

Name: SOLUTION

1a) Find the s "length of the arc" of a circle of radius 10 meters subtended by the central angle of 30° .

$$\theta = 30^\circ * \frac{1\pi \text{ radians}}{180^\circ} = \frac{\pi}{6} \text{ radians}$$

$$S = r\theta = (10 \text{ meters}) \left(\frac{\pi}{6} \right) = \frac{5\pi}{3} \text{ meters}$$

1b) Find the area of the sector of a circle of radius 10 meters subtended by the central angle of 30° .

$$\theta = 30^\circ * \frac{1\pi}{180^\circ} = \frac{\pi}{6} \text{ radians}$$

$$A = \frac{1}{2} r^2 \theta = \frac{1}{2} (10)^2 \left(\frac{\pi}{6} \right) = \frac{25}{3} \pi \text{ meters}^2$$

2) The minute hand of a clock is 7 inches long.

a) How far does the tip of the minute hand move in 20 minutes?

$$\frac{1\pi}{30 \text{ minutes}} = \frac{x}{20 \text{ minutes}} \Rightarrow x = \frac{20\pi}{30} = \frac{2}{3} \pi \text{ radians}$$

$$S = r\theta = (7 \text{ inches}) \left(\frac{2}{3} \pi \right) = 14.66 \text{ inches}$$

b) How far does the tip of the minute hand move in 25 minutes?

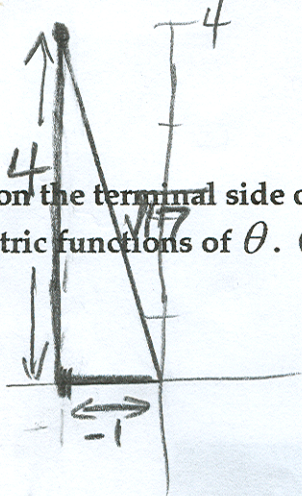
$$\frac{1\pi}{30 \text{ minutes}} = \frac{x}{25 \text{ minutes}} \Rightarrow 30x = 25\pi$$

$$x = \frac{25}{30} \pi \text{ radians}$$

$$S = r\theta = (7 \text{ inches}) \left(\frac{25}{30} \pi \right) = 18.33 \text{ inches}$$

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3) Suppose a point on the terminal side of an angle θ is $(-1, 4)$. Find the exact value of the six trigonometric functions of θ . (i.e find the following)



$$(-1)^2 + (4)^2 = c^2$$

$$1 + 16 = c^2 \Rightarrow \boxed{c = \sqrt{17}}$$

$$\sin \theta = \frac{\text{Opp}}{\text{Hyp}} = \frac{4}{\sqrt{17}} \cdot \frac{\sqrt{17}}{\sqrt{17}} = \frac{4\sqrt{17}}{17} \quad \csc \theta = \frac{\sqrt{17}}{4}$$

$$\cos \theta = \frac{\text{adj}}{\text{Hyp}} = \frac{-1}{\sqrt{17}} \cdot \frac{\sqrt{17}}{\sqrt{17}} = \frac{-\sqrt{17}}{17} \quad \sec \theta = \frac{\sqrt{17}}{-1} = -\sqrt{17}$$

$$\tan \theta = \frac{\text{Opp}}{\text{adj}} = \frac{4}{-1} = -4 \quad \cot \theta = \frac{-1}{4}$$

4a) If $\sin \theta = 0.4$, Find the value of $\sin \theta + \sin(\theta + 2\pi) + \sin(\theta + 4\pi)$

$$= 0.4 + 0.4 + 0.4 = 1.2$$

4b) Find the exact value of the following:

$$\sin 1^\circ + \sin 2^\circ + \sin 3^\circ + \sin 4^\circ + \sin 5^\circ + \dots + \sin 358^\circ + \sin 359^\circ = 0$$

Bk $\sin 1^\circ = -\sin 359^\circ$

$$\sin 2^\circ = -\sin 358^\circ$$

$$\sin 3^\circ = -\sin 357^\circ$$

and so on...

Convert the angle to decimal degrees and round to the nearest hundredth of a degree.

1) $31^{\circ}23'6'' = 31.39^{\circ}$

2) $295^{\circ}6'24'' = 295.11^{\circ}$

Convert the angle to degrees, minutes, and seconds.

3) $37.54^{\circ} = 37^{\circ}32'24''$

4) $193.32^{\circ} = 193^{\circ}19'12''$

Convert the degree measurement to radians. Express answer as multiple of π .

5) $480^{\circ} = 480^{\circ} * \frac{1\pi}{180^{\circ}} = \frac{8}{3}\pi$

6) $-144^{\circ} = -144^{\circ} * \frac{1\pi}{180^{\circ}} = -\frac{4}{5}\pi$

Convert the angle in degrees to radians. Express the answer in decimal form, rounded to two decimal places.

7) $310^{\circ} = 310^{\circ} * \frac{1\pi}{180^{\circ}} = \frac{31}{18}\pi = 5.41 \text{ Radian}$

Convert the radian measure to degrees. (Round to the nearest hundredth when necessary)

8) $\frac{8\pi}{7} * \frac{180^{\circ}}{\pi} = \frac{1440}{7} = 205.71^{\circ}$

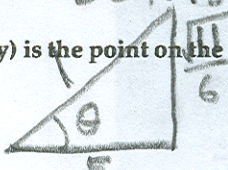
9) $\frac{43}{18}\pi * \frac{180^{\circ}}{\pi} = 430^{\circ}$

Convert the angle in radians to degrees. Round to two decimal places.

10) $4 \text{ radian} * \frac{180^{\circ}}{\pi \text{ rad}} = 229.18^{\circ}$

In the problem, t is a real number and $P=(x,y)$ is the point on the unit circle that corresponds to t . Find the exact value of the given trigonometric function.

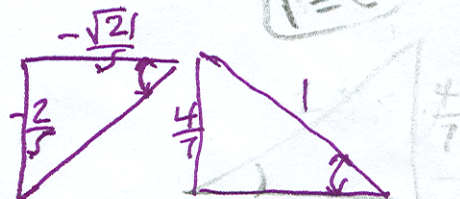
11) $(\frac{5}{6}, \frac{\sqrt{11}}{6})$; find $\sin t = \frac{\sqrt{11}}{6}$



$(\frac{5}{6})^2 + (\frac{\sqrt{11}}{6})^2 = c^2$

$1 = c$

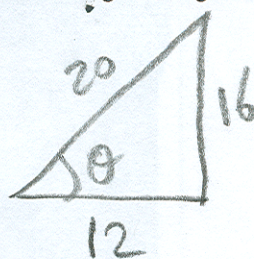
12) $(-\frac{\sqrt{21}}{5}, -\frac{2}{5})$; find $\cot t = \frac{-\sqrt{21}}{5} / \frac{-2}{5} = \frac{\sqrt{21}}{2}$



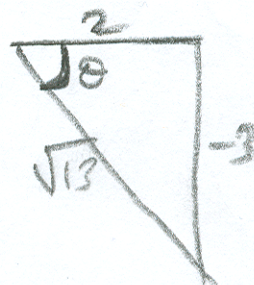
13) $(-\frac{\sqrt{33}}{7}, \frac{4}{7})$; find $\cos t = -\frac{\sqrt{33}}{7}$

A point on the terminal side of angle θ is given. Find the exact value of the given trigonometric function.

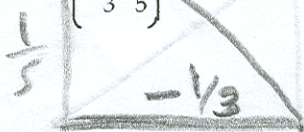
14) $(12, 16)$; Find $\sin \theta = \frac{16}{20} = \frac{4}{5}$



15) $(2, -3)$; Find $\sin \theta = \frac{-3}{\sqrt{13}} = -\frac{3\sqrt{13}}{13}$



16) $(-\frac{1}{3}, \frac{1}{5})$; Find $\cos \theta$



$\cos \theta = \frac{-1/3}{\sqrt{34}/5} = \frac{-5}{\sqrt{34}} = \frac{-5\sqrt{34}}{34}$
 $c = \sqrt{\frac{1}{25} + \frac{1}{9}} = \sqrt{\frac{34}{225}} = \frac{\sqrt{34}}{15}$

Give the exact value.

$$17) \sec 30^\circ = \frac{1}{\cos 30^\circ} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

Solve the problem.

18) Find the exact value of $\cos 60^\circ + \tan 60^\circ$. Do not use a calculator.

$$\cos 60^\circ = \frac{1}{2} + \tan 60^\circ = \sqrt{3}$$

A) $\frac{1+2\sqrt{3}}{2}$

B) $\frac{3\sqrt{3}}{2}$

C) $\frac{1+\sqrt{3}}{2}$

D) $2\sqrt{3}$

$$\frac{1}{2} + \sqrt{3} = \frac{1+2\sqrt{3}}{2}$$

19) Find the exact value of $\sin \frac{\pi}{3} - \cos \frac{\pi}{6}$. Do not use a calculator.

$$= \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} = 0$$

Find the exact value of the expression.

20) $\csc 30^\circ - \cos 30^\circ = \frac{1}{\sin 30^\circ} - \cos 30^\circ = \frac{1}{\frac{1}{2}} - \frac{\sqrt{3}}{2} = 2 - \frac{\sqrt{3}}{2} = \frac{4-\sqrt{3}}{2}$

21) $\sec 30^\circ - \sin 45^\circ = \frac{1}{\cos 30^\circ} - \sin 45^\circ = \frac{1}{\frac{\sqrt{3}}{2}} - \frac{\sqrt{2}}{2} = \frac{2}{\sqrt{3}} - \frac{\sqrt{2}}{2} = \frac{4-\sqrt{2}\sqrt{3}}{2\sqrt{3}} \left(\frac{\sqrt{3}}{\sqrt{3}} \right)$

Find the exact value of the expression. Do not use a calculator.

22) $\cos \frac{14\pi}{3} = \cos \left(\frac{12\pi}{3} + \frac{2\pi}{3} \right) = \cos \left(4\pi + \frac{2\pi}{3} \right) = \cos \frac{2\pi}{3} = -\frac{1}{2}$

23) $\sec \frac{19\pi}{4} = \sec \left(\frac{16\pi}{4} + \frac{3\pi}{4} \right) = \frac{1}{\cos \left(\frac{3\pi}{4} \right)} = \frac{1}{-\frac{\sqrt{2}}{2}} = -\frac{2}{\sqrt{2}} = -\frac{2\sqrt{2}}{2} = -\sqrt{2}$

Solve the problem.

25) Find the exact value of $\tan 150^\circ \cos 210^\circ$. Do not use a calculator.

$$= \left(-\frac{\sqrt{3}}{3} \right) \cdot \left(-\frac{\sqrt{3}}{2} \right) = \frac{+\sqrt{3}\sqrt{3}}{6} = \frac{+3}{6} = \frac{+1}{2}$$

Find the approximate value of the expression rounded to two decimal places.

26) $\sin 70^\circ = 0.94$

27) $\tan 32^\circ = 0.62$

28) $\cos \frac{5\pi}{12} = 0.26$

29) $\sec \frac{\pi}{5} = 1.24 = \frac{1}{\cos \left(\frac{\pi}{5} \right)}$

30) $\cot \frac{\pi}{7} = 2.08 = \frac{1}{\tan \left(\frac{\pi}{7} \right)}$

$$\tan 150^\circ + \cos 210^\circ = -\frac{\sqrt{3}}{3} + \left(-\frac{\sqrt{3}}{2} \right) = \frac{-2\sqrt{3} - 3\sqrt{3}}{6} = -\frac{5\sqrt{3}}{6}$$

Use the fact that the trigonometric functions are periodic to find the exact value of the expression.

31) $\cos \frac{10\pi}{3} = -\frac{1}{2} = \cos \left(\frac{6\pi}{3} + \frac{4\pi}{3} \right) = \cos \left(\frac{4\pi}{3} \right)$

32) $\sin 765^\circ = \frac{\sqrt{2}}{2}$

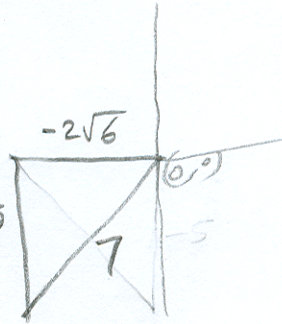
33) $\tan \frac{13\pi}{4} = 1$

Name the quadrant in which the angle θ lies.

- 34) $\tan \theta > 0$ and $\sin \theta < 0$ **Quad III**
 35) $\sin \theta > 0$ and $\cos \theta < 0$ **Quad II**
 36) $\cos \theta > 0$ and $\csc \theta < 0$ **Quad IV**
 37) $\sec \theta < 0$ and $\tan \theta < 0$ **Quad II**

Use the given values of the sine and cosine to find the function value.

38) $\sin \theta = -\frac{5}{7}$, $\cos \theta = \frac{-2\sqrt{6}}{7}$. Find $\tan \theta = \frac{-5}{-2\sqrt{6}} = \frac{5\sqrt{6}}{2\sqrt{6}\sqrt{6}} = \frac{5\sqrt{6}}{12} = 5$



39) $\sin \theta = -\frac{5}{7}$, $\cos \theta = \frac{2\sqrt{6}}{7}$. Find $\csc \theta = \frac{-7}{5}$

Find the exact value of the expression.

40) $\sec^2 65^\circ - \tan^2 65^\circ = 1$ **B/C** $\tan^2 \theta + 1 = \sec^2 \theta \Rightarrow \sec^2 \theta - \tan^2 \theta = 1$

41) $\sin^2 25^\circ + \cos^2 25^\circ = 1$ **B/C** $\sin^2 \theta + \cos^2 \theta = 1$

42) $\sin 55^\circ \csc 55^\circ = 1$ **B/C** $\csc 55^\circ = \frac{1}{\sin 55^\circ}$



Solve the problem.

43) Given $\sin \theta = \frac{1}{2}$ and $\sec \theta < 0$, find $\cos \theta$ and $\tan \theta$.

A) $\cos \theta = \sqrt{\frac{3}{2}}$, $\tan \theta = \frac{1}{\sqrt{3}}$

B) $\cos \theta = -\frac{\sqrt{3}}{2}$, $\tan \theta = -\frac{1}{\sqrt{3}}$

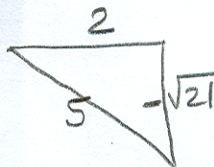
C) $\cos \theta = -\sqrt{3}$, $\tan \theta = -\frac{10}{\sqrt{3}}$

D) $\cos \theta = -\frac{\sqrt{3}}{2}$, $\tan \theta = \frac{1}{\sqrt{3}}$

Find the exact value of the requested trigonometric function of θ .

44) $\cos \theta = \frac{2}{5}$ and $\tan \theta < 0$ **Quad IV** \Rightarrow

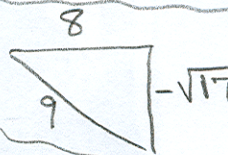
Find $\sin \theta = \frac{-\sqrt{21}}{5}$



$$\begin{aligned} 2^2 + y^2 &= 25 \\ y^2 &= 21 \\ y &= \sqrt{21} \end{aligned}$$

45) $\sec \theta = \frac{9}{8}$ and θ in quadrant IV $\Rightarrow \cos \theta = \frac{8}{9}$

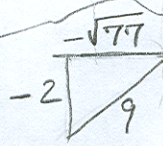
Find $\tan \theta = \frac{-\sqrt{17}}{8}$



$$\begin{aligned} 8^2 + y^2 &= 9^2 \\ y^2 &= 81 - 64 \\ y &= -\sqrt{17} \end{aligned}$$

46) $\sin \theta = -\frac{2}{9}$ and $\tan \theta > 0$ **Quad III**

Find $\sec \theta = \frac{9}{-\sqrt{77}} = \frac{-9\sqrt{77}}{77}$



$$\begin{aligned} (2)^2 + x^2 &= 81 \\ 4 + x^2 &= 81 \\ x^2 &= 77 \\ x &= \sqrt{77} \end{aligned}$$

Use the even-odd properties to find the exact value of the expression.

47) $\cos(-60^\circ) = \cos 60^\circ = \frac{1}{2}$

48) $\csc\left(-\frac{\pi}{3}\right) = \frac{1}{\sin\left(-\frac{\pi}{3}\right)} = \frac{1}{-\sin\frac{\pi}{3}} = \frac{1}{-\frac{\sqrt{3}}{2}} = \frac{-2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$

Answer Key

Testname: TRIG-REVIEW.1

49) $\tan\left(-\frac{\pi}{3}\right) = -\tan\left(\frac{\pi}{3}\right) = -\sqrt{3}$

50) $\sin(-60^\circ) = -\sin 60^\circ = -\frac{\sqrt{3}}{2}$

- 1) 31.39°
- 2) 295.11°
- 3) 37°32'24"
- 4) 193°19'12"
- 5) $\frac{8\pi}{3}$
- 6) $-\frac{4\pi}{5}$
- 7) 5.41
- 8) 205.71°
- 9) 430°
- 10) 229.18°
- 11) $\frac{\sqrt{11}}{6}$
- 12) $\frac{\sqrt{21}}{2}$
- 13) $-\frac{\sqrt{33}}{7}$
- 14) $\frac{4}{5}$
- 15) $-\frac{3\sqrt{13}}{13}$
- 16) $-\frac{5\sqrt{34}}{34}$
- 17) $\frac{2\sqrt{3}}{3}$
- 18) A
- 19) 0
- 20) $\frac{4-\sqrt{3}}{2}$
- 21) $\frac{4\sqrt{3}-3\sqrt{2}}{6}$
- 22) $-\frac{1}{2}$
- 23) $-\sqrt{2}$
- 24) $\frac{\sqrt{2}}{2}$
- 25) $-\frac{5\sqrt{3}}{6}$
- 26) 0.94
- 27) 0.62
- 28) 0.26
- 29) 1.24
- 30) 2.08

Answer Key

Testname: TRIG-REVIEW.TST

- 31) $-\frac{1}{2}$
- 32) $\frac{\sqrt{2}}{2}$
- 33) 1
- 34) Quadrant III
- 35) Quadrant II
- 36) Quadrant IV
- 37) Quadrant II
- 38) $\frac{5\sqrt{6}}{12}$
- 39) $-\frac{7}{5}$
- 40) 1
- 41) 1
- 42) 1
- 43) B
- 44) $-\frac{\sqrt{21}}{5}$
- 45) $-\frac{\sqrt{17}}{8}$
- 46) $-\frac{9\sqrt{77}}{77}$
- 47) $\frac{1}{2}$
- 48) $-\frac{2\sqrt{3}}{3}$
- 49) $-\sqrt{3}$
- 50) $-\frac{\sqrt{3}}{2}$

Radian	θ	$\sin \theta$	$\cos \theta$	$\tan \theta = \frac{\sin \theta}{\cos \theta}$
0	= 0	0	1	0
$\frac{\pi}{6}$	= 30	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
$\frac{\pi}{4}$	= 45	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$\frac{\pi}{3}$	= 60	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$\frac{\pi}{2}$	= 90	1	0	undefined
π	= 180	0	-1	0
$\frac{3\pi}{2}$	= 270	-1	0	undefined
2π	= 360	0	1	0