$$\frac{\cos\theta}{1+\sin\theta} + \frac{\sin\theta}{\cos\theta} = \frac{\cos^2\theta + \sin\theta(1+\sin\theta)}{\cos\theta(1+\sin\theta)} = \frac{\cos^2\theta + \sin^2\theta + \sin\theta}{\cos\theta(1+\sin\theta)}$$

$$= \frac{(\cancel{1+\sin\theta})}{\cos\theta(\cancel{1+\sin\theta})} = \frac{1}{\cos\theta} = \sec\theta$$

15) $\boxed{\frac{\cos\theta}{1+\sin\theta} + \tan\theta = \sec\theta}$ TRUE

Find the exact value by using a sum or difference identity.

16) $\sin\frac{\pi}{12} = \sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \sin\frac{\pi}{3}\cos\frac{\pi}{4} - \cos\frac{\pi}{3}\sin\frac{\pi}{4} = \frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2} - \frac{1}{2}\frac{\sqrt{2}}{2} = \boxed{\frac{\sqrt{2}(\sqrt{3}-1)}{4}}$

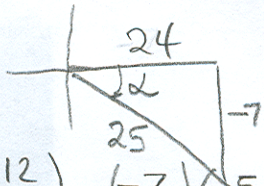
Use trigonometric identities to find the exact value.

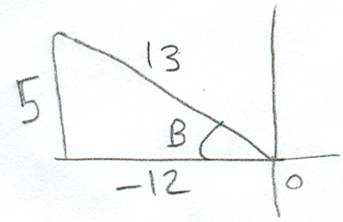
17) $\sin 25° \cos 35° + \cos 25° \sin 35° = \sin(25° + 35°) = \sin 60° = \frac{\sqrt{3}}{2}$

18) $\frac{\tan 170° - \tan 50°}{1 + \tan 170° \tan 50°} = \tan(170° - 50°) = \tan 120° = -\sqrt{3}$

Find the exact value of the expression under the given conditions.

19) $\sin\alpha = -\frac{7}{25}, \frac{3\pi}{2} < \alpha < 2\pi$; $\tan\beta = -\frac{5}{12}, \frac{\pi}{2} < \beta < \pi$





Find $\cos(\alpha + \beta)$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta = \left(\frac{24}{25}\right)\left(\frac{-12}{13}\right) - \left(\frac{-7}{25}\right)\left(\frac{5}{13}\right) = \frac{-288+35}{325} = \boxed{\frac{-253}{325}}$

Find the exact value of the trigonometric function.

20) $\sin\left(-\frac{11\pi}{12}\right) = \sin\left(\frac{-3\pi}{4} + \frac{-\pi}{6}\right) = \sin\frac{-3\pi}{4}\cos\frac{-\pi}{6} + \cos\frac{-3\pi}{4}\sin\frac{-\pi}{6}$

$\frac{-\sqrt{2}}{2}\frac{+\sqrt{3}}{2} + \frac{-\sqrt{2}}{2}\frac{-1}{2} = \boxed{\frac{\sqrt{2}-\sqrt{6}}{4}}$

Find the exact value of the expression.

21) $\frac{1 - \tan 80° \tan 70°}{\tan 80° + \tan 70°} = \frac{1}{\tan(80° + 70°)} = \frac{1}{\tan 150°} = \frac{1}{-\frac{\sqrt{3}}{3}} = \frac{-3}{\sqrt{3}} = \frac{-3\sqrt{3}}{3} = \boxed{-\sqrt{3}}$

Establish the identity.

22) $\cos(\alpha+\beta)\cos(\alpha-\beta) = ?$ $= (\cos\alpha\cos\beta - \sin\alpha\sin\beta)(\cos\alpha\cos\beta + \sin\alpha\sin\beta)$

$= \cos^2\alpha\cos^2\beta - \sin^2\alpha\sin^2\beta$

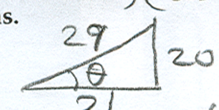
$= \underbrace{(\cos^2\alpha + \sin^2\alpha)}_{1}(\cos^2\beta - \sin^2\beta) = \boxed{\cos^2\beta - \sin^2\beta}$

23) $\cos\left[\frac{\pi}{2} + \theta\right] = ?$

$\cancel{\cos\frac{\pi}{2}\cos\theta}$ $- \sin\frac{\pi}{2}\sin\theta = -\sin\theta$

Find the exact value of the expression under the given conditions.

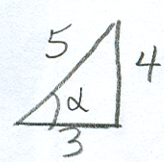
24) $\sin\theta = \frac{20}{29}, 0 < \theta < \frac{\pi}{2}$

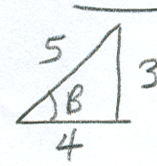


Find $\cos(2\theta)$. $= \cos^2\theta - \sin^2\theta = \left(\frac{21}{29}\right)^2 - \left(\frac{20}{29}\right)^2 = \boxed{\frac{41}{841}}$

Find the exact value of the expression.

25) $\cos\left[\tan^{-1}\frac{4}{3} - \sin^{-1}\frac{3}{5}\right] =$



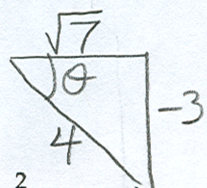


$= \cos\alpha\cos\beta + \sin\alpha\sin\beta = \frac{3}{5}\frac{4}{5} + \frac{4}{5}\frac{3}{5} = \boxed{\frac{24}{25}}$

Find the exact value of the expression under the given conditions.

26) $\csc\theta = -\frac{4}{3}, \tan\theta > 0$ Quad IV

Find $\cos(2\theta)$.



$= 1 - 2\sin^2\theta$

$= 1 - 2\left(\frac{-3}{4}\right)^2 = 1 - 2\left(\frac{9}{16}\right) = 1 - \frac{18}{16} = \frac{16-18}{16} = \frac{-2}{16} = \boxed{\frac{-1}{8}}$

$$\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{2}\cdot\frac{2}{\sqrt{3}}$$

#27

$$\frac{\sin(2\theta) + 2\sin^2\theta}{\cos(2\theta)} = \frac{2\tan\theta}{1-\tan\theta}$$

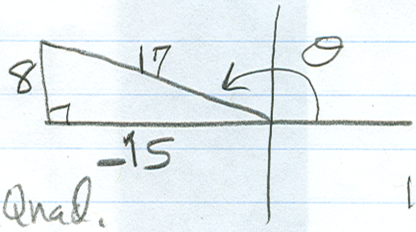
$$\frac{2\sin\theta\cos\theta + 2\sin^2\theta}{\cos^2\theta - \sin^2\theta} = \frac{2\sin\theta\cancel{(\cos\theta + \sin\theta)}}{(\cos\theta - \sin\theta)\cancel{(\cos\theta + \sin\theta)}}$$

Now divide top and bottom by $\cos\theta$

$$\frac{\frac{2\sin\theta}{\cos\theta}}{\frac{\cos\theta - \sin\theta}{\cos\theta}} = \boxed{\frac{2\tan\theta}{1-\tan\theta}}$$

#28 find $\sin\left(\frac{\theta}{2}\right)$ given that $\sec\theta = -\frac{17}{15}$ and $\frac{\pi}{2} < \theta < \pi$

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$$



$$\cos\theta = \frac{-15}{17}$$

But $\frac{\theta}{2}$ is in the first Quad.

$$17^2 = (-15)^2 + y^2$$

$$17^2 = 225 + y^2 \Rightarrow y = 8$$

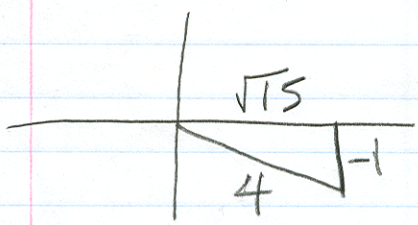
then $\sin\frac{\theta}{2} = +\sqrt{\frac{1 - \frac{-15}{17}}{2}} = +\sqrt{\frac{\frac{17+15}{17}}{2}} = \sqrt{\frac{32}{34}} = \frac{4\sqrt{2}}{\sqrt{34}} = \frac{4}{\sqrt{17}} = \boxed{\frac{4\sqrt{17}}{17}}$

#29 $\tan 75^\circ = \tan(45^\circ + 30^\circ) = \frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \tan 30^\circ}$

$$= \frac{1 + \frac{\sqrt{3}}{3}}{1 - \frac{\sqrt{3}}{3}} = \frac{\frac{3+\sqrt{3}}{3}}{\frac{3-\sqrt{3}}{3}} = \frac{3+\sqrt{3}}{3}\cdot\frac{3}{3-\sqrt{3}} = \frac{3+\sqrt{3}}{3-\sqrt{3}}\cdot\frac{3+\sqrt{3}}{3+\sqrt{3}}$$

$$= \frac{9 + 2\sqrt{3} + 3}{9 - 3} = \frac{12 + 2\sqrt{3}}{6} = \boxed{2 + \frac{\sqrt{3}}{3}}$$

#30



$$16 = 1^2 + x^2 \Rightarrow x = \sqrt{15}$$

$$\cos\left(\frac{\theta}{2}\right) = -\sqrt{\frac{1+\cos\theta}{2}} = -\sqrt{\frac{1+\frac{\sqrt{15}}{4}}{2}} = -\sqrt{\frac{4+\sqrt{15}}{4}\cdot\frac{1}{2}} = \boxed{-\frac{\sqrt{8+2\sqrt{15}}}{4}}$$

where $0 \leq \theta < 2\pi$

#31) $\sqrt{2}\cos 2\theta = 1 \implies \cos 2\theta = \frac{1}{\sqrt{2}} \implies \cos 2\theta = \frac{\sqrt{2}}{2}$

So, $2\theta = \frac{\pi}{4} + 2n\pi$ OR $2\theta = \frac{7\pi}{4} + 2n\pi$

$\theta = \frac{\pi}{8} + n\pi$; $\theta = \frac{7\pi}{8} + n\pi$

If $n=0$: $\theta = \frac{\pi}{8}, \frac{7\pi}{8}$

If $n=1$; $\theta = \frac{9\pi}{8}, \frac{15\pi}{8}$

#32) $\sin 4\theta = \frac{\sqrt{3}}{2}$ $\quad 0 \leq \theta \leq 2\pi$

$4\theta = \frac{\pi}{3} + 2n\pi$ OR $4\theta = \frac{2\pi}{3} + 2n\pi$

divide by 4

$\theta = \frac{\pi}{12} + \frac{2n\pi}{4}$ OR $\theta = \frac{2\pi}{12} + \frac{2n\pi}{4}$

$\theta = \frac{\pi}{12} + \frac{n\pi}{2}$ OR $\theta = \frac{\pi}{6} + \frac{n\pi}{2}$

If $n=0$ $\quad \theta = \frac{\pi}{12}$ and $\frac{\pi}{6}$

If $n=1$ $\quad \theta = \frac{\pi}{12} + \frac{\pi}{2} = \frac{7\pi}{12}$ and $\frac{\pi}{6} + \frac{\pi}{2} = \frac{4\pi}{6} = \frac{2\pi}{3}$

If $n=2$ $\quad \theta = \frac{\pi}{12} + \pi = \frac{13\pi}{12}$ and $\frac{\pi}{6} + \frac{\pi}{1} = \frac{7\pi}{6}$

If $n=3$ $\quad \theta = \frac{\pi}{12} + \frac{3\pi}{2} = \frac{\pi + 18\pi}{12}$ and $\frac{\pi}{6} + \frac{3\pi}{2} = \frac{\pi + 9\pi}{6} = \frac{10\pi}{6} = \frac{5\pi}{3}$

So, the solutions are:

$\frac{\pi}{6}, \frac{\pi}{12}, \frac{7\pi}{12}, \frac{2\pi}{3}, \frac{13\pi}{12}, \frac{7\pi}{6}, \frac{19\pi}{12}, \frac{5\pi}{3}$

#33) $\tan\theta = 3.7 \qquad 0 \le \theta < 2\pi$

$\theta = \tan^{-1}(3.7) = \boxed{1.31 \text{ radians}}$ and $1.31 + \pi = \boxed{4.45 \text{ radians}}$

#34) $\csc\left(\frac{\theta}{3}\right) = \frac{2\sqrt{3}}{3}$ then $\sin\left(\frac{\theta}{3}\right) = \frac{3}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{3\sqrt{3}}{2 \cdot 3} = \frac{\sqrt{3}}{2}$

then $\frac{\theta}{3} = \frac{\pi}{3} + 2K\pi \Rightarrow \boxed{\theta = \pi + 6K\pi}$

#35) $\cot\left(2\theta - \frac{\pi}{2}\right) = 1$ where $0 \le \theta < 2\pi$

$2\theta - \frac{\pi}{2} = \frac{\pi}{4} + 2K\pi$ and $2\theta - \frac{\pi}{2} = \frac{5\pi}{4} + 2K\pi$

$2\theta = \frac{\pi}{4} + \frac{\pi}{2} + 2K\pi$	$2\theta = \frac{5\pi}{4} + \frac{\pi}{2} + 2K\pi$
$2\theta = \frac{\pi + 2\pi}{4} + 2K\pi$	$2\theta = \frac{5\pi + 2\pi}{4} + 2K\pi$
$2\theta = \frac{3\pi}{4} + 2K\pi$	$2\theta = \frac{7\pi}{4} + 2K\pi$
$\theta = \frac{3\pi}{4} \div 2 + K\pi$	$\theta = \frac{7\pi}{4} \div 2 + K\pi$
$= \frac{3\pi}{4} \cdot \frac{1}{2} = \frac{3\pi}{8} + K\pi$	$\theta = \frac{7\pi}{4} \cdot \frac{1}{2} = \frac{7\pi}{8} + K\pi$

$$\boxed{\theta = \frac{3\pi}{8}, \frac{11\pi}{8}, \frac{7\pi}{8}, \frac{15\pi}{8}}$$

$0 \leq \theta < 2\pi$

#36) let $y_1 = 2 * \frac{1}{\sin X}$ and $y_2 = 5$

and use 2nd trace Intersect to find the Intersections

$\boxed{X = 0.412}$ and $\boxed{X = 2.73}$

#37) $\cos^2 X + 2\cos X + 1 = 0$ $\quad 0 \leq X < 2\pi$

We factor the above equation to

$(\cos X + 1)(\cos X + 1) = 0$

Now $\cos X + 1 = 0$

$\cos X = -1$

$\boxed{X = \pi}$

#38) $2\sin^2 X = \sin X$ $\quad 0 \leq X < 2\pi$

$2\sin^2 X - \sin X = 0$ Now, factor

$\sin X(2\sin X - 1) = 0$

$\sin X = 0$ OR $2\sin X - 1 = 0$

$\boxed{X = 0}$

$\boxed{X = \pi}$

$2\sin X = 1$

$\sin X = \frac{1}{2}$

$\boxed{X = \frac{\pi}{6} \text{ and } \frac{5\pi}{6}}$

#39) $\sec \frac{X}{2} = \cos \frac{X}{2}$ where $0 \le X < 2\pi$

$$\frac{1}{\cos \frac{X}{2}} = \cos \frac{X}{2} \Rightarrow \cos^2\left(\frac{X}{2}\right) = 1$$

then $\cos^2\left(\frac{X}{2}\right) - 1 = 0$

$$\left(\cos\left(\frac{X}{2}\right) - 1\right)\left(\cos\left(\frac{X}{2}\right) + 1\right) = 0$$

$\cos\left(\frac{X}{2}\right) = 1$ OR $\cos\left(\frac{X}{2}\right) = -1$

$\frac{X}{2} = 0$ $\quad\quad$ $\frac{X}{2} = \pi$

only solution: $\Rightarrow$ $\boxed{X = 0}$ $\quad\quad$ $\cancel{X = 2\pi}$ Not In Interval

#40) $\sin^2(2X) = 1$ where $0 \le X < 2\pi$

$$\sin^2(2X) - 1 = 0$$

$$(\sin(2X) - 1)(\sin(2X) + 1) = 0$$

$\sin(2X) - 1 = 0$ OR $\sin(2X) + 1 = 0$

$\sin(2X) = 1$ OR $\sin(2X) = -1$

$2X = \frac{\pi}{2}$ OR $\frac{5\pi}{2}$ $\quad\quad$ $2X = \frac{3\pi}{2}, \frac{7\pi}{2}$

$\boxed{X = \frac{\pi}{4}, \frac{5\pi}{4}}$ $\quad\quad$ $\boxed{X = \frac{3\pi}{4}, \frac{7\pi}{4}}$

#41 $\tan 2x - \tan x = 0$

Recall from formula sheet

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$

$$\frac{2\tan x}{1-\tan^2 x} - \tan x = 0$$

$$\frac{2\tan x - \tan x(1-\tan^2 x)}{1-\tan^2 x} = 0$$

$$\frac{2\tan x - \tan x + \tan^3 x}{1-\tan^2 x} = 0$$

$$\frac{\tan x + \tan^3 x}{1-\tan^2 x} = \frac{0}{1}$$

CROSS Multiply we get:

$$\tan x + \tan^3 x = 0$$

$$\tan x(1+\tan^2 x) = 0$$

$\tan x = 0$ OR $1+\tan^2 x = 0$

$\Downarrow$

$\tan^2 x = -1$ Impossible case (No Real Solution)

$$\boxed{\begin{array}{l} x = 0 \\ x = \pi \end{array}}$$

#42 $\cos(2x) = \sqrt{2} - \cos(2x)$ where $0 \le x < 2\pi$

$+\cos(2x) \qquad\qquad + \cos(2x)$

$$2\cos(2x) = \sqrt{2}$$

$$\cos(2x) = \frac{\sqrt{2}}{2}$$

$2x = \frac{\pi}{4} + 2k\pi$ and $2x = \frac{7\pi}{4} + 2k\pi$

$x = \frac{\pi}{8} + k\pi \qquad x = \frac{7\pi}{8} + k\pi$

If $k=0 \Rightarrow x = \frac{\pi}{8}$ and $\frac{7\pi}{8}$

If $k=1 \Rightarrow x = \frac{\pi}{8} + \pi = \frac{9\pi}{8} \qquad \frac{7\pi}{8} + \pi = \frac{15\pi}{8}$

So final solutions are:

$$x = \frac{\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{15\pi}{8}$$

#43) $\cos\theta - \sin\theta = 0$ where $0 \le \theta < 2\pi$

$\cos\theta = \sin\theta$

find all the angles where $\sin\theta$ and $\cos\theta$ are equal.

$$\boxed{\theta = \frac{\pi}{4} \quad \text{and} \quad \frac{5\pi}{4}}$$

#44) $R = \frac{V_0^2}{g} \sin 2\theta$ $\quad 0 \le \theta < \pi$

$R = 2500$ feet

$g = 32 \frac{ft}{sec^2}$

$V_0 = 400$ ft/sec

solve for θ

$2500 = \frac{400^2}{32} \sin 2\theta$

$80000 = 160000 \sin 2\theta \longrightarrow \frac{80000}{160000} = \sin 2\theta$

then $\sin 2\theta = \frac{1}{2}$

$2\theta = \frac{\pi}{6}$ OR $2\theta = \frac{5\pi}{6}$

$\theta = \frac{\pi}{12}$ OR $\theta = \frac{5\pi}{12}$

$$\boxed{\theta = \frac{\pi}{12}, \frac{5\pi}{12}, \text{OR } 15^\circ \text{ and } 75^\circ}$$