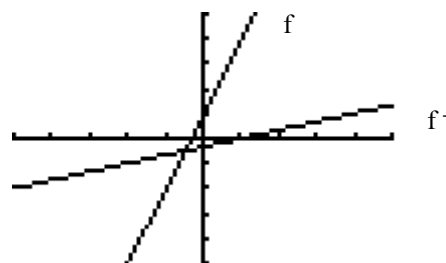


10. **Graphing an inverse function.**

For $f(x) = 3x + 2$, complete the table.

x	$f(x)$	x	$f^{-1}(x)$
-2		-4	
-1		-1	
0		2	
1		5	
2		8	

axes scaled by 2



- 11a. **Reflection Property:** For an invertible function f , the graph of f^{-1} is the reflection of the graph of f across the line $y = x$.
- 11b. **Finding the inverse equation of a function.**

$f(x) = x - 3$ ($f(x)$ subtracts 3 from x , then $f^{-1}(x)$ should add 3 to x to "undo" f .)

$$f^{-1}(x) = x + 3$$

$g(x) = \frac{x}{4}$ ($g(x)$ divides x by 4, then $g^{-1}(x)$ should multiply x by 4 to "undo" g .)

$$g^{-1}(x) = 4x$$

Note: To check $g^{-1}(x)$, graph g and $g^{-1}(x)$ to see if they are reflections of each other about the line $y = x$. ****DRAW**** the functions.

FINDING THE INVERSE FUNCTION OF A LINEAR FUNCTION ALGEBRACIALLY:

(Four-Step Process, Please See page 606 of our textbook)

<i>We Want to Find the Inverse Function of</i> $f(x) = x - 3$	<i>Now You Try to Find the Inverse Function of</i> $g(x) = 2x + 5$
STEP 1: Replace $f(x)$ with y $y = x - 3$	
STEP 2: Now, Solve for x $x - 3 = y$ $x = y + 3$	
STEP 3: Replace x with $f^{-1}(y)$ $f^{-1}(y) = y + 3$	
STEP 4: Write in terms of x $f^{-1}(x) = x + 3$	

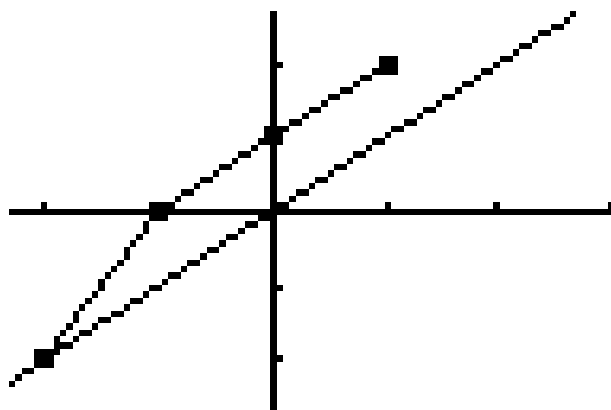
12. **Making interpretations using the inverse function.**

Let $n = f(t) = .25t - 1.67$ represent the number of people (in millions) undergoing laser eye surgery in the year that is t years since 1990.

- A. Find & interpret $f(10)$.
- B. Find an equation for f^{-1} .
- C. Find & interpret $f^{-1}(3)$.
- D. What is the slope of f^{-1} ? What does it mean in the context of this problem?

The Graph of a one-to-one function is given. Draw the graph of the inverse function. For convenience (and as a hint), the graph of $y = x$ is also given.

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