Math 180 - Reviewing Chapter 4 Name Solution

If the following defines a one-to-one function, find the inverse.

1) {(−13, 18), (15, 18), (−4, −12)} $\{(18,-13),(18,15),(-12,-4)\}$

Decide whether or not the functions are inverses of each other.

2) $f(x) = 4x + 7;\ g(x) = \frac{x}{4} - 7$ $f(g(x)) = 4\left(\frac{x}{4} - 7\right) + 7 = x - 28 + 7 = x - 21$

∴ Not an Inverse

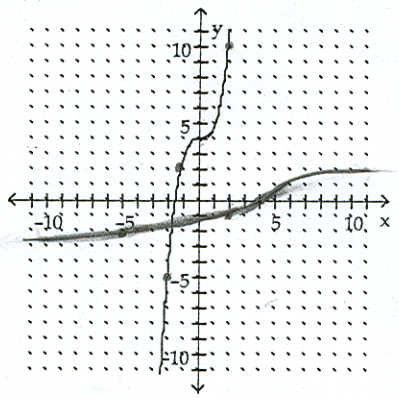
Determine the inverse of the given function.

3) {(−3, 4), (−1, 5), (0, 2), (2, 4), (5, 7)} $\{(4,-3),(5,-1)(2,0)(4,2)(7,5)\}$

The graph of a one-to-one function f is given. Draw the graph of the inverse function f^{-1} as a dashed line or curve.

4) $f(x) = x^3 + 4$

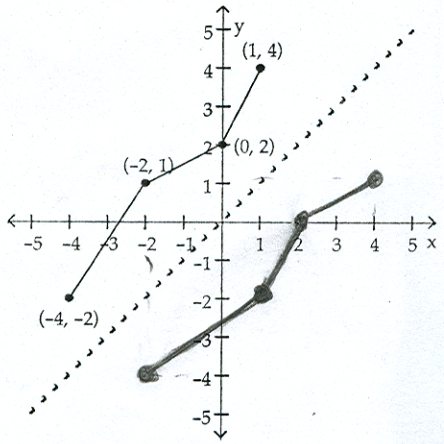
x	y
−2	−5
−1	2
0	4
2	10



x	$f^{-1}(x)$
−5	−2
2	−1
4	0
10	2

Use the graph of the given one-to-one function to sketch the graph of the inverse function. For convenience, the graph of y = x is also given.

5)



x	$f^{-1}(x)$
−2	−4
1	−2
2	0
4	1

The function f is one-to-one. Find its inverse.

6) $f(x) = 3x^2 + 2;\ x \le 0$

$y = 3x^2 + 2$

$x^2 = \frac{y-2}{3}$ → $x = +\sqrt{\frac{y-2}{3}}$

$f^{-1}(x) = \sqrt{\frac{x-2}{3}}$ $x \ge 0$

7) Determine the equation for the inverse function of $y = (x + 2)^3 - 8$.

$y = (x+2)^3 - 8$

$y + 8 = (x+2)^3$

$\sqrt[3]{(y+8)} = x + 2$ → $\sqrt[3]{y+8} - 2 = x$

$f^{-1}(x) = \sqrt[3]{x+8} - 2$

Solve the problem.

8) The profit P for selling x items is given by the equation $P(x) = 2x - 500$. Express the sales amount x as a function of the profit P. $2X = P + 500 \Rightarrow X = \frac{P+500}{2} \Rightarrow X(P) = \frac{P+500}{2}$

Approximate each value using a calculator. Express answer rounded to three decimal places.

9) $4.4^{\pi} = 105.067$

Solve the problem.

10) A rumor that there will be an open book test on chapter 4 has spread in our class of 28 students. The mathematical model for the situation is $N = 28(1 - e^{-0.16d})$ where N is the number of students who have heard the rumor and d is the number of hours that have elapsed since the rumor began.
a) How many students will have heard the rumor after 5 hours? $28(1 - e^{-0.16*5}) \cong 15$ students
b) 21 students have heard the rumor. How many hours ago did the rumor began?

$21 = 28(1 - e^{-0.16d}) \Rightarrow \frac{21}{28} = 1 - e^{-0.16d} \Rightarrow 0.25 = e^{-0.16d} \Rightarrow \ln 0.25 = -0.16d$

$\frac{\ln 0.25}{-0.16} = d$

$d \cong 9$ HRS

Graph the function.

11) Graph the function $f(x) = 2^{x-1} - 1$ (Please See Page 8)

Answer the question.

12) Define the number e.

A) The number approximately equal to 2.72.

B) The number defined by $e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$ in Calculus.

C) The number that the expression, $\left(1 + \frac{1}{n}\right)^n$, approaches as $n \to \infty$.

D) All of the above.

Solve the equation.

13) $9^{2x} \cdot 27^{(3-x)} = \frac{1}{9} \Rightarrow \left(3^2\right)^{2x} \cdot \left(3^3\right)^{(3-x)} = 3^{-2} \Rightarrow 3^{4x} \cdot 3^{9-3x} = 3^{-2} \Rightarrow 4x + 9 - 3x = -2$

$x = -11$

14) $2^{(x^2-3)} = 64 \Rightarrow 2^{x^2-3} = 2^6 \Rightarrow x^2 - 3 = 6 \Rightarrow x^2 = 9 \Rightarrow x = \pm 3$

Use a graphing calculator to solve the equation. Round your answer to two decimal places.

15) $e^x - \ln x = 4$ GRAPH: $Y_1 = e^x - \ln x$; $Y_2 = 4$ 2nd trace Intersect $x = 1.48$

Solve the given exponential equation. Round answer to three decimal places.

16) $5^{2x} + 5^{(x+1)} - 24 = 0$ $Y_1 = 5^{2x} + 5^{(x+1)} - 24$ $Y_2 = 0$ 2nd trace Intersect

$x = 0.683$

Change exponential expressions to logarithmic expression

17) $2.3^{x+5} = 15$ $\log_{2.3} 15 = x + 5 \Rightarrow \frac{\log 15}{\log 2.3} = x + 5 \Rightarrow x \cong -1.75$

Convert to logarithmic form.

18) $e^x = 15$ $\ln 15 = x$

Convert to exponential form.

19) $\log_5 \frac{1}{25} = -2$ $5^{-2} = \frac{1}{25}$

Change logarithmic expression to exponential expression.

20) $\log_b 49 = \frac{2}{3}$ $\quad b^{\frac{2}{3}} = 49 \Rightarrow \left(b^{\frac{2}{3}}\right)^{\frac{3}{2}} = (49)^{\frac{3}{2}} \Rightarrow b = 343$

Find the value of the expression.

21) $\log_9 \frac{1}{81} = \frac{\log \frac{1}{81}}{\log 9} = -2$ B/c $9^{-2} = \frac{1}{81}$

22) $\ln 1 = 0$ B/c $e^0 = 1$

Solve the problem.

23) The number of men dying of AIDS (in thousands) since 1987 is modeled by $y = 17.3 + 10.06(\ln x)$, where x represents the number of years after 1987. Use this model to predict the number of AIDS deaths among men in 1994. Express answer rounded to the nearest hundred men.

Use $x = 1994 - 1987 = 7 \Rightarrow y = 17.3 + 10.06(\ln 7) = 36.876$ thousand $= 36876 \cong 36900$ men

24) Use the function from the previous problem to find its inverse.

$y - 17.3 = 10.06 \ln x \Rightarrow \frac{y - 17.3}{10.06} = \ln x \Rightarrow e^{\frac{y-17.3}{10.06}} = x$

$f^{-1}(x) = e^{\frac{x-17.3}{10.06}}$

Find the domain of the function.

25) $f(x) = \log_{10}(x^2 - 17x + 72)$ $\quad x^2 - 17x + 72 > 0$

$(x-8)(x-9) > 0$ $\quad (-\infty, 8) \cup (9, \infty)$

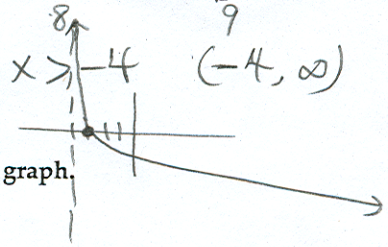
26) Determine the domain of the function $f(x) = \log_{1/2}(x + 4)$.

Sketch the graph using the transformation concept.

Then sketch the graph using the calculator.

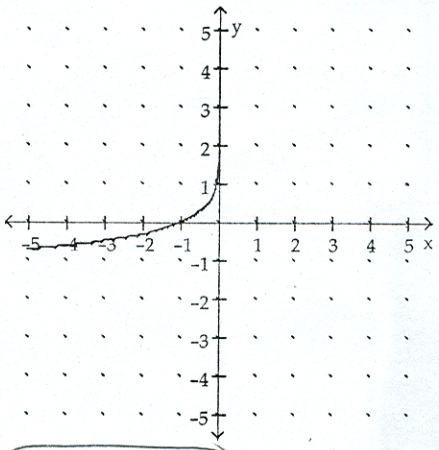
$x + 4 > 0 \Rightarrow x > -4$ $\quad (-4, \infty)$

$f(x) = \frac{\log(x+4)}{\log(1/2)}$



The graph of a logarithmic function is shown. Select the function which matches the graph.

27) Think on the transformations that took place.



A) $y = -\log(-x)$ $\quad$ B) $y = \log(x)$ $\quad$ C) $y = -\log(x)$ $\quad$ D) $y = \log(-x)$

Solve the problem.

28) $\log_x\left(\frac{8}{27}\right) = 3 \Rightarrow x^3 = \frac{8}{27}$ Now Take cube Root of both sides $x = \frac{2}{3}$

29) $\log_4 16 = x \Rightarrow 4^x = 16 \Rightarrow x = 2$

OR $\frac{\log 16}{\log 4} = 2 = x$

30) The Richter scale converts seismographic readings into numbers for measuring the magnitude of an earthquake according to this function

$M(x) = \log\left(\frac{x}{x_0}\right)$, where $x_0 = 10^{-3}$.

What would be the readings x (to the nearest tenth) for magnitudes of 4.9 and 7.9?

$M(4.9) = \log\left(\frac{4.9}{10^{-3}}\right) = 3.69 \approx 3.7$

$M(7.9) = \log\left(\frac{7.9}{10^{-3}}\right) = 3.897 \approx 3.9$

31) The formula $D = 6e^{-0.04h}$ can be used to find the number of milligrams D of a certain drug in a patient's bloodstream h hours after the drug has been given. When the number of milligrams reaches 4, the drug is to be given again. What is the time between injections?

$4 = 6e^{-0.04h} \Rightarrow \frac{4}{6} = e^{-0.04h} \Rightarrow \ln\left(\frac{4}{6}\right) = -0.04h \Rightarrow h = 10.14 \text{ HRS}$

32) $pH = -\log_{10}[H^+]$ Find the $[H^+]$ if the pH = 2.4.

$2.4 = -\log_{10}(H^+) \Rightarrow -2.4 = \log_{10}(H^+) \Rightarrow 0.00398 = H^+$

$H^+ = 3.98 \times 10^{-3}$

33) $pH = -\log_{10}[H^+]$ Find the pH if the $[H^+] = 5.7 \times 10^{-13}$.

$Ph = -\log(5.7 \times 10^{-13}) = 12.24$

Use the properties of logarithms to find the exact value of the expression. Do not use a calculator.

34) $\log 7 \quad 7^{10} = 10$ B/C $7^{10} = 7^{10}$

35) $\log_4 24 - \log_4 6 = \log_4\left(\frac{24}{6}\right) = \log_4 4 = 1$

Using the properties of logarithms, evaluate the expression.

36) $2 \ln e^{4.2} = 2(4.2) = 8.4$

Write as the sum and/or difference of logs. Do not use exponents.

37) $\log_{19} \frac{18\sqrt{r}}{s} = \log_{19} 18 + \log_{19} \sqrt{r} - \log_{19} 5 = \log_{19} 18 + \frac{1}{2}\log_{19} r - \log_{19} 5$

Express as a single logarithm.

38) $(\log_a m - \log_a n) + 3\log_a k = \log_a\left(\frac{m}{n}\right) + \log_a k^3 = \log_a\left(\frac{mk^3}{n}\right)$

Write expressions as a single logarithm.

39) $\frac{3}{4}\ln 16 - \ln(4^2 - 3^2 - 2) = \ln 16^{\frac{3}{4}} - \ln(16 - 9 - 2) = \ln 8 - \ln 5 = \ln\frac{8}{5}$

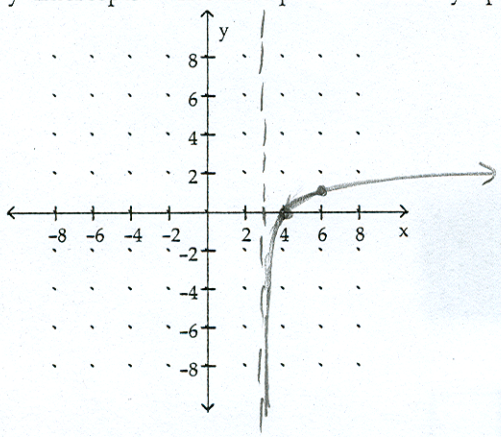
Use the Change-of-Base Formula and a calculator to evaluate the logarithm. Round your answer to two decimal places.

40) Evaluate $\log_{(2/3)} 19$.

$= \frac{\log 19}{\log\left(\frac{2}{3}\right)} = -7.26$

Graph the function using a graphing utility and the Change-of-Base Formula.

41) $y = \log_3(x - 3)$ Think on what transformation took place. Does it agree with the graph? Find x- and y-intercepts. Write the equation of the asymptote.



start with the graph of $y = \log_3 x = \frac{\log x}{\log 3}$

and shift it Right 3 units

$y = \frac{\log(x-3)}{\log 3}$

x	$y = \frac{\log(x-3)}{\log 3}$
4	0
6	1

Domain $x - 3 > 0$

$x > 3$

Solve the equation.

42) $\log 4x = \log 2 + \log(x - 1)$ $\Rightarrow \log 4x = \log(2(x-1)) \Rightarrow 4x = 2x - 2$ $x = -1$

43) $\log(3 + x) - \log(x - 4) = \log 2$ $\Rightarrow \log\left(\frac{3+x}{x-4}\right) = \log 2 \Rightarrow \frac{3+x}{x-4} = 2 \Rightarrow x = 11$

Solve the given logarithmic equation.

44) $\log_2(3x - 2) - \log_2(x - 5) = 4$ $\Rightarrow \log_2\left(\frac{3x-2}{x-5}\right) = 4 \Rightarrow 2^4 = \frac{3x-2}{x-5} \Rightarrow 16 = \frac{3x-2}{x-5}$ $x = 6$

45) $2 + \log_3(2x + 5) - \log_3 x = 4$ $\Rightarrow \log_3\left(\frac{2x+5}{x}\right) = 2 \Rightarrow 3^2 = \frac{2x+5}{x} \Rightarrow 9x = 2x + 5$

$7x = 5$ $x = \frac{5}{7}$

Solve the given exponential equation.

46) $\frac{3 \cdot 5^{2t-1}}{3} = \frac{75}{3}$ $\Rightarrow 5^{2t-1} = 25$; $5^{2t-1} = 5^2 \Rightarrow 2t - 1 = 2$ $t = \frac{3}{2}$

Solve the equation. If necessary, round your answer to two decimal places.

47) $\left(\frac{1}{4}\right)^x = 14$ $\log_{\frac{1}{4}} 14 = x \Rightarrow \frac{\log 14}{\log\left(\frac{1}{4}\right)} = x \Rightarrow x = -1.90$

Use a graphing calculator to solve the equation. Round your answer to two decimal places.

48) $\log_5 x + \log_2 x = 3$

GRAPH: $Y_1 = \frac{\log x}{\log 5} + \frac{\log x}{\log 2}$ $Y_2 = 3$ and 2nd trace Intersect $x = 4.28$

Solve the problem.

49) The size P of a small herbivore population at time t (in years) obeys the function $P(t) = 500e^{0.17t}$ if they have enough food and the predator population stays constant. After how many years will the population reach 1500?

See attached pages

50) A culture of bacteria obeys the law of uninhibited growth. If 140,000 bacteria are present initially and there are 609,000 after 6 hours, how long will it take for the population to reach one million?

51) The half-life of silicon-32 is 710 years. If 40 grams is present now, how much will be present in 200 years? (Round your answer to three decimal places.)

52) A fossilized leaf contains 14% of its normal amount of carbon 14. How old is the fossil (to the nearest year)? Use 5600 years as the half-life of carbon 14.

Please see attached pages

For Solutions see attached pages please ⇒

53) Strontium 90 decays at a constant rate of 2.44% per year. Therefore, the equation for the amount P of strontium 90 after t years is $P = P_0 e^{-0.0244t}$. How long will it take for 15 grams of strontium to decay to 5 grams? Round answer to 2 decimal places.

54) The amount of a certain drug in the bloodstream is modeled by the function $y = y_0 e^{-0.40t}$, where y_0 is the amount of the drug injected (in milligrams) and t is the elapsed time (in hours). Suppose that 10 milligrams are injected at 10:00 A.M. If a second injection is to be administered when there is 1 milligram of the drug present in the bloodstream, approximately when should the next dose be given? Express your answer to the nearest quarter hour.

55) A thermometer reading 79°F is placed inside a cold storage room with a constant temperature of 35°F. If the thermometer reads 74°F in 13 minutes, how long before it reaches 57°F? Assume the cooling follows Newton's Law of Cooling:

$U = T + (U_0 - T)e^{kt}$.

(Round your answer to the nearest whole minute.)

56) A cup of coffee is heated to 194°and is then allowed to cool in a room whose air temperature is 72°. After 11 minutes, the temperature of the cup of coffee is 140°. Find the time needed for the coffee to cool to a temperature of 102°.

57) The logistic growth model $P(t) = \frac{1240}{1 + 40.33e^{-0.325t}}$ represents the population of a bacterium in a culture tube after t hours. What was the initial amount of bacteria in the population?

58) The logistic growth model $P(t) = \frac{180}{1 + 44e^{-0.188t}}$ represents the population of a species introduced into a new territory after t years. When will the population be 80?

59) A life insurance company uses the following rate table for annual premiums for women for term life insurance. Use a graphing utility to fit an exponential function to the data. Predict the annual premium for a woman aged 70 years.

Age	35	40	45	50	55	60	65
Premium	$103	$133	$190	$255	$360	$503	$818

60) After introducing an inhibitor into a culture of luminescent bacteria, a scientist monitors the luminosity produced by the culture. Use a graphing utility to fit a logarithmic function to the data. Predict the luminosity after 20 hours.

Time, hrs	2	3	4	5	8	10	15
Luminosity	77.4	60.8	54.5	45.8	30.0	24.3	10.5

61) A mechanic is testing the cooling system of a boat engine. He measures the engine's temperature over time. Use a graphing utility to fit a logistic function to the data. What is the carrying capacity of the cooling system?

time, min	5	10	15	20	25
temperature, °F	100	180	270	300	305

#49

$P(t) = 500e^{0.17t}$

After How many years will the population reach 1500?

$\updownarrow$

$1500 = 500e^{0.17t}$

Divide both sides by 500

$3 = e^{0.17t}$

Take "ln" of Both sides and Remember $\ln e = 1$

$\ln 3 = \ln e^{0.17t}$

$\ln 3 = 0.17t \ln e^{1}$

$\frac{\ln 3}{0.17} = t \Rightarrow \boxed{t = 6.46}$ years

#50

$A = A_0 e^{kt}$

If 140000 bacteria are present initially and there are 609000 Bacteria after 6 HRS

$\updownarrow \quad \updownarrow \quad (k \times 6)$

$609000 = 140000\, e \Rightarrow \frac{609000}{140000} = e^{6k}$

$4.35 = e^{6k}$

$\ln(4.35) = \ln e^{6k}$

$\ln(4.35) = 6k \ln e^{1}$

$\frac{\ln(4.35)}{6} = k \Rightarrow \boxed{k = 0.24503}$

How long will it take for population to reach 1000000?

$A = 140000\, e^{0.24503t}$

$\updownarrow$

$1000000 = 140000\, e^{0.24503t}$

$\frac{1000000}{140000} = e^{0.24503t}$

$\ln\left(\frac{1000000}{140000}\right) = \ln e^{0.24503t}$

$1.966 = 0.24503t \ln e^{1} \Longrightarrow t = \frac{1.966}{0.24503} = \boxed{8.024 \text{ HRS}}$