$y = A\sin(wx - \phi)$ (Amplitude $= |A|$) (Period $= \frac{2\pi}{w}$) (Phase shift $= \frac{\phi}{w}$)

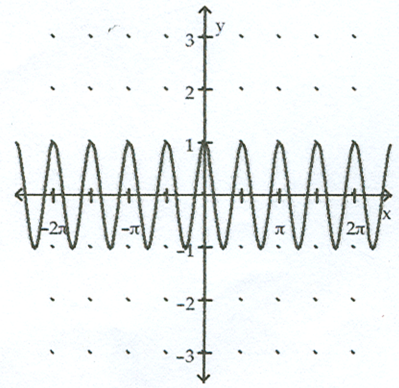
 Name________________________________

Match the function with its graph.

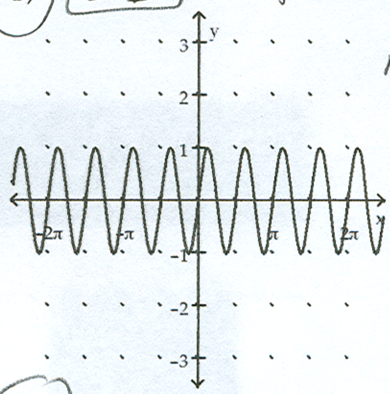
1) 1) $y = \sin 4x$ 2) $y = 4\cos x$
3) $y = 4\sin x$ 4) $y = \cos 4x$

$y = \sin 4x$, Period $= \frac{2\pi}{4} = \frac{\pi}{2}$, Amplitude $= 1$

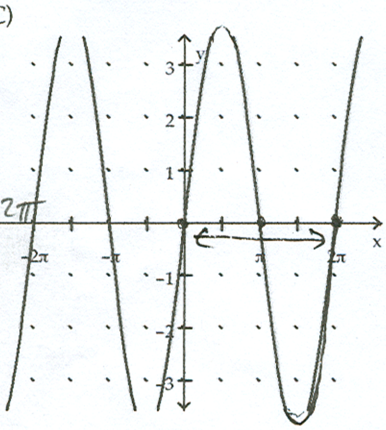
A) [4A] $y = \cos 4x$, Amp $= 1$, Period $= \frac{2\pi}{4} = \frac{\pi}{2}$



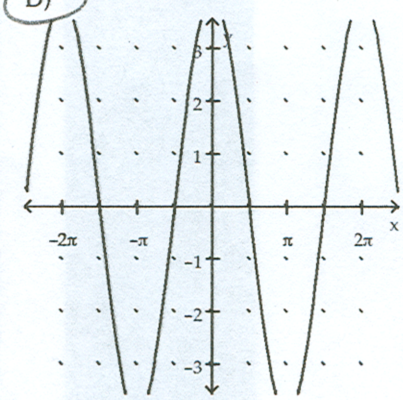
B) [1B]



C) [3C] $y = 4\sin x$, Amplitude $= 4$, Period $= \frac{2\pi}{1} = 2\pi$



D) [2D] $y = 4\cos x$, Amplitude $= 4$, Period $= \frac{2\pi}{1} = 2\pi$



Write the equation of a sine function with the given characteristics.

2) Amplitude: 4
Period: 3π

$y = A\sin(wx)$ Period $= \frac{2\pi}{w}$ $3\pi = \frac{2\pi}{w} \Rightarrow w = \frac{2\pi}{3\pi} = \frac{2}{3}$

$y = 4\sin(\frac{2}{3}x)$

Solve.

3) The current I, in amperes, ~~flowing through a particular~~ ac (alternating current) circuit at time t seconds is
$I = 240\sin(70\pi t)$
What is the period and amplitude of the current? $A = 240$ Period $= \frac{2\pi}{70\pi} = \frac{2}{70} = \frac{1}{35}$ seconds

4) For what numbers x, $0 \le x \le 2\pi$, does $\cos x = 1$? when $x = 0$ and 2π

5) For what numbers x, $0 \le x \le 2\pi$, does $\sin x = -1$? $\Rightarrow$ when $x = \frac{3\pi}{2}$

$y = A\sin(wx - \phi)$; $Amp = |A|$; $Period = \frac{2\pi}{w}$; $Phase shift = \frac{\phi}{w}$

Math 180 – Section 5.6 Name______________________________

Find the phase shift of the function.

1) $y = 3\sin(x - \frac{\pi}{2})$ $\Longrightarrow$ phase shift $= \frac{\frac{\pi}{2}}{1} = \frac{\pi}{2}$ units to the Right

2) $y = -2\sin(4x - \frac{\pi}{2})$ $\Rightarrow$ Phase shift $= \frac{\frac{\pi}{2}}{4} = \frac{\pi}{8}$ units to the Right

3) $y = 4\cos(\frac{1}{4}x + \frac{\pi}{4})$ $\Rightarrow$ phase shift $= \frac{\frac{\pi}{4}}{1/4} = \pi$ units to the Left

Find the amplitude, period, and phase shift of the sinusoidal funtion.

4) $y = -\frac{3}{4}\sin(\frac{1}{4}x + \frac{\pi}{2})$ Amp $= \frac{3}{4}$; period $= \frac{2\pi}{\frac{1}{4}} = 8\pi$; phase shift $= \frac{-\frac{\pi}{2}}{\frac{1}{4}} =$

$= \frac{\pi}{2} \cdot \frac{4}{1} = 2\pi$

Graph the function. Show at least one period.

5) $y = 2\cos(3x + \frac{\pi}{2})$

Amp $= 2$; Period $= \frac{2\pi}{3}$ Phase shift $= \frac{\phi}{w} = \frac{\frac{\pi}{2}}{3} = \frac{\pi}{6}$; see attached graph

Graph the sinusoidal function over one complete period.

6) $y = 3\sin(\frac{1}{2}x + \frac{\pi}{4})$

Amp $= 3$ Period $= \frac{2\pi}{w} = \frac{2\pi}{1/2} = 4\pi$; phase shift $= \frac{\phi}{w} = \frac{\frac{\pi}{4}}{\frac{1}{2}} = \frac{\pi}{2}$

Write the equation of a sine function with the given characteristics.

7) Amplitude: 4
Period: 6π
Phase Shift: $\frac{\pi}{6}$

$6\pi = \frac{2\pi}{w} \Rightarrow w = \frac{2\pi}{6\pi} = \frac{1}{3}$

Phase shift $= \frac{\phi}{w} \longrightarrow \frac{\pi}{6} = \frac{\phi}{\frac{1}{3}} \Rightarrow \phi = \frac{\pi}{18}$

$y = 4\sin(\frac{1}{3}x - \frac{\pi}{18})$

8) Amplitude: 4
Period: π
Phase Shift: -2

Period $= \frac{2\pi}{w}$

$\pi = \frac{2\pi}{w} \longrightarrow \pi w = 2\pi \Longrightarrow w = 2$

Phase shift $= \frac{\phi}{w}$

$-2 = \frac{\phi}{2} \Rightarrow \phi = -4$

$y = A\sin(wx - \phi)$

$y = 4\sin(2x + 4)$

Match the function with its graph.

6) 1) $y = -2 \sin (2x)$ 2) $y = -2 \sin \left(\frac{1}{2}x\right)$

3) $y = 2 \cos (2x)$ 4) $y = 2 \cos \left(\frac{1}{2}x\right)$

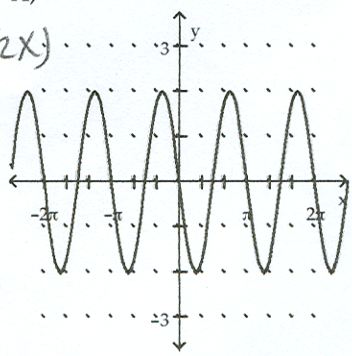
#1A

$y=-2\sin(2x)$

Amp $= 2$

Period $= \frac{2\pi}{2} = \pi$

A)



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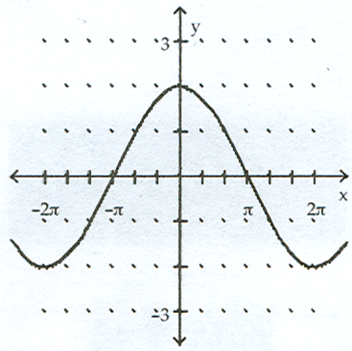


#4B

$y=2\cos(\frac{1}{2}x)$

Amp $= 2$

Period $= \frac{2\pi}{\frac{1}{2}} = 4\pi$

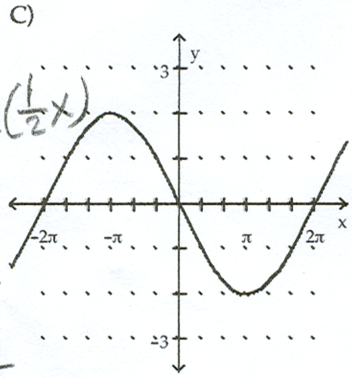
#2C

$y=-2\sin(\frac{1}{2}x)$

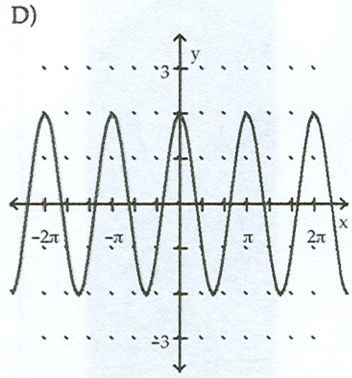
Amp $= 2$

Period $= \frac{2\pi}{\frac{1}{2}} = 4\pi$

C)



D)



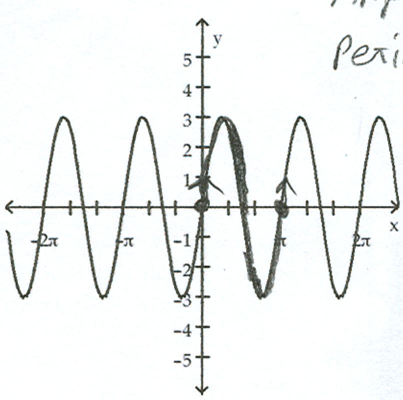
#3D $y= 2\cos(2x)$

Amp $= 2$

Period $= \frac{2\pi}{2} = \pi$

Find an equation for the graph.

7)



Amp $= 3$

Period $= \frac{2\pi}{w}$ and period $= \pi$

$\frac{2\pi}{w} = \pi \implies \pi w = 2\pi$

$w = 2$

$y = 3\sin(2x)$

Solve.

8) For what numbers x, $0 \le x \le 2\pi$, does $\sin x = 0$?

$\sin x = 0$ when $x = 0$, $x = \pi$, $x = 2\pi$

 Name______________________________

Find the value of the expression.

1) $\sin^{-1}\frac{\sqrt{2}}{2} = \frac{\pi}{4}$ $\quad -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$ 1) ____________

2) $\cos^{-1}\frac{\sqrt{2}}{2} = \frac{\pi}{4}$ $\quad 0 \leq \theta \leq \pi$ 2) ____________

3) $\tan^{-1}1 = \frac{\pi}{4}$ $\quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$ 3) ____________

4) $\sin^{-1}0 = 0$ 4) ____________

5) $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\pi}{4}$ 5) ____________

6) $\tan^{-1}-1 = -\frac{\pi}{4}$ 6) ____________

7) $\sin^{-1}-0.5 = -\frac{\pi}{6}$ 7) ____________

8) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$ 8) ____________

9) $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}$ 9) ____________

10) $\sin^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{3}$ 10) ____________

11) $\cos^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{6}$ 11) ____________

12) $\tan^{-1}0 = 0$ 12) ____________

Use a calculator to find the value of the expression in radian measure rounded to 2 decimal places.

13) $\sin^{-1}(0.4) = 0.41$ 13) ____________

14) $\cos^{-1}\left(\frac{1}{6}\right) = 1.40$ 14) ____________

15) $\tan^{-1}(1.5) = 0.98$ 15) ____________

16) $\sin^{-1}\left(\frac{\sqrt{5}}{3}\right) = 0.84$ 16) ____________

Find the exact value of the expression.

17) $\cos(\cos^{-1}(-0.9372))$ $= -0.9372$ 17) ____________

18) $\sin[\sin^{-1}(-0.6)]$ $= -0.6$ 18) ____________

19) $\tan[\tan^{-1}(0.2)]$ $= 0.2$ 19) ____________

20) $\cos(\cos^{-1}(0.45))$ $= 0.45$ 20) ____________

21) $\tan[\tan^{-1}(2.35)]$ $= 2.35$ 21) ____________

22) True or false? Why? $(\cos(\cos^{-1}(3)) = 3$ False 22) ____________

Cosine of an Angle can not be 3

23) True or false? Why? $\sin^{-1}(\sin(\frac{3\pi}{4})] = \frac{3\pi}{4}$ False 23) ____________

$\sin\frac{3\pi}{4} = \frac{\sqrt{2}}{2}$ and $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$ Recall Range of $\sin^{-1}(x)$ is $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

24) True or false? Why? $\tan^{-1}(\tan(\frac{5\pi}{6})) = \frac{5\pi}{6}$ False 24) ____________

$\tan\frac{5\pi}{6} = \frac{-\sqrt{3}}{3}$ and $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right) = \frac{-\pi}{6}$

25) True or false? $\sin[\sin^{-1}(2)] = 2$ 25) ____________

sine of an Angle can not be 2.

Solve the problem.

26) The formula 26) ____________

$$D = 24\left[1 - \frac{\cos^{-1}(\tan i \tan \theta)}{\pi}\right]$$

can be used to approximate the number of hours of daylight when the declination of the sun is $i°$ at a location $\theta°$ north latitude for any date between the vernal equinox and autumnal equinox. To use this formula, $\cos^{-1}(\tan i \tan \theta)$ must be expressed in radians. Approximate the number of hours of daylight in Fargo, North Dakota, (46°52′north latitude) for vernal equinox ($i = 0°$). $= 46.87°$

$\cos^{-1}(\tan 0° \tan 46.87°) = 1.57$ Radians

$D = 24\left[1 - \frac{1.57}{\pi}\right] = 12.00$ hours

Name________________________________

$y = \sin^{-1}(x) \quad -\frac{\pi}{2} \le y \le \frac{\pi}{2}$

$y = \tan^{-1}(x) \quad -\frac{\pi}{2} < y < \frac{\pi}{2}$

$y = \cos^{-1}(x) \quad 0 \le y \le \pi$

Find the value of the expression.

1) $\sin^{-1} 0.5 = \frac{\pi}{6}$

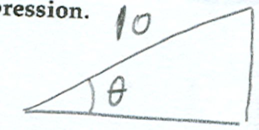
2) $\tan^{-1} -1 = -\frac{\pi}{4}$

3) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$

4) $\sin^{-1} \frac{\sqrt{2}}{2} = \frac{\pi}{4}$

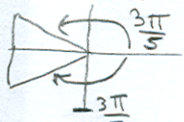
Find the exact value of the expression.

5) $\sin[\sin^{-1}(0.7)]$

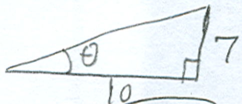


$\Rightarrow \sin(\sin^{-1}(0.7)) = \frac{7}{10} = 0.7$

6) $\cos^{-1}\left[\cos\left(-\frac{3\pi}{5}\right)\right] = \frac{3\pi}{5}$



7) $\tan[\tan^{-1}(0.7)] = 0.7$



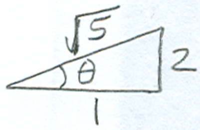
$\tan\theta = \frac{7}{10} = 0.7$

Use a calculator to find the value of the expression in radian measure rounded to 2 decimal places.

8) $\cos^{-1}\left(\frac{1}{6}\right) = 1.40$

Find the exact value of the expression.

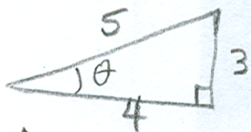
9) $\sin(\tan^{-1} 2)$



$\sin(\tan^{-1} 2) = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

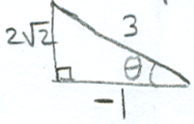
10) $\cos\left[\sin^{-1}\frac{3}{5}\right]$

$= 1 + y^2$



$\cos\left(\sin^{-1}\left(\frac{3}{5}\right)\right) = \frac{4}{5}$

11) $\tan\left[\cos^{-1}\left(-\frac{1}{3}\right)\right] \Rightarrow$



$\tan\left(\cos^{-1}\left(-\frac{1}{3}\right)\right) = \frac{2\sqrt{2}}{-1} = -2\sqrt{2}$

Establish the identity.

12) $\sin^2\theta + \tan^2\theta + \cos^2\theta = ?$

$1 + \tan^2\theta = \sec^2\theta$

Recall $\sin^2\theta + \cos^2\theta = 1$

$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$

$\tan^2\theta + 1 = \sec^2\theta$

13) $\sec\theta - \frac{1}{\sec\theta} = ?$

$\Rightarrow \frac{1}{\cos\theta} - \cos\theta = \frac{1 - \cos^2\theta}{\cos\theta} = \frac{\sin^2\theta}{\cos\theta} = \frac{\sin\theta}{\cos\theta}\sin\theta = \tan\theta\sin\theta$

Establish the identity.

14) $(1 + \cot\theta)(1 - \cot\theta) - \csc^2\theta = -2\cot^2\theta$

$1 - \cot\theta + \cot\theta - \cot^2\theta - \csc^2\theta$

$1 - \csc^2\theta - \cot^2\theta$

$-\cot^2\theta - \cot^2\theta = -2\cot^2\theta$

$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$

$1 + \cot^2\theta = \csc^2\theta$

$1 - \csc^2\theta = -\cot^2\theta$

$$\frac{\cos\theta}{1+\sin\theta}+\frac{\sin\theta}{\cos\theta}=\frac{\cos^2\theta+\sin\theta(1+\sin\theta)}{\cos\theta(1+\sin\theta)}=\frac{\cos^2\theta+\sin^2\theta+\sin\theta}{\cos\theta(1+\sin\theta)}$$

15) $\frac{\cos\theta}{1+\sin\theta}+\tan\theta=\sec\theta$ TRUE

$$=\frac{(1+\sin\theta)}{\cos\theta(1+\sin\theta)}=\frac{1}{\cos\theta}=\sec\theta$$

Find the exact value by using a sum or difference identity.

16) $\sin\frac{\pi}{12}=\sin\left(\frac{\pi}{3}-\frac{\pi}{4}\right)=\sin\frac{\pi}{3}\cos\frac{\pi}{4}-\cos\frac{\pi}{3}\sin\frac{\pi}{4}=\frac{\sqrt3}{2}\frac{\sqrt2}{2}-\frac{1}{2}\frac{\sqrt2}{2}=\frac{\sqrt2(\sqrt3-1)}{4}$

Use trigonometric identities to find the exact value.

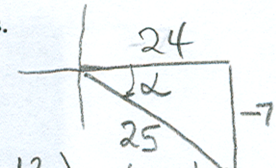
17) $\sin 25°\cos 35°+\cos 25°\sin 35°=\sin(25°+35°)=\sin 60°=\frac{\sqrt3}{2}$

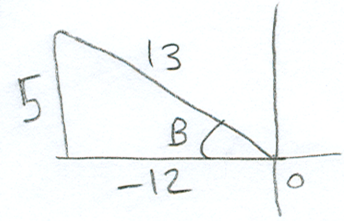
18) $\frac{\tan 170°-\tan 50°}{1+\tan 170°\tan 50°}=\tan(170°-50°)=\tan 120°=-\sqrt3$

Find the exact value of the expression under the given conditions.

19) $\sin\alpha=-\frac{7}{25},\ \frac{3\pi}{2}<\alpha<2\pi;\quad \tan\beta=-\frac{5}{12},\ \frac{\pi}{2}<\beta<\pi$

Find $\cos(\alpha+\beta)$





$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta=\left(\frac{24}{25}\right)\left(\frac{-12}{13}\right)-\left(\frac{-7}{25}\right)\left(\frac{5}{13}\right)=\frac{-288+35}{325}=\frac{-253}{325}$$

Find the exact value of the trigonometric function.

20) $\sin\left(-\frac{11\pi}{12}\right)=\sin\left(\frac{\pi}{4}-\frac{\pi}{3}\right)=\sin\frac{\pi}{4}\cos\frac{\pi}{3}-\cos\frac{\pi}{4}\sin\frac{\pi}{3}$

$$=\frac{\sqrt2}{2}\frac{1}{2}-\frac{\sqrt2}{2}\frac{\sqrt3}{2}=\frac{\sqrt2-\sqrt6}{4}$$

Find the exact value of the expression.

21) $\frac{1-\tan 80°\tan 70°}{\tan 80°+\tan 70°}=\frac{1}{\tan(80°+70°)}=\frac{1}{\tan 150°}=\frac{1}{-\frac{\sqrt3}{3}}=\frac{-3}{\sqrt3}=\frac{-3\sqrt3}{3}=-\sqrt3$

Establish the identity.

22) $\cos(\alpha+\beta)\cos(\alpha-\beta)=?=(\cos\alpha\cos\beta-\sin\alpha\sin\beta)(\cos\alpha\cos\beta+\sin\alpha\sin\beta)$

$$=\cos^2\alpha\cos^2\beta-\sin^2\alpha\sin^2\beta$$

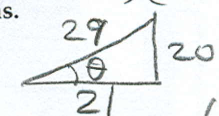
23) $\cos\left[\frac{\pi}{2}+\theta\right]=?$

$\cos\frac{\pi}{2}\cos\theta-\sin\frac{\pi}{2}\sin\theta=-\sin\theta$

$$=(\cos^2\alpha+\sin^2\alpha)(\cos^2\beta-\sin^2\beta)=\cos^2\beta-\sin^2\beta$$

Find the exact value of the expression under the given conditions.

24) $\sin\theta=\frac{20}{29},\ 0<\theta<\frac{\pi}{2}$

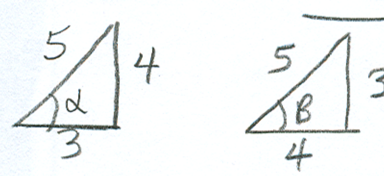


Find $\cos(2\theta)$. $=\cos^2\theta-\sin^2\theta=\left(\frac{21}{29}\right)^2-\left(\frac{20}{29}\right)^2=\frac{41}{841}$

Find the exact value of the expression.

25) $\cos\left[\tan^{-1}\frac{4}{3}-\sin^{-1}\frac{3}{5}\right]=$

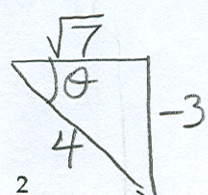
$$=\cos\alpha\cos\beta+\sin\alpha\sin\beta=\frac{3}{5}\frac{4}{5}+\frac{4}{5}\frac{3}{5}=\frac{24}{25}$$



Find the exact value of the expression under the given conditions.

26) $\csc\theta=-\frac{4}{3},\ \tan\theta>0$ Quad IV

Find $\cos(2\theta)$.



$$=1-2\sin^2\theta$$

$$=1-2\left(\frac{-3}{4}\right)^2=1-2\left(\frac{9}{16}\right)=1-\frac{18}{16}=\frac{16-18}{16}=\frac{-2}{16}=\frac{-1}{8}$$